# MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

**MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - I - PAPER - 3 (E)**

**Time : 2 Hours** **Model Answer Paper** **Max. Marks : 40**

**A.1. Solve the following : (Any 5)**

(i) A natural number is 'x'.

y It's reciprocal is $\frac{1}{x}$

As per the given condition,

$x + \frac{1}{x} = \frac{37}{6}$ ½

Multiplying throughout by 6x we get,

$6x^2 + 6 = 37x$

y $6x^2 - 37x + 6 = 0$ ½

(ii) The roots of the quadratic equation are 0 and – 6

Let ~ = 0 and = – 6

y ~ + = 0 + (– 6) = 0 – 6 = – 6

~ . = 0 × – 6 = 0 ½

We know that,

$x^2 - (\sim + \ ) x + \sim . \ = 0$

y $x^2 - (-6) x + 0 = 0$

y $x^2 + 6x = 0$

y The required quadratic equation is $x^2 + 6x = 0$. ½

(iii) If one of the root of the quadratic equation is $2\sqrt{3} - 4$,

then the other root is $2\sqrt{3} + 4$

Let ~ = $2\sqrt{3} - 4$ and = $2\sqrt{3} + 4$ ½

y ~ + = $2\sqrt{3} - 4 + 2\sqrt{3} + 4 = 4\sqrt{3}$

and ~. = $\left(2\sqrt{3} - 4\right)\left(2\sqrt{3} + 4\right)$

= $\left(2\sqrt{3}\right)^2 - (4)^2$

= $(4 \times 3) - 16$

= $12 - 16$

= $-4$ ½

(iv) Mean $(\bar{x}) = \frac{pf_i x_i}{pf_i}$

$= \frac{680}{30}$

$= \boxed{22.67}$ **1**

(v)

| **No. of rooms occupied** | **Frequency ($f_i$) (No. of days)** | **Cumulative frequency less than type** |
|---|---|---|
| 0 - 10 | 5 | 5 |
| 10 - 20 | 27 | 32 ← *c.f.* |
| 20 - 30 | 17 ← *f* | 49 |
| 30 - 40 | 11 | 60 |
| 40 - 50 | 9 | 69 |
| 50 - 60 | 1 | 70 |
| **Total** | **70 ← N** | |

Here total frequency = $pf_i$ = N = 70

∴ $\frac{N}{2} = \frac{70}{2} = 35$

Cumulative frequency (less than type) which is just greater than 35 is 49. Therefore corresponding class 20 - 30 is median class.

L = 20, N = 70, *c.f.* = 32, f = 17, h = 10 **1**

(vi) Let total sales done be Rs. x

Sales done due to salesman A = Rs. 18000

Measure of central angle for salesman A = 90°

As per the given condition,

$\frac{90}{360} \times x = 18000$

∴ $x = \frac{18000 \times 360}{90}$

∴ $x = 18000 \times 4$

∴ $x = 72000$

∴ $\boxed{\text{Total sales done is Rs. } 72000}$ **1**

**A.2. Solve the following : (Any 4)**

(i) $y^2 + 8y + 5 = 0$

Comparing with $ay^2 + by + c = 0$ we have a = 1, b = 8, c = 5

$a = b^2 - 4ac$ ½

$= (8)^2 - 4\ (1)\ (5)$

$= 64 - 20$

$= 44$ ½

y $a > 0$

Hence roots of the quadratic equation are real and unequal. **1**

(ii) Class width (h) = 4

| **Age in years ($x_i$)** | **Class Mark ($f_i$)** | **No. of people** | $f_i x_i$ |
|---|---|---|---|
| 7 - 11 | 9 | 5 | 45 |
| 11 - 15 | 13 | 9 | 117 |
| 15 - 19 | 17 | 13 | 221 |
| 19 - 23 | 21 | 21 | 441 |
| 23 - 27 | 25 | 16 | 400 |
| 27 - 31 | 29 | 15 | 435 |
| 31 - 35 | 33 | 12 | 396 |
| 35 - 39 | 37 | 9 | 333 |
| **Total** | | **100** | **2388** |

