A.P. SET CODE
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MT - z

2017 ___ __ 1100 - MT - z - MATHEMATICS (71) ALGEBRA - SET - D (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. Solve ANY FIVE of the following :

(i) $t_1 = 3$, $t_2 = 6$, $t_3 = 12$, $t_4 = 24$

$t_2 - t_1 = 6 - 3 = 3$

$t_3 - t_2 = 12 - 6 = 6$

$t_4 - t_3 = 24 - 12 = 12$

$\because$ The difference between two consecutive terms is not constant.

$\therefore$ **The sequence is not an A.P.** **1**

(ii) $x^2 - 2x + 1 = 0$, $x = 1$

Putting $x = 1$ in L.H.S. we get,

L.H.S. $= (1)^2 - 2(1) + 1$ ½

$= 1 - 2(1) + 1$

$= 2 - 2$

$= 0$

$= $ R.H.S.

$\therefore$ L.H.S. = R.H.S.

Thus equation is satisfied.

So 1 is the root of the given quadratic equation. ½

(iii) $3x^2 + 2x - 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 3$, $b = 2$, $c = -1$

$a = b^2 - 4ac$

$= (2)^2 - 4(3)(-1)$

$= 4 + 12$

$= 16$

$\therefore$ $a = 16$ **1**

(iv) Substituting $x = 2$ and $y = 5$ in the L.H.S. of the equation $3x - y = 1$ ½

L.H.S. $= 3x - y$

$= 3(2) - 5$

$= 6 - 5$

$= 1$

$= $ R.H.S.

	y x = 2 and y = 5 satisfies the equation 3x – y = 1 Hence (2, 5) lies on the graph of the equation 3x – y = 1.	½
(v)	S = {HH, HT, TH, TT} A = {HT} y Aà = {HH, TH, TT}	1
(vi)	Mean = 101, Median = 100 [Given] We know, Mean – Mode = 3 (Mean – Median) y 101 – Mode = 3 (101 – 100) y 101 – Mode = 3 (1) y 101 – 3 = Mode y Mode = 98	½ ½
A.2.	**Solve ANY FOUR of the following :**	
(i)	**Given :** For an A.P. $t_4 = 12$, d = – 10 **Find :** General term { t_n } **Sol.** $t_n = a + (n – 1) d$ y $t_4 = a + (4 – 1) d$ y $12 = a + 3 (– 10)$ y $a = 12 + 30$ y $a = 42$ $t_n = a + (n – 1) d$ y $t_n = 42 + (n – 1) (– 10)$ y $t_n = 42 – 10n + 10$ y $t_n = 52 – 10n$ y The general term of A.P. is 52 – 10n.	 1 1
(ii)	$y^2 – (k – 4)y – 4k = 0$. By putting y = k in L.H.S. we get L.H.S. = $(k)^2 – (k – 4)(k) – 4k$ = $k^2 – (k^2 – 4k) – 4k$ = $k^2 – k^2 + 4k – 4k$ = 0 = R.H.S. y L.H.S. = R.H.S. Thus equation is satisfied. So k is the root of the given quadratic equation.	 ½ 1 ½

(iii) Let total number of persons be x.

No. of persons with blood group B = 600 persons

Percentage of persons with blood group B = 30 %

y $\frac{30}{100} \times x = 600$ **1**

y $x = \frac{600 \times 100}{30}$

y $x = 2000$

y Total numbers of persons are 2000. **1**

(iv) $3x - y = 7$

$x + 4y = 11$

$D = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - (-1 \times 1) = 12 + 1 = 13$

$D_x = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = (7 \times 4) - (-1 \times 11) = 28 + 11 = 39$

$D_y = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = (3 \times 11) - (7 \times 1) = 33 - 7 = 26$ **1**

