MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - I - PAPER - 5 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Solve the following : (Any 5)

(i) The length of a rectangle is 'x' cm.

y It's breadth is (x – 3) cm.

As per the given condition,

$\because$ Area of rectangle = Length × Breadth

y $70 = x \times (x - 3)$

y $70 = x^2 - 3x$

y $0 = x^2 - 3x - 70$

y $\boxed{x^2 - 3x - 70 = 0}$ **1**

(ii) The roots of the quadratic equation are – 5 and 9

Let $\sim = -5$ and $\beta = 9$ **½**

y $\sim + \beta = -5 + 9 = 4$

$\sim . \beta = -5 \times 9 = -45$

We know that,

$x^2 - (\sim + \beta)x + \sim . \beta = 0$

y $x^2 - 4x + (-45) = 0$

y $x^2 - 4x - 45 = 0$

y $\boxed{\text{The required quadratic equation is } x^2 - 4x - 45 = 0}$ **½**

(iii) If one of the root of the quadratic equation is $\sqrt{5} - \sqrt{3}$, then the other root is $\sqrt{5} + \sqrt{3}$.

Let $\sim = \sqrt{5} - \sqrt{3}$ and $\beta = \sqrt{5} + \sqrt{3}$ **½**

$\therefore \alpha + \beta = \sqrt{5} - \sqrt{3} + \sqrt{5} + \sqrt{3} = 2\sqrt{5}$

and $\sim . \beta = \left(\sqrt{5} - \sqrt{3}\right) \times \left(\sqrt{5} + \sqrt{3}\right)$

$= \left(\sqrt{5}\right)^2 - (\sqrt{3})^2$

$= 5 - 3$

$= 2$ **½**

(iv) $pf_iu_i = -28$, $pf_i = 100$, A = 25, h = 4

$$\bar{u} = \frac{pf_iu_i}{pf_i}$$

$$= \frac{-28}{100}$$

$$= -0.28 \quad \text{½}$$

Mean $(\bar{x})$ = A + h. $\bar{u}$

= 25 + 4(– 0.28)

= 25 – 1.12

= 23.88 ½

(v)

Classes (Age in years)	**Frequency (f_i) (No. of patients)**	**Cumulative frequency less than type**
10 - 20	60	60
20 - 30	42	102 ← *c.f.*
30 - 40	55 ← *f*	157
40 - 50	70	227
50 - 60	53	280
60 -70	20	300
Total	**300 ← N**	

Here total frequency = pf_i = N = 300

$$\frac{N}{2} = \frac{300}{2} = 150$$

Cumulative frequency (less than type) which is just greater than 150 is 157. Therefore corresponding class 30 - 40 is median class.

L = 30, N = 300, *c.f.* = 102, f = 55, h = 10 **1**

(vi) Let total number of persons be x.

No. of persons with blood group B = 600 persons

Percentage of persons with blood group B = 30%

$$\frac{30}{100} \times x = 600$$

$$x = \frac{600 \times 100}{30}$$

$$x = 2000$$

Total numbers of persons are 2000. **1**

A.2. **Solve the following : (Any 4)**

(i) $2y^2 + 11y - 7 = 0$

Comparing with $ay^2 + by + c = 0$ we have $a = 2, b = 11, c = -7$ ½

$a = b^2 - 4ac$

$= (11)^2 - 4\ (2)\ (-7)$

$= 121 + 56$

$= 177$ ½

y $a > 0$

Hence roots of the quadratic equation are real and unequal. **1**

(ii) Class width (h) = 4

Age in years (x_i)	**Class Mark (f_i)**	**No. of people**	$f_i x_i$
7 - 11	9	5	45
11 - 15	13	9	117
15 - 19	17	13	221
19 - 23	21	21	441
23 - 27	25	16	400
27 - 31	29	15	435
31 - 35	33	12	396
35 - 39	37	9	333
Total		**100**	**2388**

1

$$\text{Mean} = \frac{\text{p}\, f_i x_i}{\text{p}\, f_i}$$

y $$\text{Mean} = \frac{2388}{100}$$

y Mean = 23.88 years. **1**

(iii) If one of the root of the quadratic equation is $2 + \sqrt{5}$, then the other root is $2 - \sqrt{5}$.