**1**

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

y $\text{Mean} = \frac{2388}{100}$

y Mean = 23.88 years. **1**

(iii) If one of the root of the quadratic equation is $\sqrt{3} - \sqrt{7}$ then the other root is $\sqrt{3} + \sqrt{7}$

Let $\sim = \sqrt{3} - \sqrt{7}$ and $= \sqrt{3} + \sqrt{7}$

y $\sim + = \sqrt{3} - \sqrt{7} + \sqrt{3} + \sqrt{7}$

$= 2\sqrt{3}$ ½

~ . $= \left(\sqrt{3} - \sqrt{7}\right)\left(\sqrt{3} + \sqrt{7}\right)$

$= \left(\sqrt{3}\right)^2 - \left(\sqrt{7}\right)^2$

$= 3 - 7$

$= -4$ ½

We know that,

$x^2 - (\sim + \ ) x + \sim . = 0$

y $x^2 - 2\sqrt{3} + (-4) = 0$

y $x^2 - 2\sqrt{3} - 4 = 0$

y The required quadratic equation is $x^2 - 6x - 3 = 0$ **1**

(iv)

| Time (in min) | No. of persons | |
|---|---|---|
| 30 - 40 | 4 | |
| 40 - 50 | 6 | ì $f_1$ |
| 50 - 60 | 19 | ì $f_m$ |
| 60 - 70 | 14 | ì $f_2$ |
| 70 - 80 | 8 | |
| 80 - 90 | 7 | |
| 90 - 100 | 2 | |

Here the maximum frequency $f_m = 19$

The corresponding class 50 - 60 is the modal class.

