A.P. SET CODE

B

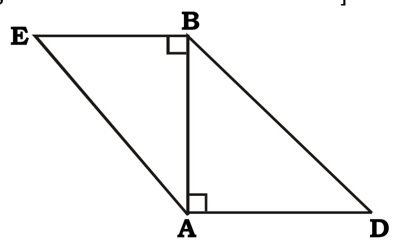
MT - x

2017 ___ __ 1100 - MT - x - MATHEMATICS (71) GEOMETRY- SET - B (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. Solve ANY FIVE of the following :

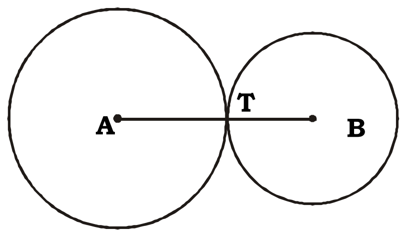
(i) $\frac{\text{A (aABE)}}{\text{A (aBAD)}} = \frac{\text{BE}}{\text{AD}}$ [Triangles with common base] ½



y $\frac{\text{A (aABE)}}{\text{A (aBAD)}} = \frac{6}{9}$

y $\boxed{\frac{\text{A (aABE)}}{\text{A (aBAD)}} = \frac{2}{3}}$ ½

(ii)



Diameter = 8 cm

y Radius AT = 4 cm

Diameter = 6 cm

y Radius BT = 3 cm

A - T - B [If two circles are touching circles, then the common point lies on the line joining their centres] ½

y AB = AT + BT [A - T - B]

y AB = 4 + 3

y $\boxed{\text{AB = 7 cm}}$ ½

(iii) ∵ cos (–¯) = cos ¯

y cos (– 30) = cos 30 ½

y $\boxed{\text{cos (– 30)} = \frac{\sqrt{3}}{2}}$ ½

(iv) $3y = 2x + 7$

y $y = \frac{2}{3}x + \frac{7}{3}$ [Dividing throughout by 3] ½

Comparing the above equation with slope intercept form

$y = mx + c$, we get $c = \frac{7}{3}$

y **y-intercept of line is $\frac{7}{3}$** ½

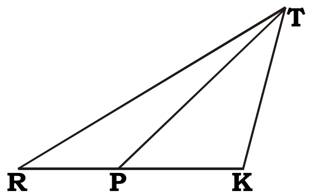
(v) $F + V = E + 2$ ½

y $6 + V = 10 + 2$

y $V = 12 - 6$

y **$V = 6$** ½

(vi)



aTRP and aTPK have a common vertex T and their bases RP and PK lie on the same line RK ½

y Their heights are equal

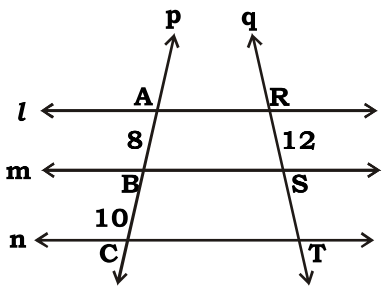
$\frac{A\,(aTRP)}{A\,(aTPK)} = \frac{RP}{PK}$ [Triangles having equal heights]

y $\frac{A\,(aTRP)}{A\,(aTPK)} = \frac{3}{2}$

y **A (aTRP) : A (aTPK) = 3 : 2** ½

A.2. Solve ANY FOUR of the following :

(i)



line l || line m || line n [Given]

y On transversals p and q,

$\frac{AB}{BC} = \frac{RS}{ST}$ [By Property of Intercepts made by three parallel lines] 1

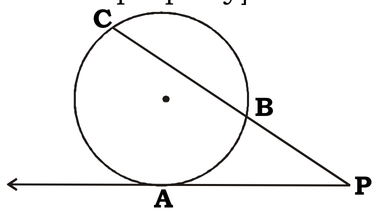
y $\frac{8}{10} = \frac{12}{ST}$ [Given]

y $ST = \frac{12 \times 10}{8}$ ½

y **ST = 15 units** ½

(ii) Line PBC is a secant intersecting the circle at points B and C and line PA is a tangent to the circle at point A.