Let ~ = $2 + \sqrt{5}$ and = $2 - \sqrt{5}$

y ~ + = $2 + \sqrt{5} + 2 - \sqrt{5} = 4$ ½

and ~. = $\left(2 + \sqrt{5}\right) \times \left(2 - \sqrt{5}\right)$

$= (2)^2 - \left(\sqrt{5}\right)^2$

$= 4 - 5$

$= -1$ ½

We know that,

$x^2 - (\sim + \ )x + \sim. \ = 0$

y $x^2 - 4x + (-1) = 0$

y $x^2 - 4x - 1 = 0$

y The required quadratic equation is $x^2 - 4x - 1 = 0$ **1**

(iv) Class width (h) = 10

Speed in (km / hr)	**No. of students**
20 - 30	6
30 - 40	80 ì f_1
40 - 50	156 ì f_m
50 - 60	98 ì f_2
60 - 70	60

Here the maximum frequency $f_m = 156$

The corresponding class 40 - 50 is the modal class.

L = 40, $f_m = 156$, $f_1 = 80$, $f_2 = 98$, h = 10

Mode = $L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$ **½**

$= 40 + \left(\frac{156 - 80}{2(156) - 80 - 98}\right) \times 10$

$= 40 + \left(\frac{76}{312 - 178}\right) \times 10$ **½**

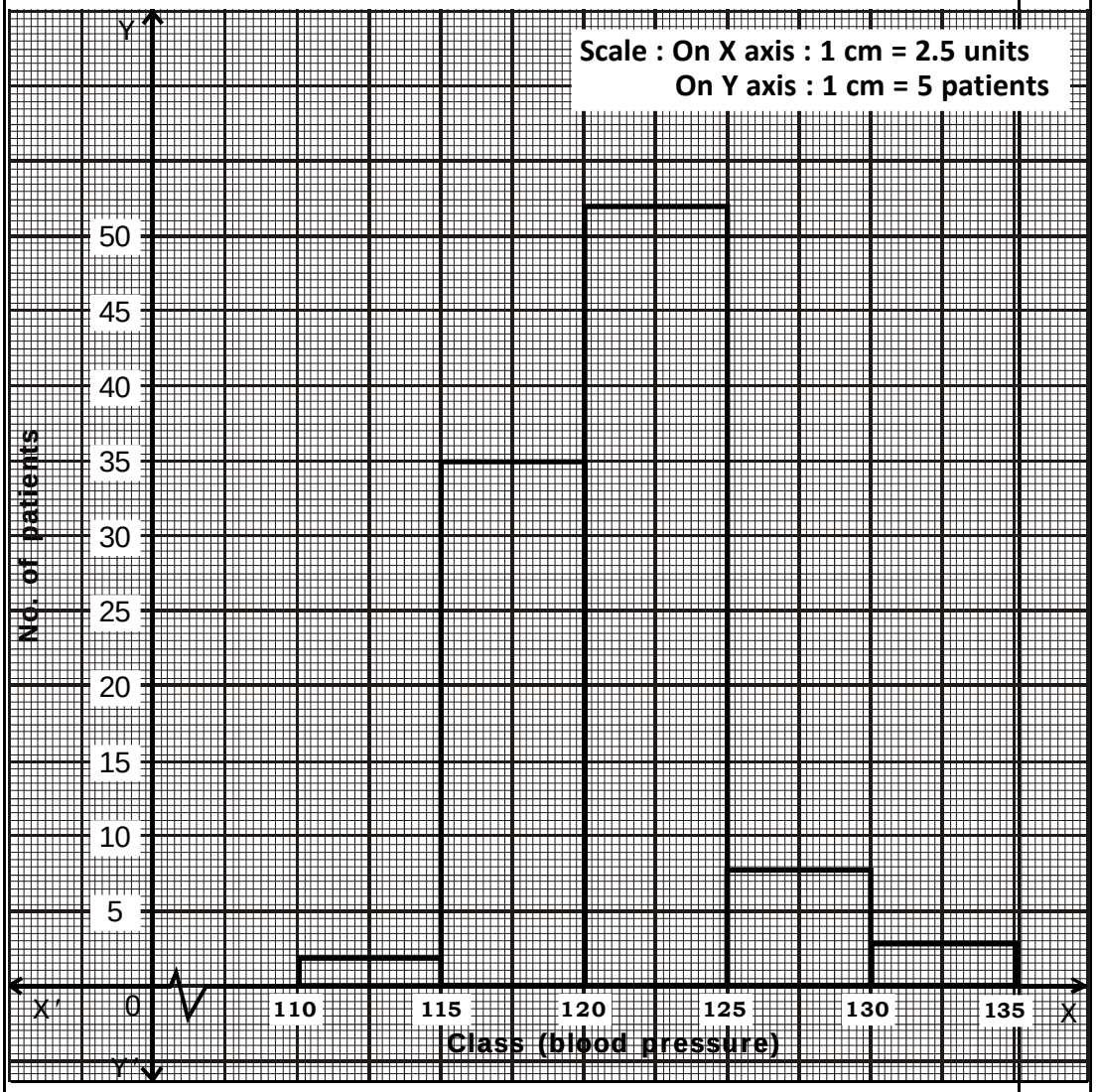
$= 40 + \left(\frac{76}{134}\right) \times 10$