$L = 50, f_m = 19, f_1 = 6, f_2 = 14, h = 10$

Mode $= L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$ ½

$= 50 + \left(\frac{19 - 6}{2(19) - 6 - 14}\right) \times 10$

$= 50 + \left(\frac{13}{38 - 20}\right) \times 10$ ½

$= 50 + \left(\frac{13}{18}\right) \times 10$

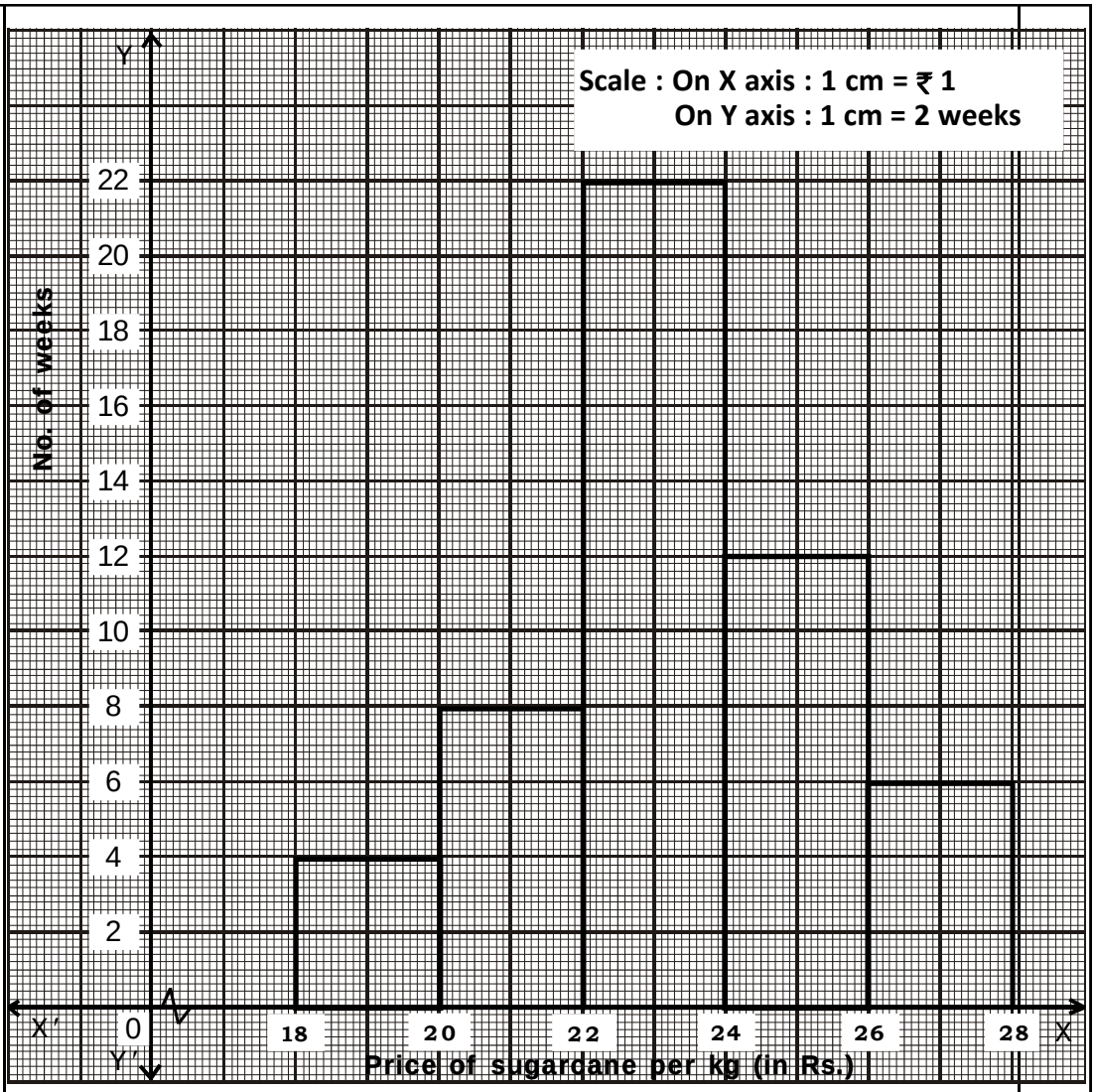
$= 50 + \frac{65}{9}$

$= 50 + 7.22$

$= 57.22$

y Mode of time is 57.22 minutes. **1**

(v)




**2**

(vi)

| **Blood group** | **Percentage** | **Measure of central angle** |
|---|---|---|
| O | 45 | $\frac{45}{100} \times 360^\circ = 162^\circ$ |
| A | 20 | $\frac{20}{100} \times 360^\circ = 72^\circ$ |
| B | 30 | $\frac{30}{100} \times 360^\circ = 108^\circ$ |
| AB | 5 | $\frac{5}{100} \times 360^\circ = 18^\circ$ |
| **Total** | **100** | **360°** |

**2**

**A.3.** **Solve the following : (Any 3)**

(i) $(m-1)x^2 - 2(m-1)x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = m-1$, $b = -2(m-1)$, $c = 1$ ½

$a = b^2 - 4ac$

$= [-2(m-1)]^2 - 4(m-1)(1)$

$= 4(m^2 - 2m + 1) - 4m + 4$

$= 4m^2 - 5m + 4 - 4m + 4$

$= 4m^2 - 12m + 8$

$\because$ The roots of the given equation are real and equal. ½

$\therefore$ a must be zero.

$\therefore \quad 4m^2 - 12m + 8 = 0$

$\therefore \quad m^2 - 3m + 2 = 0$ [Dividing throughout by 4] ½

$\therefore \quad m^2 - 2m - m + 2 = 0$

$\therefore \quad m(m-2) - 1(m-2) = 0$

$\therefore \quad (m-2)(m-1) = 0$

$\therefore \quad m - 2 = 0$ or $m - 1 = 0$

$\therefore \quad m = 2$ or $m = 1$ **1**

If $m = 1$

$a = 1 - 1$ [$\because a = m - 1$]

$a = 0$

But $a \neq 0$

$\therefore$ m = 1 is not acceptable.

