A.P. SET CODE
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MT - z

2017 ___ __ 1100 - **MT - z** - MATHEMATICS (71) GEOMETRY- SET - D (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. **Solve ANY FIVE of the following :**

(i) $(25)^2 = 625$ ......(i)

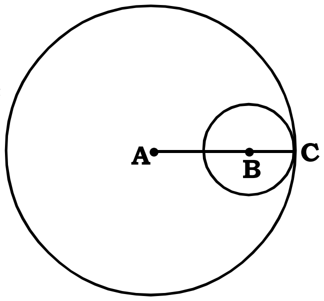
$(7)^2 + (24)^2 = 49 + 576$

$7^2 + 24^2 = 625$ ......(ii)

y $(25)^2 = (7)^2 + (24)^2$ [From (i) and (ii)] ½

y The given sides form a right angled triangle. [By Converse of Pythagoras theorem] ½

(ii) A - B - C [If two circles are touching circles, then the common point lies on a line joining their centres] ½



AB + BC = AC

y AB + 3 = 8

y AB = 8 – 3

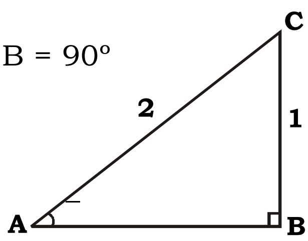
y AB = 5 cm ½

(iii) **Given :** In aABC,

BC = 1, AC = 2 and ĎB = 90º ½

To find : sin ¯

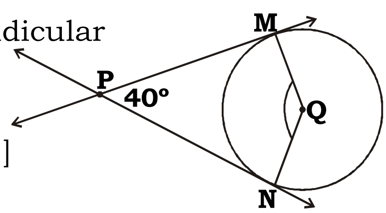
We know that,



$\sin ¯ = \frac{BC}{AC}$

$\sin ¯ = \frac{1}{2}$ ½

(iv)	Inclination of the line (¯) = 60°	
	y Slope of the line = tan ¯	½
	= tan 60	
	= $\sqrt{3}$	
	y **Slope of the line is $\sqrt{3}$.**	½
(v)	By Euler's formula,	
	F + V = E + 2	½
	y 12 + V = 30 + 2	
	y V = 32 – 12	
	y **V = 20**	½
(vi)	Given, the side of a square is 16 cm	
	We know that,	
	diagonal of square = $\sqrt{2} \times$ side	
	= $\sqrt{2} \times 16$	½
	= $16\sqrt{2}$	
	y **The diagonal of a square is $16\sqrt{2}$ cm.**	½
A.2.	**Solve ANY FOUR of the following :**	
(i)	aABC ~ aDEF [Given]	
	y $\frac{A\ (aABC)}{A\ (aDEF)} = \frac{AB^2}{DE^2}$ [Areas of similar triangles]	½
	y $\frac{9}{64} = \frac{AB^2}{(5.6)^2}$ [Given]	
	y $\frac{3}{8} = \frac{AB}{5.6}$ [Taking square roots]	½
	y AB = $\frac{3 \times 5.6}{8}$	½
	y AB = 3 × 0.7	
	y **AB = 2.1 cm**	½
(ii)	In ☐MQNP,	
	m ĎMPN = 40° [Given]	½
	m ĎPMQ = 90° } [Radius is perpendicular	½
	m ĎPNQ = 90° } to the tangent]	½
	y **m ĎMQN = 140°** [Remaining angle]	½



(iii) Radius of a circle = 0.7 cm

Area of the sector = 0.49 m² ½

Area of the sector = $\frac{r}{2} \times l$

y 0.49 = $\frac{0.7}{2} \times l$ ½

y $\frac{49}{100} = \frac{7}{20} \times l$

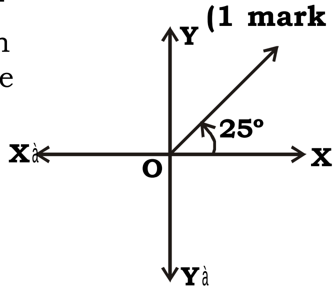
y $\frac{49 \times 20}{100 \times 7} = l$ ½

y l = 1.4

y **The length of the arc is 1.4 m.** ½

(iv) Since the initial arm rotates in anticlockwise direction and the angle is more than 0° but less than 90°, terminal arm lies in I quadrant. 1