By Cramer's rule,

$x = \frac{D_x}{D} = \frac{39}{13} = 3$

$y = \frac{D_y}{D} = \frac{26}{13} = 2$

y x = 3 and y = 2 is the solution of given simultaneous equations. **1**

(v) A die is thrown

y S = { 1, 2, 3, 4, 5, 6 }

y n (S) = 6

A is the event that a prime number comes up

y A = { 2, 3, 5 }

y n (A) = 3 **1**

B is the event that a number divisible by 3 comes up

y B = { 3, 6 }

y n (B) = 2 **1**

(vi) $S_n = \frac{n^2 (n+1)^2}{4}$

y $S_1 = \frac{1^2 (1+1)^2}{4} = \frac{1 (2)^2}{4} = \frac{1 \times 4}{4} = 1$

y $S_2 = \frac{2^2 (2+1)^2}{4} = \frac{4 (3)^2}{4} = 9$

y $S_3 = \frac{3^2 (3+1)^2}{4} = \frac{9 \times 4^2}{4} = 9 \times 4 = 36$ **1**

We know that,

$t_1 = S_1 = 1$

$t_2 = S_2 - S_1 = 9 - 1 = 8$

$t_3 = S_3 - S_2 = 36 - 9 = 27$

y The first three terms of the sequence are 1, 8 and 27. **1**

A.3. Solve ANY THREE of the following :

(i) Since the no. of seats in each row of the auditorium are 20, 22, 24, ...

The no. of seats in each row form an A.P.

No. of seats in first row (a) = 20 **½**

Difference in no. of seats in two successive rows is (d) = 2 **½**

No. of seats in 25th row = t_{25} = ?