$\therefore$ $\boxed{m = 1}$ ½

(ii) Class width (h) = 10, Assumed mean (A) = 45

| **Speed (in km/hr.)** | **Class Mark ($x_i$)** | **$d_i = x_i - A$** | **No. of students ($f_i$)** | **$f_i d_i$** |
|---|---|---|---|---|
| 20 - 30 | 25 | – 20 | 6 | – 120 |
| 30 - 40 | 35 | – 10 | 80 | – 800 |
| 40 - 50 | 45 ← A | 0 | 156 | 0 |
| 50 - 60 | 55 | 10 | 98 | 980 |
| 60 - 70 | 65 | 20 | 60 | 1200 |
| **Total** | | | **400** | **1260** |

**1**

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$$

$$\therefore \quad \bar{d} = \frac{1260}{400}$$

$\therefore \quad \bar{d} = 3.15$ **1**

Mean $(\bar{x}) = A + \bar{d}$

$= 45 + 3.15$

$= 48.15$

$\therefore$ Mean of driving speed of vehicle is 48.15 km/hr. **1**

(iii)

$7y^4 - 25y^2 + 12 = 0$

$\therefore 7(y^2)^2 - 25y^2 + 12 = 0$

Substituting $y^2 = m$ we get,

$7m^2 - 25m + 12 = 0$ **1**

$\therefore 7m^2 - 21m - 4m + 12 = 0$

$\therefore 7m(m - 3) - 4(m - 3) = 0$

$\therefore (m - 3)(7m - 4) = 0$

$\therefore m - 3 = 0$ or $7m - 4 = 0$

$m = 3$ or $7m = 4$

$\therefore m = 3$ or $m = \frac{4}{7}$ **1**

Resubstituting $m = y^2$ we get,

$y^2 = 3$ or $y^2 = \frac{4}{7}$

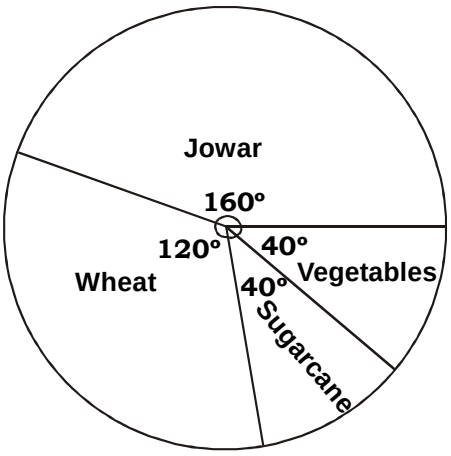
Taking square root on both the sides, we get,

$y = \pm\sqrt{3}$ or $y = \pm\frac{2}{\sqrt{7}}$ **1**

(iv)

| **Crop** | **Area in hectare** | **Measure of central angle** |
|---|---|---|
| Jowar | 8000 | $\frac{8000}{18000} \times 360° = 160°$ |
| Wheat | 6000 | $\frac{6000}{18000} \times 360° = 120°$ |
| Sugarcane | 2000 | $\frac{2000}{18000} \times 360° = 40°$ |
| Vegetables | 2000 | $\frac{2000}{18000} \times 360° = 40°$ |
| **Total** | **18000** | **360°** |

**1**




**2**

| | | |
|---|---|---|
| (v) | Let ~ and be the roots of a quadratic equation. | |
| | ∵ ~ + = 3 and ~³ + ³ = 63 [Given] | |
| | We know that, | |
| | $x^2 - (\sim + )x + \sim. = 0$ .....(i) | ½ |
| | Also, ~³ + ³ = (~ + )³ – 3 ~. (~ + ) | |
| y | 63 = (3)³ – 3 ~. (3) [∵ ~ + = 3 | |
| | and ~³ + ³ = 63] | ½ |
| y | 63 = 27 – 9 ~. | |
| y | 9 ~. = 27 – 63 | |
| y | 9 ~. = – 36 | |
| y | $\sim. = \frac{-36}{9}$ | |
| y | ~. = – 4 | |
| y | $x^2 - (\sim + )x + \sim. = 0$ [From (i)] | 1 |
| y | $x^2 - 3x + (-4) = 0$ [∵ ~ + = 3 and | |
| | ~. = – 4] | |
| y | $x^2 - 3x - 4 = 0$ | |
| y | The required quadratic equation is $x^2 - 3x - 4 = 0$. | 1 |
| **A.4.** | **Solve the following : (Any 2)** | |
| (i) | Let the smaller positive integer be x | |
| y | The bigger positive integer is x + 2 | |
| | As per the given condition, | |
| | $(x + 2)^3 - x^3 = 56$ | |
| y | $x^3 + 3 \times (x^2) \times (2) + 3 \times (x) \times 2^2 + 2^3 - x^3 - 56 = 0$ | 1 |
| y | $x^3 + 6x^2 + 12x + 8 - x^3 - 56 = 0$ | |
| y | $6x^2 + 12x - 48 = 0$ | |
| | Dividing throughout by 6 we get, | |
| | $x^2 + 2x - 8 = 0$ | 1 |
| y | $x^2 + 4x - 2x - 8 = 0$ | |
| y | $x(x + 4) - 2(x + 4) = 0$ | |
| y | $(x + 4)(x - 2) = 0$ | |
| y | $x + 4 = 0$ or $x - 2 = 0$ | |
| y | $x = -4$ or $x = 2$ | 1 |
| ∵ | x is a positive integer | |
| y | x ÷ – 4 | |
| | Hence x = 2 | |
| y | x + 2 = 2 + 2 = 4 | |
| y | The two positive integers are 2 and 4. | 1 |

