MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - I - PAPER - 4 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Solve the following : (Any 5)

(i) Two numbers are 'y' and y – 3.
As per the given condition,
y (y – 3) = 42
y $y^2 - 3y - 42 = 0$ **1**

(ii) The roots of the quadratic equation are 3 and 10
Let ~ = 3 and = 10
y ~ + = 3 + 10 = 13
~ . = 3 × 10 = 30 **½**
We know that,
$x^2 - (\sim +)\, x + \sim. = 0$
y $x^2 - 13x + 30 = 0$
y The required quadratic equation is $x^2 - 13x + 30 = 0$ **½**

(iii) If one of the root of the quadratic equation is $2 + \sqrt{5}$, then the other root is $2 - \sqrt{5}$.
Let ~ = $2 + \sqrt{5}$ and = $2 - \sqrt{5}$
y ~ + = $2 + \sqrt{5} + 2 - \sqrt{5} = 4$ **½**

and ~. = $\left(2 + \sqrt{5}\right) \times \left(2 - \sqrt{5}\right)$
= $(2)^2 - \left(\sqrt{5}\right)^2$
= 4 – 5
= – 1 **½**

(iv) $\bar{u} = \frac{pf_i u_i}{pf_i}$

$= \frac{36}{100}$

= 0.36 **½**

Mean $(\bar{x})$ = $A + h.\bar{u}$
= 41 + 3(0.36)
= 41 + 1.08
= 42.08 ½

(v)

Weight (in gms)	No. of packets
200 - 201	12 ì f_1
201 - 202	26 ì f_m
202 - 203	20 ì f_2
203 - 204	9
204 - 205	2
205 - 206	1

Here the maximum frequency f_m = 26.
The corresponding class 201 - 202 is the modal class.
L = 201, f_m = 26, f_1 = 12, f_2 = 20, h = 1 **1**

(iv)

Name of the Candidate	Measure of central angle ($\bar{}$)	Number of votes
Suja	60°	$\frac{60}{360} \times 720 = 120$

Minimum number of votes is 120 obtained by Suja. **1**

A.2. Solve the following : (Any 4)

(i) $2y^2 - 7y - 3 = 0$
Comparing with $ay^2 + by + c = 0$ we have a = 2, b = – 7, c = – 3
$a = b^2 - 4ac$
$= (-7)^2 - 4\ (2)\ (-3)$ ½
$= 49 + 24$
$= 73$
y a > 0 ½
Hence roots of the quadratic equation are real and unequal. **1**

(ii) Class width (h) = 10

No. of days (x_i)	Class Mark (f_i)	No. of employees	$f_i x_i$
0 - 10	5	5	25
10 - 20	15	7	105
20 - 30	25	11	275
30 - 40	35	4	140
40 - 50	45	3	135
Total		**30**	**680**

1

Mean $= \dfrac{\text{p } f_i x_i}{\text{p } f_i}$

y Mean $= \dfrac{680}{30}$

y Mean $= 22.67$ (approximately)

y Mean of medical leave is 22.67 days. **1**

(iii) The one of the root of the quadratic equation is $\sqrt{7} - \sqrt{2}$ then the other root is $\sqrt{7} + \sqrt{2}$

y $\sim = \sqrt{7} - \sqrt{2}$ and $= \sqrt{7} + \sqrt{2}$

y $\sim + = \sqrt{7} - \sqrt{2} + \sqrt{7} + \sqrt{2} = 2\sqrt{7}$ **½**

$\sim . = \left(\sqrt{7} - \sqrt{2}\right)\left(\sqrt{7} + \sqrt{2}\right)$

$= \left(\sqrt{7}\right)^2 - \left(\sqrt{2}\right)^2$

$= 7 - 2$

$= 5$ **½**

We know that,

$x^2 - (\sim +) x + \sim. = 0$

y $x^2 - 2\sqrt{7} + 5 = 0$

y The required quadratic equation is $x^2 - 2\sqrt{7} + 5 = 0$. **1**

(iv)

No. of units of electricity	No. of household	
0 - 20	4	
20 - 40	16	
40 - 60	47	ì f_1
60 - 80	65	ì f_m
80 - 100	8	ì f_2

Here the maximum frequency $f_m = 65$.