y $CP \times BP = AP^2$ [Tangent secant property] ½



y $CP \times 9 = (12)^2$

y $CP \times 9 = 144$

y $CP = \frac{144}{9}$

y $CP = 16$ units ½

$CP = BC + BP$ [∵ C - B - P] ½

y $16 = BC + 9$

y $BC = 16 - 9$

y $\boxed{BC = 7 \text{ units}}$ ½

(iii) Perimeter of one face of a cube = 24 cm

Perimeter of one face of a cube = $4l$

y $4l = 24$

y $l = \frac{24}{4}$

y $l = 6$ cm ½

Total surface area of a cube $= 6l^2$ ½

$= 6\ (6)^2$

$= 6 \times 6 \times 6$

$= 216\ cm^2$

Volume of the cube $= l^3$ ½

$= 6^3$

$= 216\ cm^3$.

y **Total area of the 6 faces is 216 cm^2 and volume of the cube is 216 cm^3.** ½

(iv) The terminal arm passes through P (5, – 12)

x = 5 and y = – 12

$r = \sqrt{x^2 + y^2}$ ½

$= \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

y $r = 13$ units ½

Let the angle be ¯

y sin ¯ = $\frac{y}{r}$ = $\frac{-12}{13}$ **1**

(v) Length of the cuboidal water tank (l) = 2 m

its breadth (b) = 1.6 m

and its height (h) = 1.8 m ½

Volume of cuboidal water tank = $l \times b \times h$ ½

= $2 \times 1.6 \times 1.8$

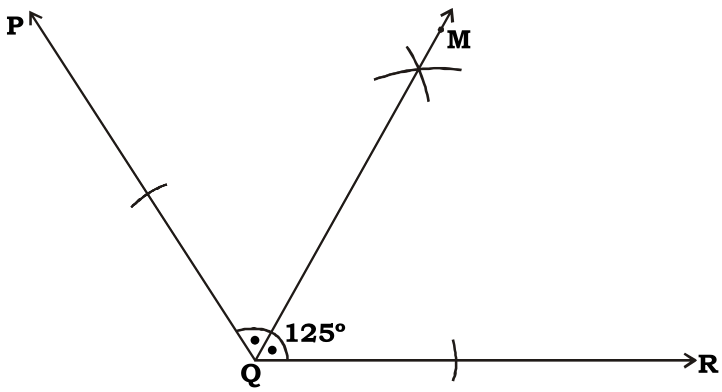
= 5.76 m^3

= 5.76×1000 litres ½

[$l \text{ m}^3$ = 1000 litres]

y **Volume of cuboidal water tank is 5760 litres.** ½

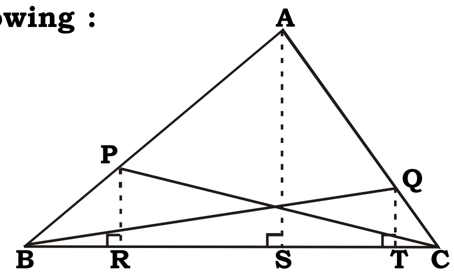
(vi)



1 mark for drawing ĎPQR
1 mark for drawing bisector of ĎPQR

A.3. Solve ANY THREE of the following :

(i)



(a) $\frac{A\ (aABC)}{A\ (aPBC)} = \frac{AS}{PR}$ [Triangles with common base] **1**

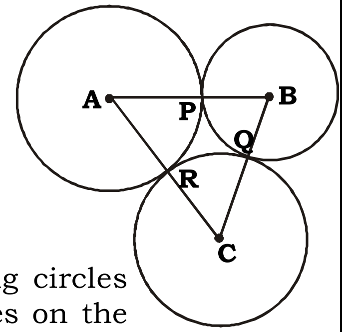
(b) $\frac{\text{A (aABS)}}{\text{A (aASC)}} = \frac{\text{BS}}{\text{SC}}$ [Triangles having equal heights] **1**

(c) $\frac{\text{A (aPRC)}}{\text{A (aBQT)}} = \frac{\text{RC} \times \text{PR}}{\text{BT} \times \text{QT}}$ [The ratio of the areas of two triangles equal to the ratio of the products of their bases and corresponding heights]

Similarly,

(d) $\frac{\text{A (aBPR)}}{\text{A (aCQT)}} = \frac{\text{BR} \times \text{PR}}{\text{CT} \times \text{QT}}$ **1**

(ii) **Given :** Circles with centres A, B and C touch each other pairwise externally at points P, Q and R respectively.
AB = 3 cm, BC = 3cm, CA = 4cm.