(ii)

| Classes (Age in years) | Frequency ($f_i$) (No. of patients) | Cumulative frequency less than type |
|---|---|---|
| 10 - 20 | 60 | 60 |
| 20 - 30 | 42 | 102 ì *c.f.* |
| 30 - 40 | 55 ì *f* | 157 |
| 40 - 50 | 70 | 227 |
| 50 - 60 | 53 | 280 |
| 60 -70 | 20 | 300 |
| **Total** | **300 ì N** | |

**1**

Here total frequency = $\sum f_i$ = N = 300

y $\frac{N}{2} = \frac{300}{2} = 150$

Cumulative frequency (less than type) which is just greater than 150 is 157. Therefore corresponding class 30 - 40 is median class.

L = 30, N = 300, *c.f.* = 102, f = 55, h = 10 **1**

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$

$$= 30 + \left(\frac{300}{2} - 102\right)\frac{10}{55}$$ **1**

$$= 30 + (150 - 102)\ \frac{10}{55}$$

$$= 30 + (48)\ \frac{10}{55}$$

$$= 30 + (48)\ \frac{2}{11}$$

$$= 30 + \frac{96}{11}$$

$$= 30 + 8.73$$

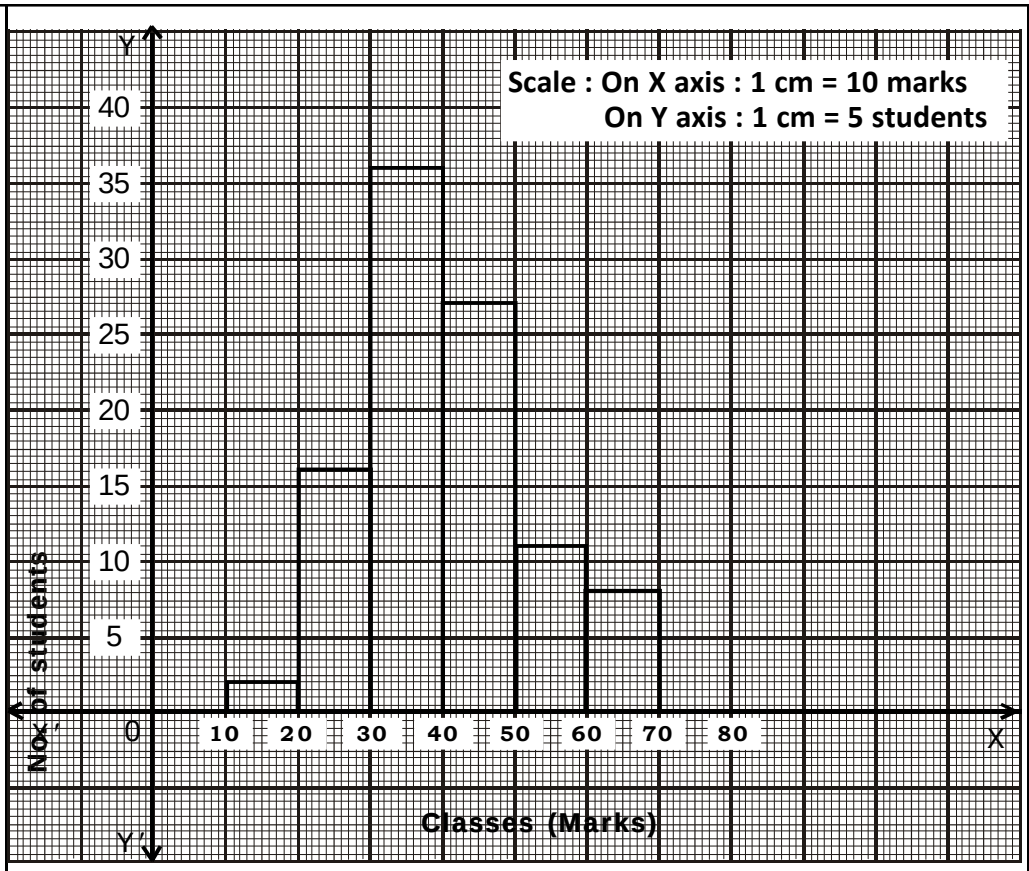
$$= 38.73$$

y Median of age is 38.73 years. **1**

(iii)

| Class marks | Classes No. of students | Frequency |
|---|---|---|
| 5 | 0 - 10 | 0 |
| 15 | 10 - 20 | 2 |
| 25 | 20 - 30 | 16 |
| 35 | 30 - 40 | 36 |
| 45 | 40 - 50 | 27 |
| 55 | 50 - 60 | 11 |
| 65 | 60 - 70 | 8 |
| 75 | 70 - 80 | 0 |

**1**




3

**A.5.** **Solve the following : (Any 2)**

(i) Let the number of days required by John alone to complete the work be x days

y No. of days requred by ganesh alone is (x + 10 ) days.

Also number of days required by both to complete the same work is 12 days