$= 40 + 5.67$

$= 45.67$

y Mode of driving speed is 45.67 km/hr. **1**

(v) **2**



(vi)

Name of the Candidate	**Measure of central angle (°)**	**Number of votes**	
Albert	100°	$\frac{100}{360} \times 720 = 200$	½
Nashima	120°	$\frac{120}{360} \times 720 = 240$	½
Winner Nashima defeated the nearest contestant Albert by 40 votes.			**1**

A.3. **Solve the following : (Any 3)**

(i) $(m-1)x^2 - 2(m-1)x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we have

$a = m-1,\ b = -2(m-1),\ c = 1$ ½

$a = b^2 - 4ac$

$= [-2(m-1)]^2 - 4(m-1)(1)$

$= 4(m^2 - 2m + 1) - 4m + 4$

$= 4m^2 - 5m + 4 - 4m + 4$

$= 4m^2 - 12m + 8$ ½

∵ The roots of the given equation are real and equal.

∴ a must be zero.

$4m^2 - 12m + 8 = 0$

$m^2 - 3m + 2 = 0$ [Dividing throughout by 4]

$m^2 - 2m - m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$(m-2)(m-1) = 0$

$m - 2 = 0$ or $m - 1 = 0$

$m = 2$ or $m = 1$ **1**

If $m = 1$

$a = 1 - 1$ [∵ $a = m - 1$]

$a = 0$

But $a \neq 0$

∴ m = 1 is not acceptable.

$\boxed{m = 1}$ **1**

(ii) Class width (h) = 10, Assumed Mean (A) = 55

Marks (x_i)	**Class Mark (f_i)**	**$d_i = x_i - A$**	**No. of students**	**$f_i d_i$**
0 - 10	5	– 50	3	– 150
10 - 20	15	– 40	5	– 200
20 - 30	25	– 30	7	– 210
30 - 40	35	– 20	10	– 200
40 - 50	45	– 10	12	– 120
50 - 60	55 ← A	0	15	0
60 - 70	65	10	12	120
70 - 80	75	20	6	120
80 - 90	85	30	2	60
90 - 100	95	40	8	320
Total			**80**	**– 260**