(1 mark for figure)



(v) Length of arc (l) = 176 m

y measure of arc (¯) = 36°

$l = \frac{¯}{360} \times$ 2®r ½

y 176 = $\frac{36}{360} \times$ 2®r ½

y 176 = $\frac{1}{10} \times$ 2®r ½

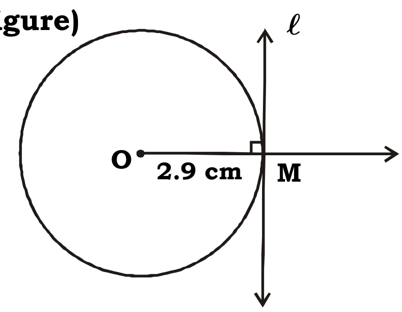
y 176 × 10 = 2®r

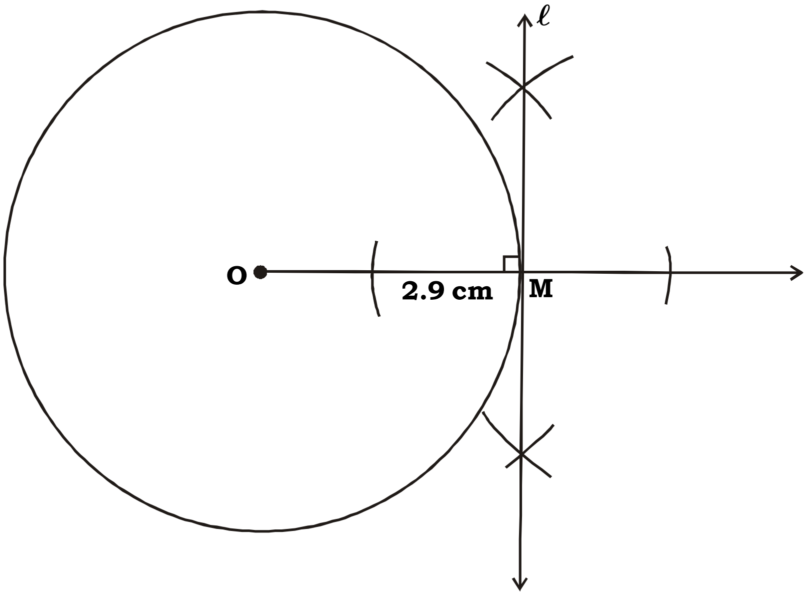
y 2®r = 1760

But, circumference = 2®r

y **Circumference of the circle is 1760 m.** ½

(vi) **(Analytical Figure)**

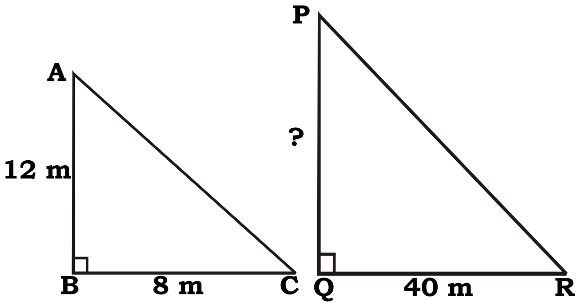




1 mark for drawing circle
1 mark for drawing tangent

A.3. **Solve ANY THREE of the following :**

(i) In the adjoining figure, seg AB and seg PQ represents the vertical stick and the tower respectively and seg BC and seg QR represents the shadow cast by them respectively. **1**



aABC ~ aPQR ½

y $\frac{AB}{PQ} = \frac{BC}{QR}$ [c.s.s.t.] ½

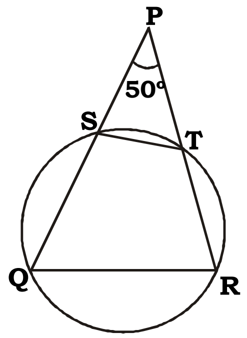
y $\frac{12}{PQ} = \frac{8}{40}$ [Given]

y $PQ = \frac{40 \times 12}{8}$ ½

y $PQ = 60$

y **Height of the tower is 60 m.** ½

(ii)