$t_n = a + (n + 1) d$

y $t_{25} = a + (25 - 1) d$ **1**

y $t_{25} = 20 + 24 (2)$

y $t_{25} = 20 + 48$ **½**

y $t_{25} = 68$

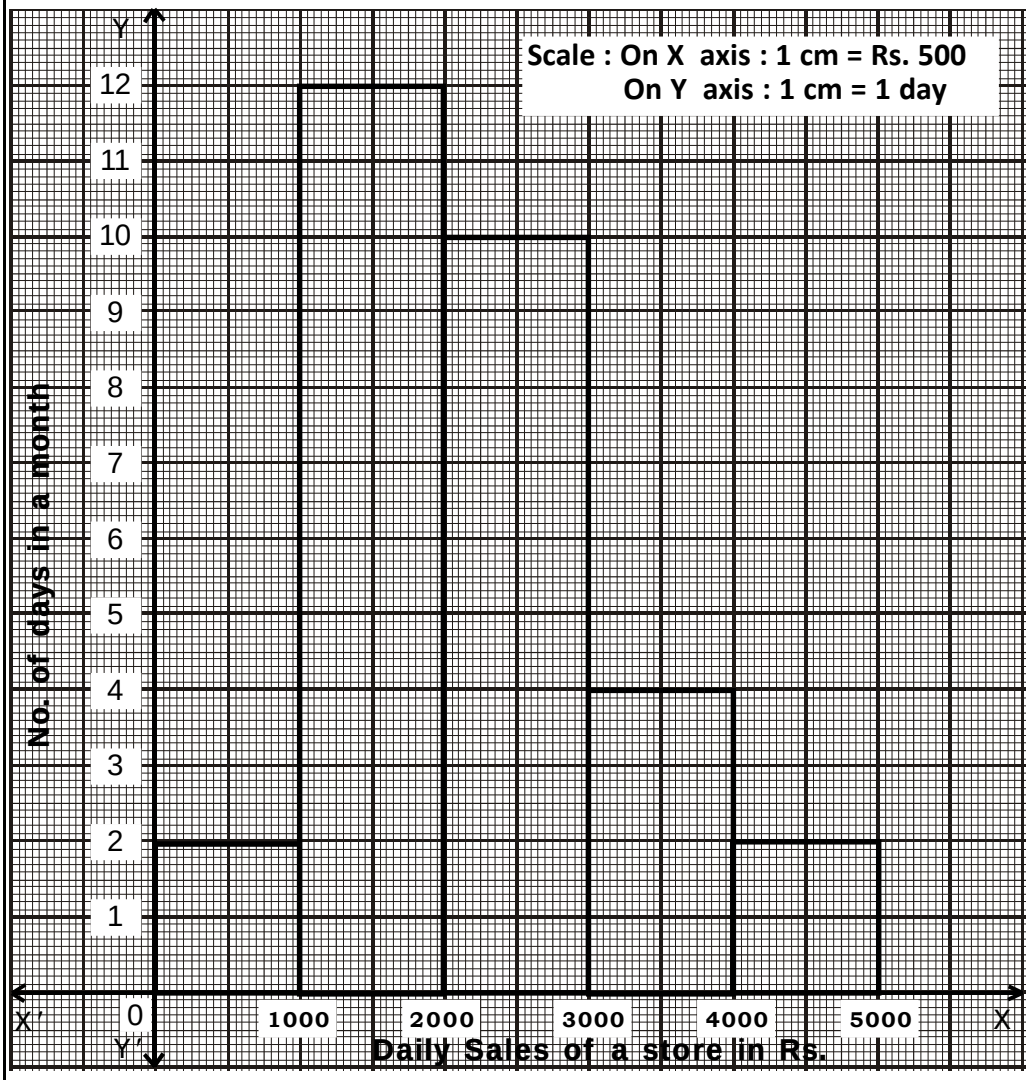
y There are 68 seats in 25th row. **½**

(ii) $x^2 - 2 (c + 1) x + c^2 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have A = 1, B = – 2 (c + 1), $C = c^2$ **½**

	a = $B^2 - 4AC$	½
	= $[-2(c+1)]^2 - 4(1)(c^2)$	
	= $(-2)^2 (c+1)^2 - 4c^2$	
	= $4(c^2 + 2c + 1) - 4c^2$	
	= $4c^2 + 8c + 4 - 4c^2$	½
	= $8c + 4$	
	∵ The roots of the given equation are real and equal.	
	∴ a must be zero.	½
	∴ $8c + 4 = 0$	
	∴ $8c = -4$	
	∴ $c = \frac{-4}{8}$	
	∴ $\boxed{c = \frac{-1}{2}}$	1
(iii)	When a coin tossed three times	
	S = { HHH, HTH, THH, TTH, HHT, HTT, THT, TTT }	
	n (S) = 8	1
	(a) Let A be the event of getting head on middle coin	
	A = { HHH, THH, HHT, THT }	
	n (A) = 4	
	$P(A) = \frac{n(A)}{n(S)}$	
	∴ $P(A) = \frac{4}{8}$	
	∴ $\boxed{P(A) = \frac{1}{2}}$	1
	(ii) Let B be the event of getting exactly one tail	
	B = { HTH, THH, HHT }	
	n (B) = 3	
	$P(B) = \frac{n(B)}{n(S)}$	
	∴ $\boxed{P(B) = \frac{3}{8}}$	1

(iv)



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(v)

Mode of investment	Percentage of residents	Measure of central angle
Shares	10	$\frac{10}{100} \times 360^\circ = 36^\circ$
Mutual funds	20	$\frac{20}{100} \times 360^\circ = 72^\circ$
Real estate	35	$\frac{35}{100} \times 360^\circ = 126^\circ$
Gold	30	$\frac{30}{100} \times 360^\circ = 108^\circ$
Government bonds	5	$\frac{5}{100} \times 360^\circ = 18^\circ$
Total	**100**	**360°**

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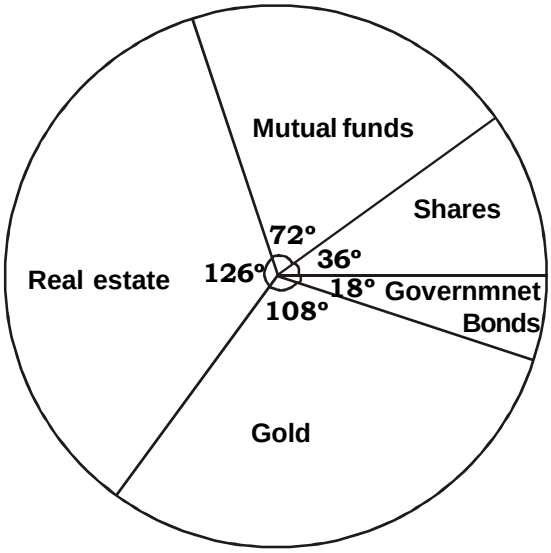