To Find : Radii of the circles with centre A, B and C.

Sol. A - P - B
B - Q - C
A - R - C
[If two circles are touching circles then the common point lies on the line joining their centres] **½**

Let, AP = AR = x
BP = BQ = y
CQ = CR = z
[Radii of the same circle]

AP + BP = AB [∵ A - P - B]
y x + y = 3 ....(i)
BQ + CQ = BC [∵ B - Q - C] **½**
y y + z = 3 ....(ii)
AR + CR = AC [∵ A - R - C]
y x + z = 4 ......(iii)
Adding (i), (ii) and (iii),
y x + y + y + z + x + z = 3 + 3 + 4
y 2x + 2y + 2z = 10
y 2(x + y + z) = 10
y x + y + z = 5 ......(iv) **½**
Substituting (i) in (iv),
3 + z = 5
y z = 5 – 3
y z = 2 **½**
Substituting (ii) in (iv),
x + 3 = 5
y x = 5 – 3
y x = 2 **½**

Substituting (iii) in (iv),

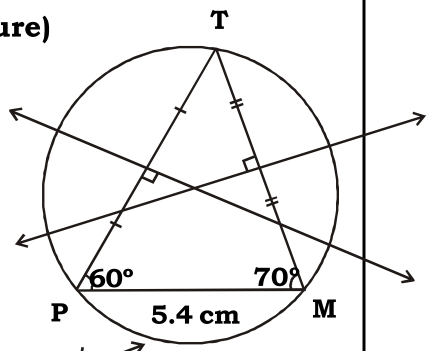
$y + 4 = 5$

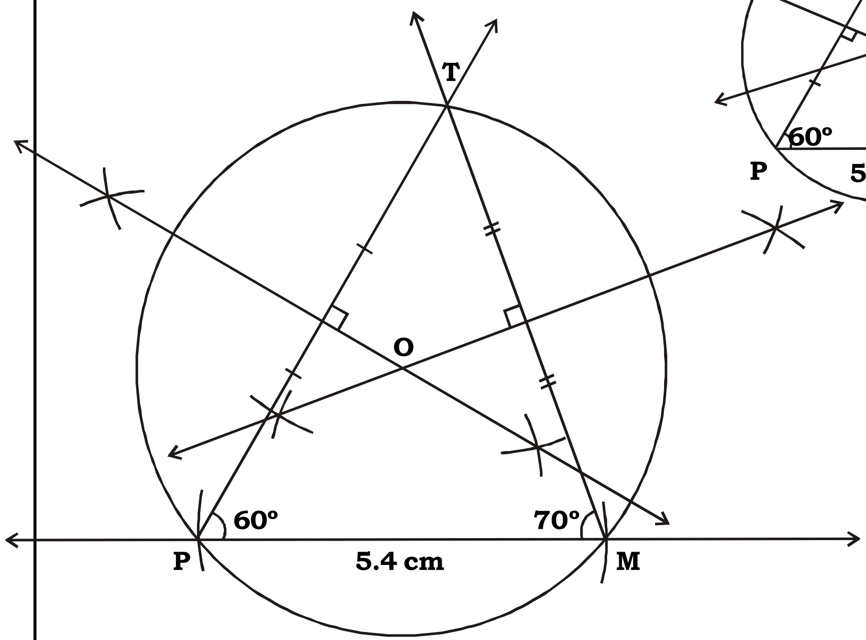
y $y = 5 - 4$

y $y = 1$

y **Radii of circles with centres A, B and C are 2 cm, 1 cm and 2 cm respectively.** ½