In aPQR

seg PQ] seg PR [Given]

y ĎPQR] ĎPRQ ......(i) [Isosceles triangle theorem] ½

m ĎPQR + m ĎPRQ + m ĎQPR = 180° [Sum of the measures of angles of a triangle is 180°] ½

y m ĎPRQ + m ĎPRQ + 50 = 180° [From (i) and Given]

2m ĎPRQ = 180° – 50°

∴ 2m ĎPRQ = 130°

∴ m ĎPRQ = 65° ......(ii) ½

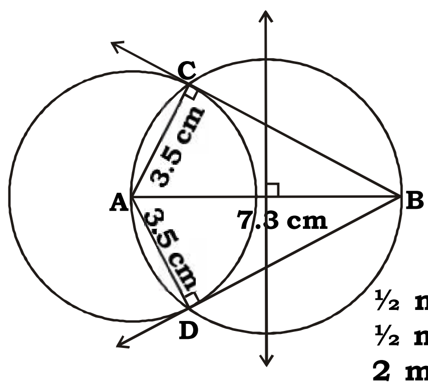
□SQRT is cyclic

y ĎPST] ĎTRQ [An exterior angle of cyclic quadrilateral is congruent to the angle opposite to adjacent interior angle] 1

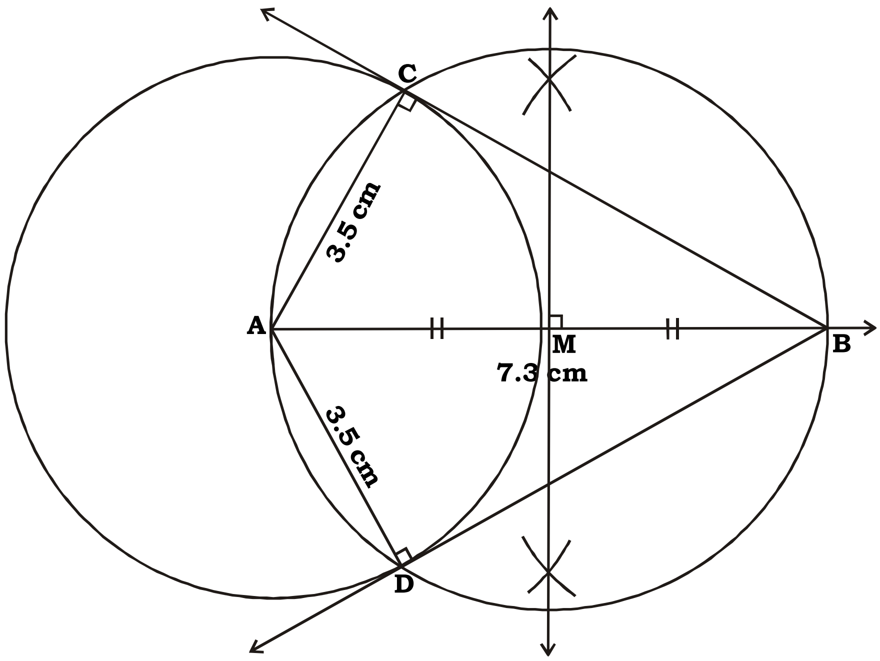
y ĎPST] ĎPRQ [P - T - R] ½

y **ĎPST = 65°** [From (ii)]

(iii) **(Analytical Figure)**



½ mark for analytical figure
½ mark for the circle with radius 3.5 cm
2 mark for drawing the two tangents



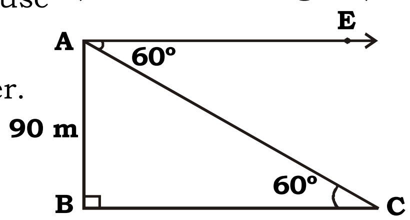
(½ mark for figure)

(iv) AB represents the height of lighthouse

C represents the position of ship. ½

A represents the position of observer.