y Work done by John in 1 day = $\frac{1}{x}$

y Work done by Ganesh in 1 day = $\frac{1}{x+10}$ **1**

y Work done by both in 1 day = $\frac{1}{12}$

As per the given condition,

$$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$$ **1**

y $$\frac{x+10+x}{x(x+10)} = \frac{1}{12}$$

y $\frac{2x + 10}{x^2 + 10x} = \frac{1}{12}$

y $12\ (2x + 10) = 1\ (x^2 + 10x)$

y $24x + 120 = x^2 + 10x$

y $0 = x^2 + 10x - 24x - 120$ **1**

y $x^2 - 14x - 120 = 0$

y $x^2 - 20x + 6x - 120 = 0$

y $x\ (x - 20) + 6\ (x - 20) = 0$

y $(x - 20)\ (x + 6) = 0$

y $x - 20 = 0$ or $x + 6 = 0$

y $x = 20$ or $x = -6$ **1**

∵ The numbers of days cannot be negative

y $x \div -6$

Hence $x = 20$

y $x + 10 = 20 + 10 = 30$

y Ganesh alone worked for 30 days. **1**

(ii) Class width (h) = 10, Assumed mean (A) = 45

| **Class mark** | **Class Mark** $(x_i)$ | $\mathbf{d_i = x_i - A}$ | $\mathbf{u_i = \frac{d_i}{h}}$ | **No. of students** $(f_i)$ | $f_i u_i$ |
|---|---|---|---|---|---|
| 0 - 10 | 5 | – 40 | – 4 | 3 | – 12 |
| 10 - 20 | 15 | – 30 | – 3 | 5 | – 15 |
| 20 - 30 | 25 | – 20 | – 2 | 7 | – 14 |
| 30 - 40 | 35 | – 10 | – 1 | 10 | – 10 |
| 40 - 50 | 45 ← A | 0 | 0 | 12 | 0 |
| 50 - 60 | 55 | 10 | 1 | 15 | 15 |
| 60 - 70 | 65 | 20 | 2 | 12 | 24 |
| 70 - 80 | 75 | 30 | 3 | 6 | 18 |
| 80 - 90 | 85 | 40 | 4 | 2 | 8 |
| 90 - 100 | 95 | 50 | 5 | 8 | 40 |
| **Total** | | | | **80** | **54** **2** |