(iii) **(Rough Figure)**

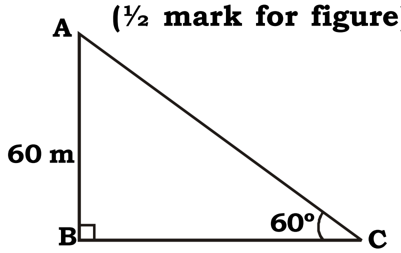




½ mark for rough figure
1 mark for aPMT
1 mark for the two perpendicular bisectors
½ mark for the circumcirle

(iv) seg AB represents the distance of a kite from ground.
AB = 60 m ½
seg AC represents the length of the string
m ĎACB = 60°
In right angled aABC,

(½ mark for figure)



$\sin 60° = \frac{AB}{AC}$ [By definition] ½

y $\frac{\sqrt{3}}{2} = \frac{60}{AC}$

	y AC = $\frac{120}{\sqrt{3}}$	
	y AC = $\frac{120\sqrt{3}}{3}$	½
	y AC = $40\sqrt{3}$ m	
	y AC = 40 × 1.73	½
	y AC = 69.2 m	
	y The length of the string, assuming that there is no slack in the string is 69.2 m.	½
(v)	Let, A ≡ (– 1, 1), B ≡ (– 9, 6), C ≡ (– 2, 14), D ≡ (6, 9)	
	Slope of a line = $\frac{y_2 - y_1}{x_2 - x_1}$	½
	Slope of side AB = $\frac{6 - 1}{-9 - (-1)}$	
	= $\frac{5}{-9 + 1}$	
	= $\frac{5}{-8}$	
	y Slope of line AB = $\frac{-5}{8}$	½
	Slope of line CD = $\frac{9 - 14}{6 - (-2)}$	½
	= $\frac{-5}{6 + 2}$	
	y Slope of line CD = $\frac{-5}{8}$	½
	y Slope of line AB and slope of line CD are equal.	
	y line AB || line CD	½
	y The line joining (– 1, 1) and (– 9, 6) is parallel to the line joining (– 2, 14) and (6, 9).	½

A.4. Solve ANY TWO of the following :

(i)

$$\text{L.H.S.} = \sqrt{\frac{\operatorname{cosec} x - 1}{\operatorname{cosec} x + 1}}$$

$$= \sqrt{\frac{(\operatorname{cosec} x - 1)(\operatorname{cosec} x - 1)}{(\operatorname{cosec} x + 1)(\operatorname{cosec} x - 1)}}$$ ½

$$= \sqrt{\frac{(\operatorname{cosec} x - 1)^2}{\operatorname{cosec}^2 x - 1}}$$

$$= \sqrt{\frac{(\operatorname{cosec} x - 1)^2}{\cot^2 x}}$$ $\left[\begin{array}{l} 1 + \cot^2 x = \operatorname{cosec}^2 x \\ \text{y } \cot^2 x = \operatorname{cosec}^2 x - 1 \end{array}\right]$ ½

$$= \frac{\operatorname{cosec} x - 1}{\cot x}$$

$$= \frac{\operatorname{cosec} x}{\cot x} - \frac{1}{\cot x}$$ ½

$$= \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} - \tan x$$ ½

$$= \frac{1}{\cos x} - \tan x$$ ½

$$= \sec x - \tan x$$

$$= \frac{(\sec x - \tan x)(\sec x + \tan x)}{(\sec x + \tan x)}$$ ½

$$= \frac{\sec^2 x - \tan^2 x}{\sec x + \tan x}$$ ½

$$= \frac{1}{\sec x + \tan x}$$ $\left[\begin{array}{l} 1 + \tan^2 x = \sec^2 x \\ \text{y } \sec^2 x - \tan^2 x = 1 \end{array}\right]$ ½