The corresponding class 60 - 80 is the modal class.

L = 60, $f_m = 65$, $f_1 = 47$, $f_2 = 8$, h = 20 **½**

Mode $= L + \left(\dfrac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$

$= 60 + \left(\dfrac{65 - 47}{2(65) - 47 - 8}\right) \times 20$ **½**

$= 60 + \left(\dfrac{18}{130 - 55}\right) \times 20$

$= 60 + \left(\dfrac{18}{75}\right) \times 20$

$= 60 + \left(\frac{18}{15}\right) \times 4$

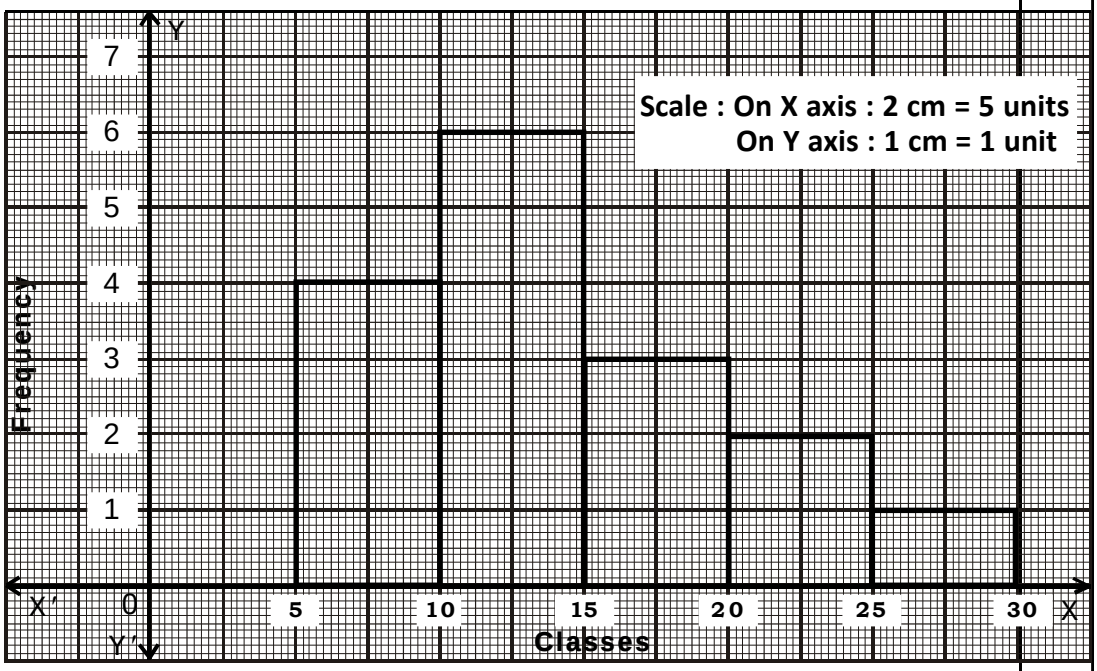
$= 60 + \left(\frac{6}{5}\right) \times 4$

$= 60 + 4.8$

$= 64.8$

y Mode of electricity units is 64.8 units. **1**

(v)



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(vi)

Items	**Measure of central angle**	**Expenditure (in Rs.)**
Cement	75°	$\frac{75}{360} \times 540000 = 112500$
Bricks	50°	$\frac{50}{360} \times 540000 = 75000$
Labour	100°	$\frac{100}{360} \times 540000 = 150000$
Timber	90°	$\frac{90}{360} \times 540000 = 135000$
Steel	45°	$\frac{45}{360} \times 540000 = 67500$
Total	**360°**	**540000**

2

A.3. **Solve the following : (Any 3)**

(i) $x^2 - 2(c+1)x + c^2 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have $A = 1$, $B = -2(c+1)$, $C = c^2$ ½

$a = B^2 - 4AC$

$= [-2(c+1)]^2 - 4(1)(c^2)$ ½

$= (-2)^2 (c+1)^2 - 4c^2$

$= 4(c^2 + 2c + 1) - 4c^2$

$= 4c^2 + 8c + 4 - 4c^2$

$= 8c + 4$ **1**

$\because$ The roots of the given equation are real and equal.