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A.4. Solve ANY TWO of the following :

(i) Let three consecutive terms in an A.P. be

$a - d, a, a + d$ **½**

As per the first given condition,

$a - d + a + a + d = -3$

y $3a = -3$

y $a = -1$ **1**

As per the second given condition,

$(a - d)^3 a^3 (a + d)^3 = 512$

y $[(a - d) a (a + d)]^3 = 512$ **½**

Taking cube roots on both sides

$(a - d) a (a + d) = \sqrt[3]{512}$

y $(a - d) a (a + d) = 8$ **½**

y $a (a - d) (a + d) = 8$

y $a (a^2 - d^2) = 8$

y $-1 [(-1)^2 - d^2] = 8$

y $-1 (1 - d^2) = 8$

y $d^2 - 1 = 8$

y $d^2 = 8 + 1$

y $d^2 = 9$

y $d = \pm 3$ **½**

If $d = 3$

y $a - d = -1 - 3 = -4$ $\quad a - d = -1-(-3) = 2$

y $a + d = -1 + 3 = 2$ $\quad a + d = -1-3 = 4$ **½**

y The three consecutive terms of A.P. are $-4, -1, 2$ or $2, -1, -4$ **½**

(ii) $kx + y = k - 2$

Comparing with $a_1x + b_1y = c_1$ we get, $a_1 = k$, $b_1 = 1$, $c_1 = k - 2$ ½

$9x + ky = k$

Comparing with $a_2x + b_2y = c_2$ we get, $a_2 = 9$, $b_2 = k$, $c_2 = k$ ½

y $\frac{a_1}{a_2} = \frac{k}{9}$

y $\frac{b_1}{b_2} = \frac{1}{k}$

y $\frac{c_1}{c_2} = \frac{k-2}{k}$ 1

The simultaneous equations have infinitely many solutions.

y $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

y $\frac{k}{9} = \frac{1}{k} = \frac{k-2}{k}$

y $\frac{k}{9} = \frac{1}{k}$

y $k^2 = 9$

y $k = \pm 3$ [Taking square roots on both sides] 1

$k = 3$ satisfies both conditions hence $k = 3$ is the value for which the given simultaneous equations have infinitely many solutions. 1

(iii) Two dice are thrown

y S = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

y n (S) = 36 1

(a) Let A be the event that sum of numbers on their upper faces is divisible by 9

A = { (3, 6), (4, 5), (5, 4), (6, 3) }

n (A) = 4

P (A) = $\frac{n(A)}{n(S)}$

y P (A) = $\frac{4}{36}$

y P (A) = $\frac{1}{9}$ 1

(b) Let B be the event that sum of number on their upper faces is at the most 3.

B = { (1, 1), (1, 2), (2, 1) }

n (B) = 3

$P(B) = \frac{n(B)}{n(S)}$

y $P(B) = \frac{3}{36}$

y $\boxed{P(B) = \frac{1}{12}}$ **1**

(c) Let C be the event that number on the upper face of the first die is less than the number on the upper face of second die.

C = { (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6) }

n (C) = 15

$P(C) = \frac{n(C)}{n(S)}$

y $P(C) = \frac{15}{36}$

y $\boxed{P(C) = \frac{5}{12}}$ **1**

A.5. Solve ANY TWO of the following :

(i) Let the number of days required by John alone to complete the work be x days

y No. of days required by ganesh alone is (x + 10) days.