$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$

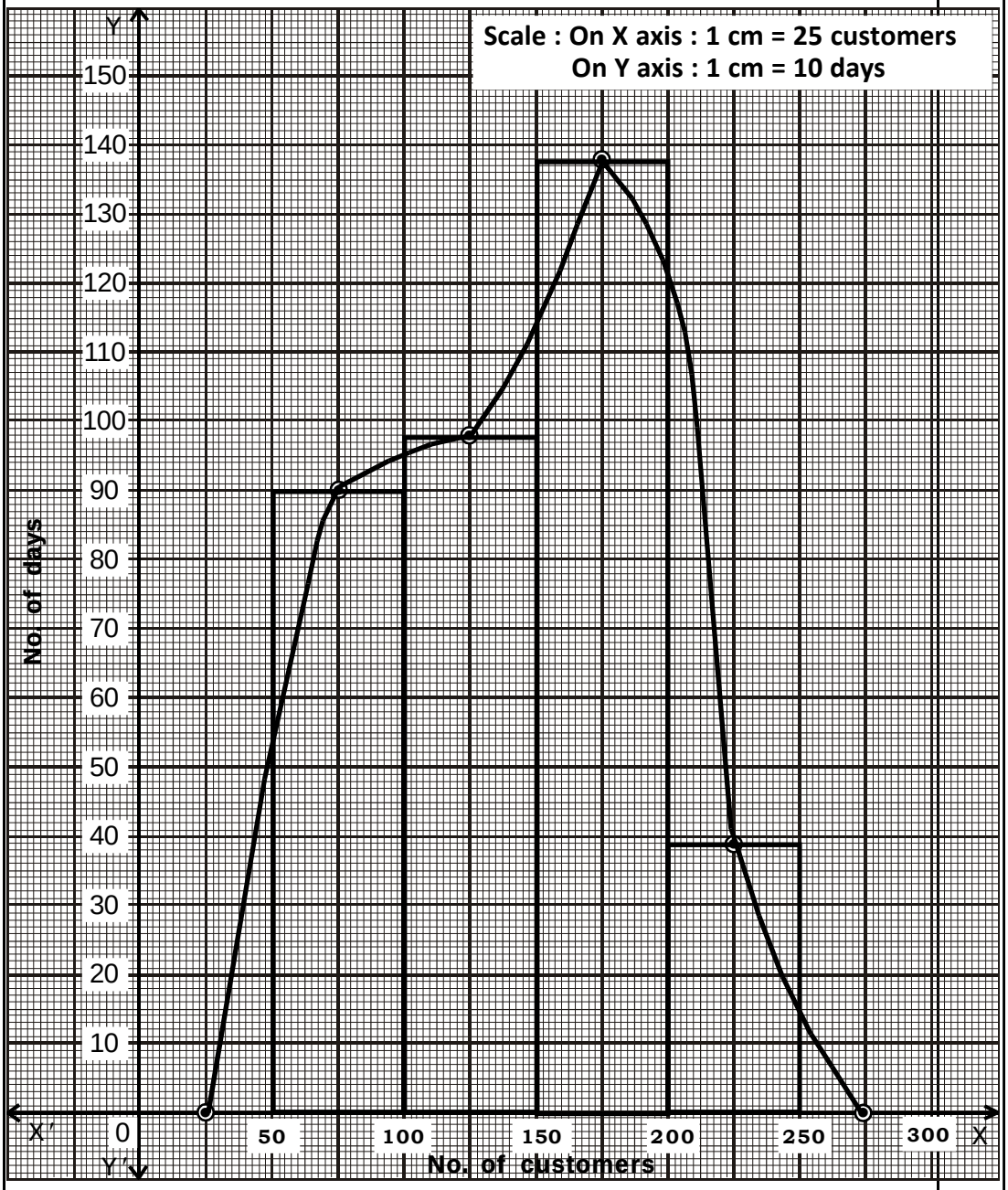
y $\bar{u} = \frac{54}{80}$

y $\bar{u} = 0.675$ **1**

Mean $(\bar{x})$ = $A + h\bar{u}$
= 45 + 10 (6.25)
= 45 + 6.75
= 51.75 **1**

y Mean of marks obtained by students is 51.75 marks. **1**

(iii)




**5**

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