∠EAC is the angle of depression.



AB = 90 m

m ∠EAC = 60°

∠EAC ≅ ∠ACB [Converse of alternate angles test] ½

∴ m ∠ACB = 60°

In right angled ΔABC,

$\tan 60° = \frac{AB}{BC}$ [By definition] ½

∴ $\sqrt{3} = \frac{90}{BC}$

∴ $BC = \frac{90}{\sqrt{3}}$

∴ $BC = \frac{90\sqrt{3}}{3}$ ½

∴ $BC = 30\sqrt{3}$ m

∴ BC = 30 × 1.73

∴ BC = 51.9 m

∴ **The ship is 51.9 m far from the lighthouse.** ½

(v) Let,A ø $(-3, 11)$ ø (x_1, y_1)

B ø $(6, 2)$ ø (x_2, y_2)

C ø$=$$(k, 4)$ ø (x_3, y_3)

∵ Points A, B and C are collinear

Slope of line AB = Slope of line BC ½

y $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_3 - y_2}{x_3 - x_2}$ ½

y $\dfrac{2 - 11}{6 - (-3)} = \dfrac{4 - 2}{k - 6}$

y $\dfrac{-9}{6 + 3} = \dfrac{2}{k - 6}$ ½

y $\dfrac{-9}{9} = \dfrac{2}{k - 6}$

y $-1 = \dfrac{2}{k - 6}$ ½

y $-(k - 6) = 2$

y $-k + 6 = 2$

y $-k = 2 - 6$ ½

y $-k = -4$

y $k = 4$

y **The value of k is 4** ½

A.4. Solve ANY TWO of the following :

(i) L.H.S. $= \dfrac{\cos^2 -}{1 - \tan -} + \dfrac{\sin^3 -}{\sin - - \cos -}$

$= \dfrac{\cos^2 -}{1 - \dfrac{\sin -}{\cos -}} + \dfrac{\sin^3 -}{\sin - - \cos -}$ ½

$= \dfrac{\cos^2 -}{\dfrac{\cos - - \sin -}{\cos -}} + \dfrac{\sin^3 -}{\sin - - \cos -}$ ½

$= \dfrac{\cos^3 -}{\cos - - \sin -} + \dfrac{\sin^3 -}{\sin - - \cos -}$ ½

$= \dfrac{\cos^3 -}{\cos - - \sin -} - \dfrac{\sin^3 -}{\cos - - \sin -}$ ½

$= \dfrac{\cos^3 - - \sin^3 -}{\cos - - \sin -}$ ½

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \cos\theta . \sin\theta + \sin^2\theta)}{(\cos\theta - \sin\theta)}$$ ½

$= \cos^2\theta + \sin^2\theta + \sin\theta . \cos\theta$

$= 1 + \sin\theta . \cos\theta$ $[\because \sin^2\theta + \cos^2\theta = 1]$ ½

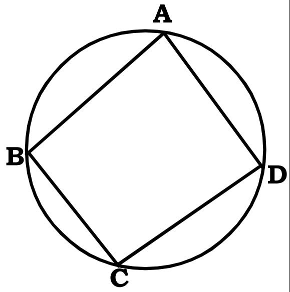
= R.H.S.