$\therefore$ a must be zero.

$\therefore 8c + 4 = 0$

$\therefore 8c = -4$

$\therefore c = \frac{-4}{8}$

$\therefore \boxed{c = \frac{-1}{2}}$ **1**

(ii) Class width (h) = 5, Assumed mean (A) = 22.5

No. of trees (x_i)	**Class Mark**	**$d_i = x_i - A$**	**No. of societies (f_i)**	**$f_i d_i$**
10 - 15	12.5	– 10	2	– 20
15 - 20	17.5	– 5	7	– 35
20 - 25	22.5 ← A	0	9	0
25 - 30	27.5	5	8	40
30 - 35	32.5	10	6	60
35 - 40	37.5	15	4	60
Total			**36**	**105** **1**

$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$

$\therefore \bar{d} = \frac{105}{36}$

$\therefore \bar{d} = 2.92$ **1**

Mean $(\bar{x}) = A + \bar{d}$

$= 22.5 + 2.92$

$= 25.42$

$\therefore$ Mean of trees planted by the societies is 25.42 trees. **1**

(iii)

$3x^4 - 13x^2 + 10 = 0$

y $3(x^2)^2 - 13x^2 + 10 = 0$

Substituting $x^2 = m$ we get,

$3m^2 - 13m + 10 = 0$

y $3m^2 - 3m - 10m + 10 = 0$ **1**

y $3m(m-1) - 10(m-1) = 0$

y $(m-1)(3m-10) = 0$

y $m - 1 = 0$ or $3m - 10 = 0$

y $m = 1$ or $3m = 10$

y $m = 1$ or $m = \frac{10}{3}$

Resubstituting $m = x^2$ we get,

$x^2 = 1$ or $x^2 = \frac{10}{3}$ **1**

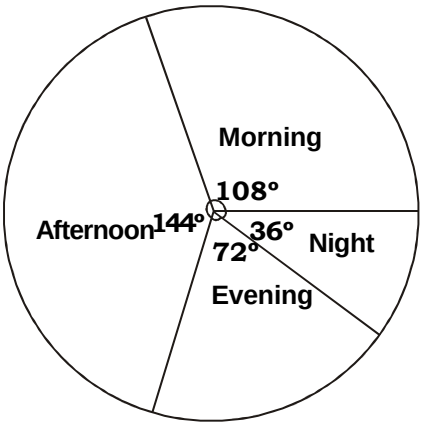
Taking square root on both the sides we get,

$x = \pm 1$ or $x = \pm\sqrt{\frac{10}{3}}$ **1**

(iv)

Part of day	**Percentage of electricity used**	**Measure of central angle**
Morning	30	$\frac{30}{100} \times 360^\circ = 108^\circ$
Afternoon	40	$\frac{40}{100} \times 360^\circ = 144^\circ$
Evening	20	$\frac{20}{100} \times 360^\circ = 72^\circ$
Night	10	$\frac{10}{100} \times 360^\circ = 36^\circ$
Total	**100**	**360°**

1



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(v) ~ and are the roots of a quadratic equation

$\because$ $\sim + = 5$ and $\sim^3 + {}^3 = 35$ [Given]

We know that,

$x^2 - (\sim +)\,x + \sim . = 0$ .....(i) ½

Also, $\sim^3 + {}^3 = (\sim +)^3 - 3 \sim . (\sim +)$

y $35 = (5)^3 - 3 \sim . (5)$ [$\because$ $\sim + = 5$ and $\sim^3 + {}^3 = 35$] ½

y $35 = 125 - 15 \sim .$

y $15 \sim . = 125 - 35$

y $15 \sim . = 90$

y $\sim . = \frac{90}{15}$

y $\sim . = 6$

$\because$ $x^2 - (\sim +)x + \sim . = 0$ [From (i)] **1**

y $x^2 - 5x + 6 = 0$

y The required quadratic equation is $x^2 - 5x + 6 = 0$. **1**

A.4. Solve the following : (Any 2)