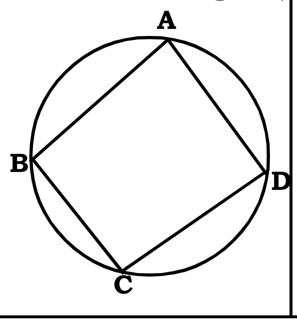
$= \text{R.H.S.}$

y $$\sqrt{\frac{\mathbf{cosec\ x - 1}}{\mathbf{cosec\ x + 1}}} = \frac{\mathbf{1}}{\mathbf{sec\ x + tan\ x}}$$

(ii) **Given :** ☐ABCD is a cyclic **(½ mark for figure)** ½

To Prove : m ĎABC + m ĎADC = 180°

m ĎBAD + m ĎBCD = 180°



Proof :

m ĎABC $= \frac{1}{2}$ m (arc ADC) .....(i)

m ĎADC $= \frac{1}{2}$ m (arc ABC) .....(ii) [Inscribed angle theorem] 1

Adding (i) and (ii), we get

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC)

y m ĎABC + m ĎADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)] ½

y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y **m ĎABC + m ĎADC = 180° ....(iii)**

In ☐ABCD,

m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°

[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(iii) A ø (5, 4),

B ø (– 3, – 2),

C ø (1, – 8)

seg AD is the median of seg BC

y D is midpoint of seg BC

y D ø $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ ½

ø $\left(\frac{-3 + 1}{2}, \frac{-2 + (-8)}{2}\right)$

ø $\left(\frac{-2}{2}, \frac{-2 - 8}{2}\right)$

ø $\left(-1, \frac{-10}{2}\right)$

ø (– 1, – 5) ½

By two point form,

The equation of median AD

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

y $$\frac{x - 5}{5 - (-1)} = \frac{y - 4}{4 - (-5)}$$

y $\frac{x - 5}{5 + 1} = \frac{y - 4}{4 + 5}$

y $\frac{x - 5}{6} = \frac{y - 4}{9}$

y $9 (x - 5) = 6 (y - 4)$

y $9x - 45 = 6y - 24$

y $9x - 6y - 45 + 24 = 0$

y $9x - 6y - 21 = 0$

y $3x - 2y - 7 = 0$ [Dividing throughout by 3] ½

y The equation of median AD is $3x - 2y - 7 = 0$

Slope of line AC $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

$= \frac{-8 - 4}{1 - 5}$

$= \frac{-12}{-4}$

$= 3$ ½

∵ Slope of parallel lines are equal

Slope of the line parallel to line AC is 3

The line passes through B (– 3, – 2)

y The equation of the line parallel to line AC passing through point B by the slope point form is

$y - y_1 = m (x - x_1)$ ½

y $y - (-2) = 3 [x - (-3)]$

y $y + 2 = 3 (x + 3)$

y $y + 2 = 3x + 9$

y $3x - y + 9 - 2 = 0$

y $3x - y + 7 = 0$

y The equation of the line parallel to AC passing through point B is $3x - y + 7 = 0$ ½

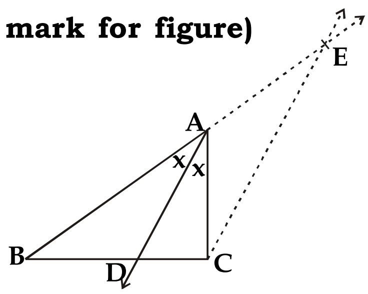
A.5. **Solve ANY TWO of the following :**

(i) **Given :** In aABC,

ray AD is the bisector of ĎBAC such that B - D - C.

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$

(½ mark for figure) ½



Construction : Draw a line passing through C, parallel to line AD and intersecting line BA at point E, B - A - E. ½

Proof : In △BEC,

line AD || side CE [Construction] ½

∴ $\frac{BD}{DC} = \frac{AB}{AE}$ ....(i) [By B.P.T.] ½

line CE || line AD [Construction]

∴ On transversal BE,

∠BAD ≅ ∠AEC ....(ii) [Converse of corresponding angles test] ½

Also, On transversal AC,

∠DAC ≅ ∠ACE ....(iii) [Converse of alternate angles test]