$\therefore$ $$\frac{\cos^2\theta}{1 - \tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta . \cos\theta$$ ½

(ii) **Given :** □ABCD is a cyclic **(½ mark for figure)**

To Prove : m ∠ABC + m ∠ADC = 180° ½

m ∠BAD + m ∠BCD = 180°



Proof :

m ∠ABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ∠ADC = $\frac{1}{2}$ m (arc ABC) .....(ii) [Inscribed angle theorem] 1

Adding (i) and (ii), we get

m ∠ABC + m ∠ADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC)

$\therefore$ m ∠ABC + m ∠ADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)] ½

$\therefore$ m ∠ABC + m ∠ADC = $\frac{1}{2}$ × 360° [$\because$ Measure of a circle is 360°] ½

$\therefore$ **m ∠ABC + m ∠ADC = 180° ....(iii)**

In □ABCD,

m ∠BAD + m ∠BCD + m ∠ABC + m ∠ADC = 360°

[$\because$ Sum of measure of angles of a quadrilateral is 360°] ½

$\therefore$ m ∠BAD + m ∠BCD + 180° = 360° [From (iii)]

$\therefore$ **m ∠BAD + m ∠BCD = 180°** ½

(iii) A ≡ (5, 4),

B ≡ (– 3, – 2),

C ≡ (1, – 8)

seg AD is the median of seg BC

$\therefore$ D is midpoint of seg BC

y D ø $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

ø $\left(\frac{-3 + 1}{2}, \frac{-2 + (-8)}{2}\right)$

ø $\left(\frac{-2}{2}, \frac{-2 - 8}{2}\right)$

ø $\left(-1, \frac{-10}{2}\right)$

ø (– 1, – 5) ½

By two point form,

The equation of median AD

$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$ ½

y $\frac{x - 5}{5 - (-1)} = \frac{y - 4}{4 - (-5)}$

y $\frac{x - 5}{5 + 1} = \frac{y - 4}{4 + 5}$

y $\frac{x - 5}{6} = \frac{y - 4}{9}$ ½

y $9 (x - 5) = 6 (y - 4)$

y $9x - 45 = 6y - 24$

y $9x - 6y - 45 + 24 = 0$

y $9x - 6y - 21 = 0$

y $3x - 2y - 7 = 0$ [Dividing throughout by 3] ½

y The equation of median AD is $3x - 2y - 7 = 0$

Slope of line AC $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

$= \frac{-8 - 4}{1 - 5}$

$= \frac{-12}{-4}$

$= 3$

∵ Slope of parallel lines are equal

Slope of the line parallel to line AC is 3

The line passes through B (– 3, – 2) ½

y The equation of the line parallel to line AC passing through point B by the slope point form is

$y - y_1 = m(x - x_1)$ ½

y $y - (-2) = 3[x - (-3)]$

y $y + 2 = 3(x + 3)$

y $y + 2 = 3x + 9$

y $3x - y + 9 - 2 = 0$

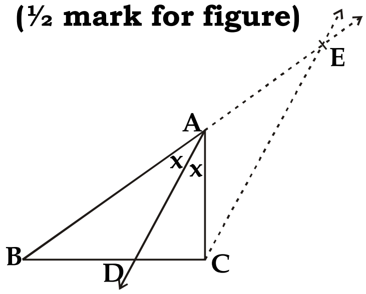
y $3x - y + 7 = 0$

y **The equation of the line parallel to AC passing through point B is $3x - y + 7 = 0$** ½

A.5. **Solve ANY TWO of the following :**

(i) **Given :** In aABC,
ray AD is the bisector of ĎBAC
such that B - D - C. ½

(½ mark for figure)



To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$

Construction : Draw a line passing through C, parallel to line AD and intersecting line BA at point E, B - A - E. ½

Proof : In aBEC,

line AD || side CE [Construction]

y $\frac{BD}{DC} = \frac{AB}{AE}$ ....(i) [By B.P.T.] ½

line CE || line AD [Construction]