(i) Let the number be x

y It's reciprocal is $\frac{1}{x}$ ½

As per the given condition,

$$4(x) + 3\left(\frac{1}{x}\right) = 7$$

y $4x + \frac{3}{x} = 7$ **1**

Multiplying throughout by x we get,

$4x^2 + 3 = 7x$

y $4x^2 - 7x + 3 = 0$ **1**

y $4x^2 - 4x - 3x + 3 = 0$

y $4x(x - 1) - 3(x - 1) = 0$

y $(x - 1)(4x - 3) = 0$

y $x - 1 = 0$ or $4x - 3 = 0$

y $x = 1$ or $4x = 3$ **1**

y $x = 1$ or $x = \frac{3}{4}$

y The number is 1 or $\frac{3}{4}$. ½

(ii)

Classes (I.Q.)	Frequency (f_i) (No. of candidates)	Cumulative frequency less than type
70 - 80	7	7
80 - 90	16	23 ì *c.f.*
90 - 100	20 ì *f*	43
100 - 110	17	60
110 - 120	11	71
120 - 130	7	78
130 - 140	2	80
Total	**80 ì N**	

1

Here total frequency = $\sum f_i = N = 80$

y $\frac{N}{2} = \frac{80}{2} = 40$

Cumulative frequency (less than type) which is just greater than 40 is 43. Therefore corresponding class 90 - 100 is median class.

L = 90, N = 80, *c.f.* = 23, f = 20, h = 10 **1**

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$

$$= 90 + \left(\frac{80}{2} - 23\right)\frac{10}{20} \quad \textbf{1}$$

$$= 90 + (40 - 23)\ \frac{1}{2}$$

$$= 90 + (17)\frac{1}{2}$$

$$= 90 + 8.5$$

$$= 98.5$$

y Median of I.Q.is 98.5. **1**

(iii)

Area in hecters	Continuous classes	Class mark	No. of farmers
11 - 20	10.5 - 20.5	15.5	58
21 - 30	20.5 - 30.5	25.5	103
31 - 40	30.5 - 40.5	35.5	208
41 - 50	40.5 - 50.5	45.5	392
51 - 60	50.5 - 60.5	55.5	112
61 - 70	60.5 - 70.5	65.5	34
71 - 80	70.5 - 80.5	75.5	12