Also number of days required by both to complete the same work is 12 days **½**

y Work done by John in 1 day = $\frac{1}{x}$

y Work done by Ganesh in 1 day = $\frac{1}{x+10}$ **½**

y Work done by both in 1 day = $\frac{1}{12}$ **½**

As per the given condition,

$$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$$ ½

y $$\frac{x + 10 + x}{x(x+10)} = \frac{1}{12}$$

y $$\frac{2x + 10}{x^2 + 10x} = \frac{1}{12}$$

y $12(2x + 10) = 1(x^2 + 10x)$

y $24x + 120 = x^2 + 10x$

y $0 = x^2 + 10x - 24x - 120$ **1**

y $x^2 - 14x - 120 = 0$

y $x^2 - 20x + 6x - 120 = 0$

y $x(x - 20) + 6(x - 20) = 0$

y $(x - 20)(x + 6) = 0$

y $x - 20 = 0$ or $x + 6 = 0$ **1**

y $x = 20$ or $x = -6$

∵ The numbers of days cannot be negative

y $x \div -6$

Hence $x = 20$

y $x + 10 = 20 + 10 = 30$

y Ganesh alone worked for 30 days. **1**

(ii) Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

y Speed of the boat upstream = $(x - y)$ km/hr
and speed of the boat downstream = $(x + y)$ km/hr

We know that, Time = $\frac{\text{Distance}}{\text{Speed}}$

As per the first condition,

$$\frac{8}{x-y} + \frac{32}{x+y} = 6 \quad \text{.....(i)}$$ ½

As per the second condition,

$$\frac{20}{x-y} + \frac{16}{x+y} = 7 \quad \text{.....(ii)}$$ ½

Substituting $\frac{1}{x-y} = m$ and $\frac{1}{x+y} = n$ in (i) and (ii) we get,

$8m + 32n = 6$ .....(iii)

$20m + 16n = 7$ .....(iv) ½

Multiplying (iv) by 2 we get,

$40m + 32n = 14$ ……(v)

Subtracting (v) from (iii),

$8m + 32n = 6$

$40m + 32n = 14$

(–) (–) (–)

$-32m = -8$

y $m = \frac{-8}{-32}$

y $m = \frac{1}{4}$ ½

Substituting $m = \frac{1}{4}$ in (iii),

$8\left(\frac{1}{4}\right) + 32n = 6$

y $2 + 32n = 6$

y $32n = 6 - 2$

y $32n = 4$

y $n = \frac{4}{32}$

y $n = \frac{1}{8}$ ½

Resubstituting the values of m and n we get,

$m = \frac{1}{x - y}$

y $\frac{1}{4} = \frac{1}{x - y}$

y $x - y = 4$ ……(vi) ½

$n = \frac{1}{x + y}$

y $\frac{1}{8} = \frac{1}{x + y}$

y $x + y = 8$ ……(vii) ½

Adding (vi) and (vii),

$x - y = 4$

$x + y = 8$

$2x = 12$

y $x = \frac{12}{2}$

y $x = 6$ ½

Substituting x = 6 in (vii)

$6 + y = 8$

y $y = 8 - 6$

y $y = 2$

y The speed of boat in still water is 6 km/hr and speed of stream is 2 km/ hr. **1**

(iii)

Bonus paid (in Rs.)	**Frequency (f_i) (No. of students)**	**Cumulative frequency less than type**
Below 500	4	4
500 - 600	8	12
600 - 700	12	24 ì *c.f.*
700 - 800	17 ì *f*	41
800 - 900	10	51
900 - 1000	7	58
1000 - 1100	2	60
Total	**60 ì N**	**1**

Here total frequency = pf_i = N = 60

y $\frac{N}{2} = \frac{60}{2} = 30$

Cumulative frequency (less than type) which is just greater than 30 is 41. Therefore corresponding class 700 - 800 is median class. ½

L = 700, N = 60, *c.f.* = 24, f = 17, h = 100 **1**

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$

$$= 700 + \left(\frac{60}{2} - 24\right)\frac{100}{17}$$ **1**

$$= 700 + (6)\ \frac{100}{17}$$

$$= 700 + \frac{600}{17}$$

$$= 700 + 35.29$$ **1**

$$= 735.29$$

y Median of bonus paid is Rs.735.29. ½

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