y On transversal BE,

ĎBAD]ĎAEC ....(ii) [Converse of corresponding angles test] ½

Also, On transversal AC,

ĎDAC] ∠ACE ....(iii) [Converse of alternate angles test] ½

But, ĎBAD] ∠DAC ....(iv) [∵ ray AD bisects ĎBAC] ½

In aAEC,

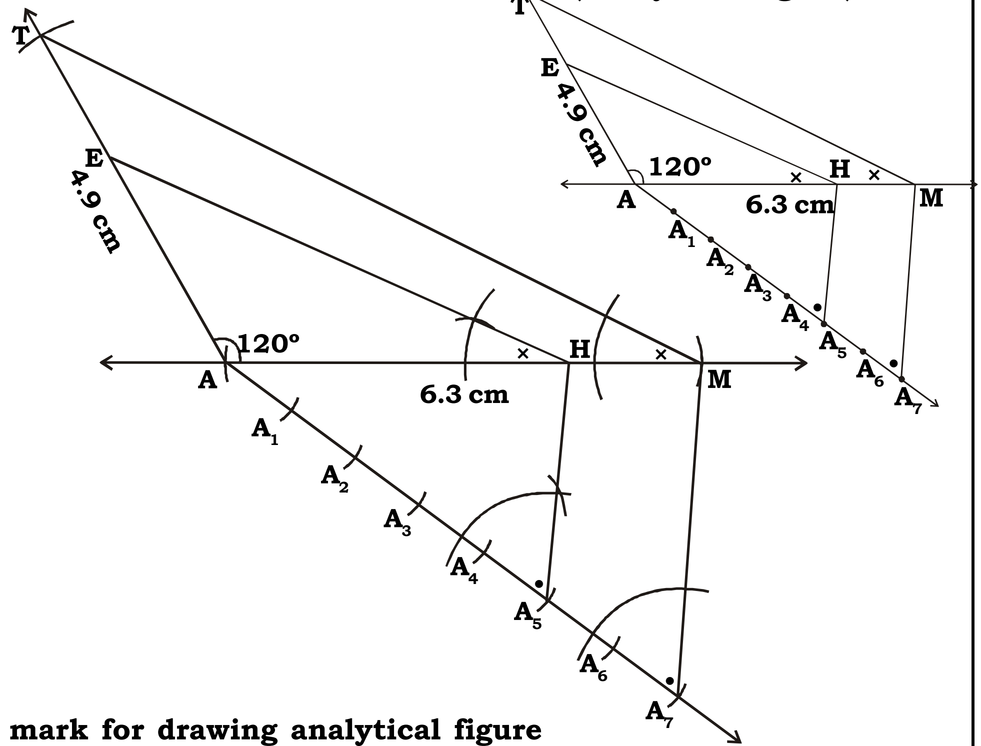
ĎAEC]ĎACE [From (ii), (iii) and (iv)]

y seg AC] seg AE [Converse of Isosceles triangle theorem] 1

y AC = AE ....(v)

y $\mathbf{\frac{BD}{DC} = \frac{AB}{AC}}$ [From (i) and (v)] ½

(ii)



(Analytical figure)

½ mark for drawing analytical figure
1 mark for ∆AMT
½ mark for constructing 7 congruent parts
1½ mark for constructing $\angle HA_5A \cong \angle MA_7A$
1½ mark for constructing $\angle EHA \cong \angle TMA$

(iii)

Diameter of marble = 1.4 cm

∴ its radius (r) = $\frac{1.4}{2}$

= 0.7 cm ½

Volume of a marble = $\frac{4}{3}\pi r^3$ ½

= $\frac{4}{3} \times \pi \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$ cm³ ½

∵ Marbles are submerged fully in the water, water level rises by 5.6 cm

∴ Height of water displaced (h) = 5.6 cm

Diameter of beaker = 7 cm

∴ Its radius (r_1) = $\frac{7}{2}$ cm ½

Volume of water displaced

$= ®r_1^2h$ ½

$= ® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$ cm^3 ½

Number of marbles

$= \frac{\text{Volume of water displaced}}{\text{Volume of marble}}$ ½

$= ® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} \div \left(\frac{4}{3} \times ® \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}\right)$ ½

$= ® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} \times \frac{3}{4} \times \frac{1}{®} \times \frac{10}{7} \times \frac{10}{7} \times \frac{10}{7}$ ½

$= 150$

y **Number of marbles is 150.** ½

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