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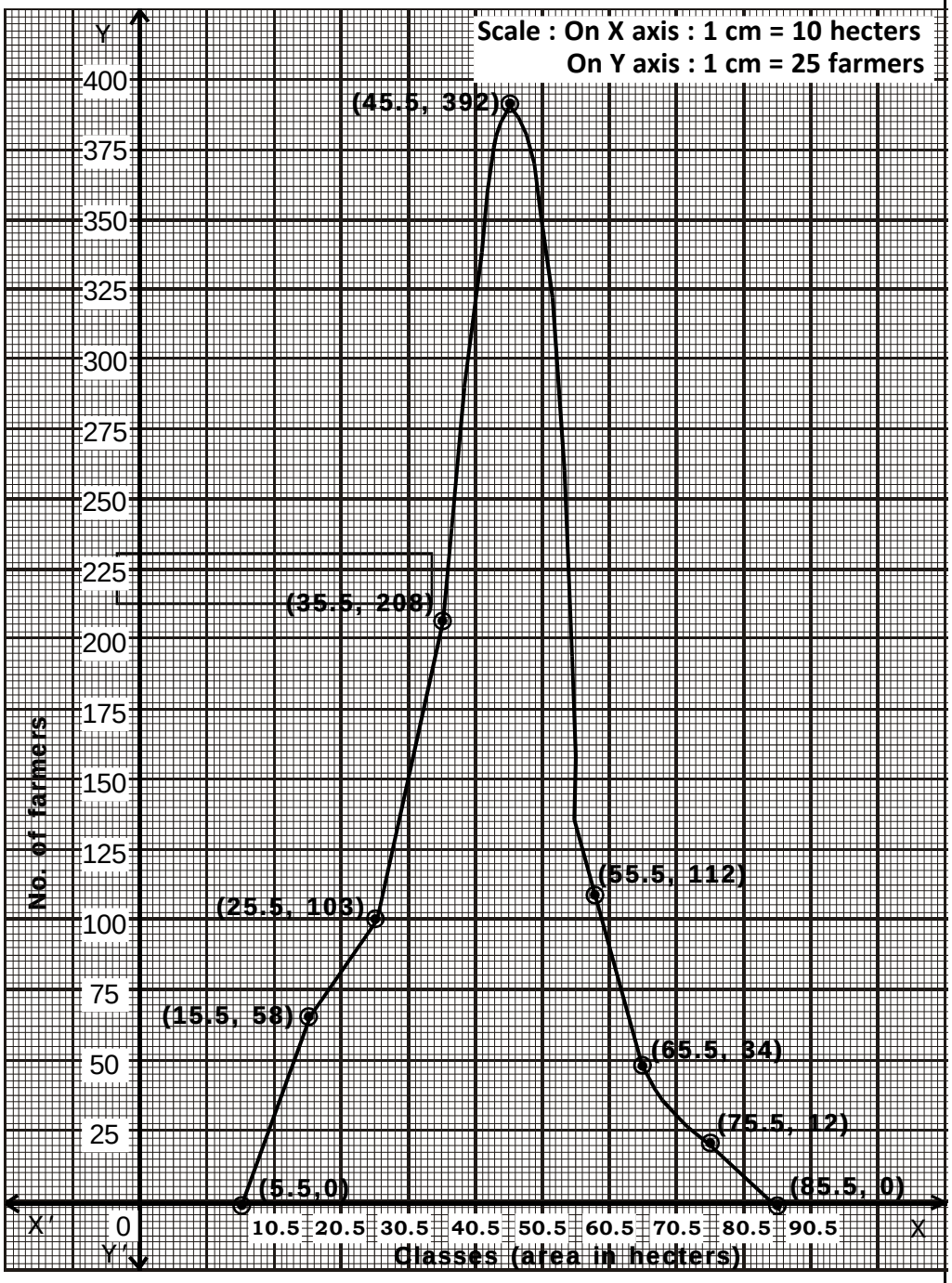

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A.5. **Solve the following : (Any 2)**

(i) Let the usual speed of train be x km/hr.

$\because$ Distance = Time × Speed

Speed = $\frac{\text{Distance}}{\text{Time}}$ y Time = $\frac{\text{Distance}}{\text{Speed}}$

Distance covered by train = 440 km

y Time taken by train = $\left(\frac{440}{x}\right)$ hrs

New speed of train = (x + 8) km/hr

y New time taken by train = $\left(\frac{440}{x+8}\right)$ hrs. **1**

As per the given condition,

$\frac{440}{x} - \frac{440}{x+8} = \frac{1}{2} \quad \left[\because 30 \text{ min} = \frac{1}{2}\text{hr}\right]$ **1**

y $440\left[\frac{1}{x} - \frac{1}{x+8}\right] = \frac{1}{2}$

y $\frac{x+8-x}{x(x+8)} = \frac{1}{2} \times \frac{1}{440}$

y $\frac{8}{x^2+8x} = \frac{1}{880}$

y $8(880) = 1(x^2 + 8x)$

y $7040 = x^2 + 8x$

y $0 = x^2 + 8x - 7040 = 0$ **1**

y $x^2 + 88x - 80x - 7040 = 0$

y $x(x + 88) - 80(x + 88) = 0$

y $(x + 88)(x - 80) = 0$

y $x + 88 = 0$ or $x - 80 = 0$

y $x = -88$ or $x = 80$ **1**

∵ The speed of train can never be negative.

y $x \div -88$

Hence x = 80

y The usual speed of train is 80 km/hr. **1**

(ii) Class width (h) = 5, Assumed mean (A) = 37.5

Classes	**Class Mark**	$\mathbf{d_i = x_i - A}$	$\mathbf{u_i = \frac{d_i}{h}}$	**No. of advertisements**	$f_i u_i$
25 - 30	27.5	– 10	– 2	10	– 20
30 - 35	32.5	– 5	– 1	32	– 32
35 - 40	37.5 ← A	0	0	15	0
40 - 45	42.5	5	1	9	9
45 - 50	47.5	10	2	7	14
50 - 55	52.5	15	3	2	6
Total				**75**	**– 23**