But, ∠BAD ≅ ∠DAC ....(iv) [∵ ray AD bisects ∠BAC] ½

In △AEC,

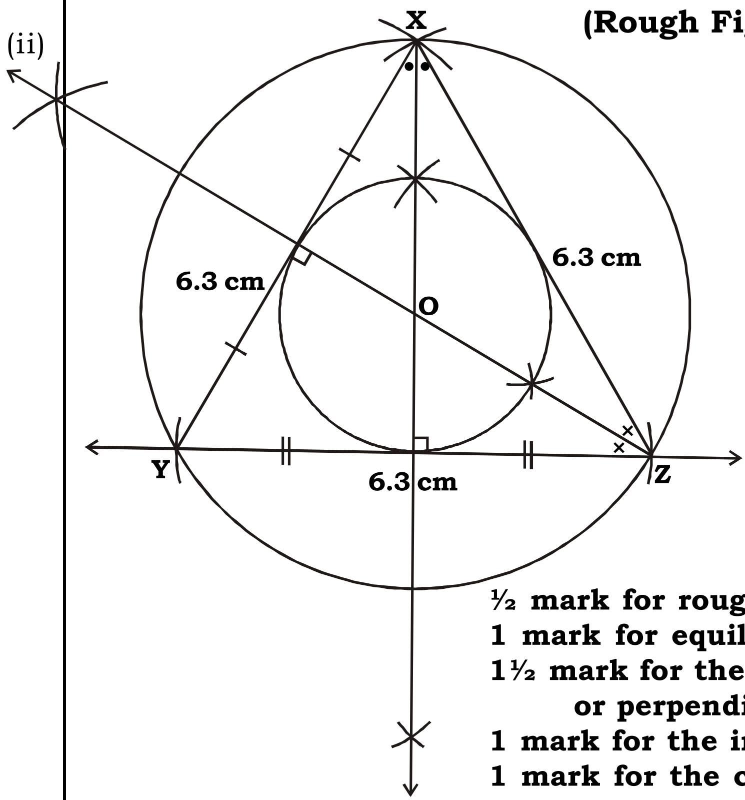
∠AEC ≅ ∠ACE [From (ii), (iii) and (iv)]

∴ seg AC ≅ seg AE [Converse of Isosceles triangle theorem]

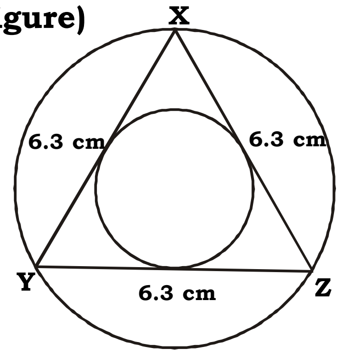
∴ AC = AE ....(v) 1

∴ $\mathbf{\frac{BD}{DC} = \frac{AB}{AC}}$ [From (i) and (v)] ½

(ii)



(Rough Figure)



½ mark for rough figure
1 mark for equilateral △XYZ
1½ mark for the two angle bisectors or perpendicular bisectors
1 mark for the incircle
1 mark for the circumcircle

(iii)

Diameter of marble = 1.4 cm

y its radius (r) = $\frac{1.4}{2}$

= 0.7 cm ½

Volume of a marble = $\frac{4}{3}$®r^3 ½

= $\frac{4}{3} \times ® \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$ cm^3 ½

∵ Marbles are submerged fully in the water, water level rises by 5.6 cm

y Height of water displaced (h) = 5.6 cm

Diameter of beaker = 7 cm

y Its radius (r_1) = $\frac{7}{2}$ cm ½

Volume of water displaced = ®r_1^2h ½

= $® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$ cm^3 ½

Number of marbles

= $\frac{\text{Volume of water displaced}}{\text{Volume of marble}}$ ½

= $® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} \div \left(\frac{4}{3} \times ® \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}\right)$ ½

= $® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} \times \frac{3}{4} \times \frac{1}{®} \times \frac{10}{7} \times \frac{10}{7} \times \frac{10}{7}$ ½

= 150

y **Number of marbles is 150.** ½

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