2

$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$

y $\bar{u} = \frac{-23}{75}$

y $\bar{u} = -0.306$ **1**

Mean $(\bar{x})$ = $A + h\bar{u}$
= 37.5 + 5 (– 0.306) **1**
= 37.5 – 1.533
= 35.97

y Mean of duration of advertisement is 35.97 seconds. **1**

(iii)

Electricity bill in a month in Rs.	Class mark	No. of families
200 - 400	300	362
400 - 600	500	490
600 - 800	700	185
800 - 1000	900	63

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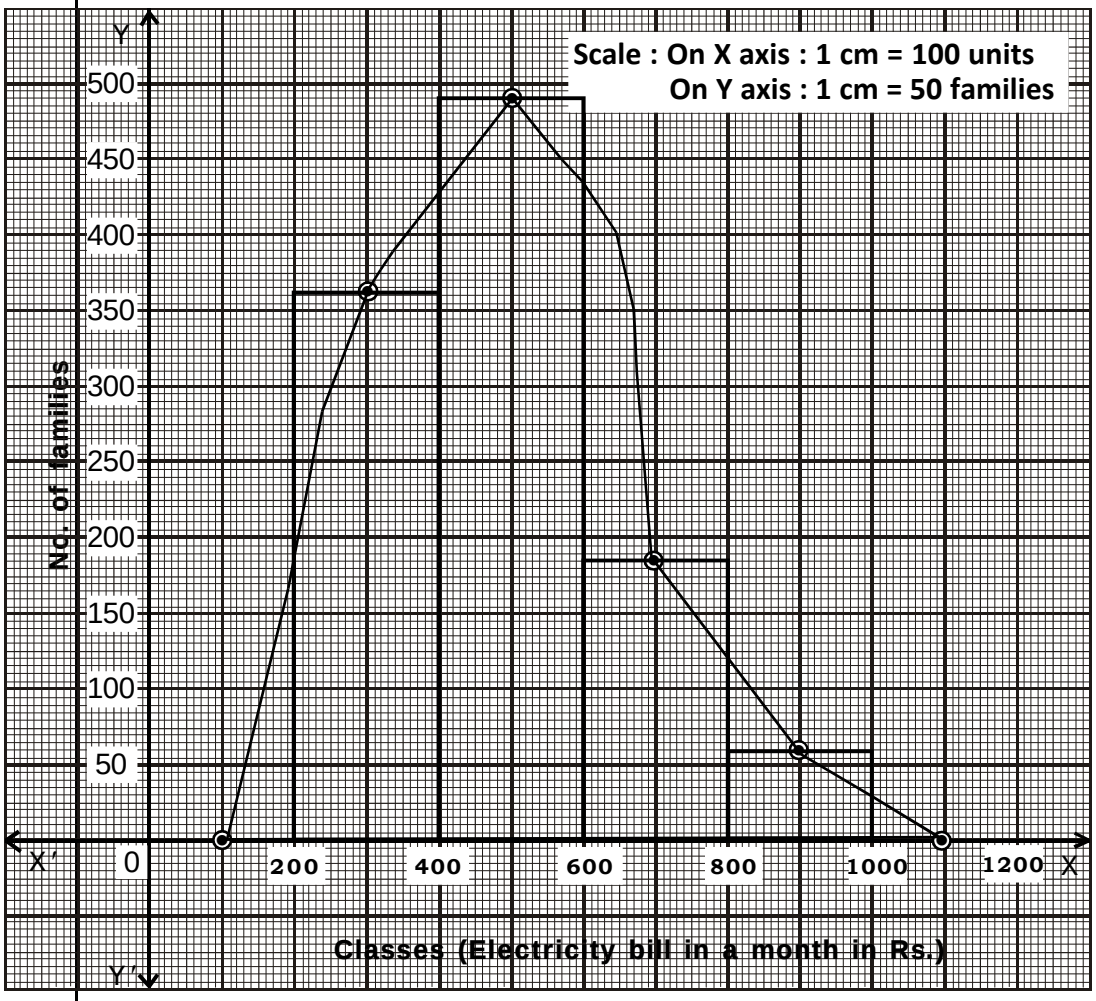

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