1

$$\bar{d} = \frac{p f_i d_i}{p f_i}$$

y $$\bar{d} = \frac{-260}{80}$$

y $\bar{d} = -3.25$ **1**

Mean $(\bar{x}) = A + \bar{d}$

$= 55 + (-3.25)$

$= 55 - 3.25$

$= 51.75$

y Mean of marks obtained 51.75 marks. **1**

(iii) $$x^2 + \frac{12}{x^2} = 7$$

Multiplying throughout by x^2 we get,

$x^4 + 12 = 7x^2$

y $(x^2)^2 - 7x^2 + 12 = 0$

Substituting $x^2 = m$ we get,

$m^2 - 7m + 12 = 0$ **1**

y $m^2 - 4m - 3m + 12 = 0$

y $m(m-4) - 3(m-4) = 0$

y $(m-4)(m-3) = 0$

y $m - 4 = 0$ or $m - 3 = 0$

y $m = 4$ or $m = 3$ **1**

Resubstituting $m = x^2$ we get,

$x^2 = 4$ or $x^2 = 3$

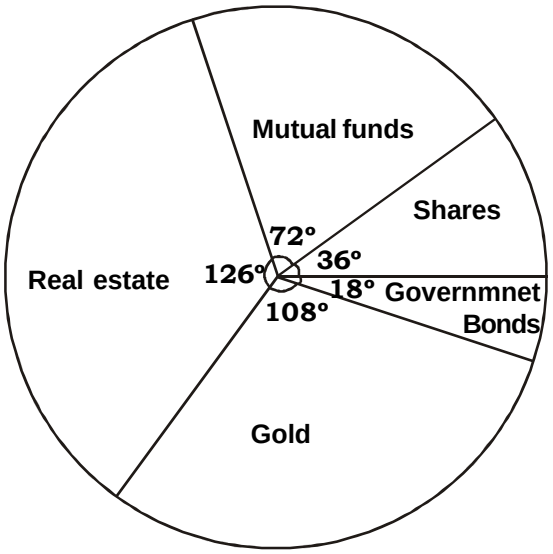
Taking square root on both the sides we get,

$x = \pm 2$ or $x = \pm\sqrt{3}$ **1**

(iv)

Mode of investment	**Percentage of residents**	**Measure of central angle**
Shares	10	$\frac{10}{100} \times 360^\circ = 36^\circ$
Mutual funds	20	$\frac{20}{100} \times 360^\circ = 72^\circ$
Real estate	35	$\frac{35}{100} \times 360^\circ = 126^\circ$
Gold	30	$\frac{30}{100} \times 360^\circ = 108^\circ$
Government bonds	5	$\frac{5}{100} \times 360^\circ = 18^\circ$
Total	**100**	**360°**

1



2

(v) Let ~ and be the roots of a quadratic equation.

$\because$ ~ + = 3 and ~³ + ³ = 63 [Given]

We know that,

$x^2 - (\sim +)x + \sim. = 0$ .....(i) **½**

Also, ~³ + ³ = (~ +)³ – 3 ~. (~ +)

y 63 = (3)³ – 3 ~. (3) [$\because$ ~ + = 3 and ~³ + ³ = 63] **½**

y 63 = 27 – 9 ~.

y 9 ~. = 27 – 63

y 9 ~. = – 36

y $\sim. = \frac{-36}{9}$

y ~. = – 4 **1**

y $x^2 - (\sim +)x + \sim. = 0$ [From (i)]

y $x^2 - 3x + (-4) = 0$ [$\because$ ~ + = 3 and ~. = – 4]

y $x^2 - 3x - 4 = 0$

y The required quadratic equation is $x^2 - 3x - 4 = 0$. **1**

A.4. Solve the following : (Any 2)

(i) Let the other natural number be x.

y The first natural number is x + 5.

As per the given condition,

$x^2 + (x + 5)^2 = 73$ **1**

y $x^2 + x^2 + 10x + 25 - 73 = 0$

y $2x^2 + 10x - 48 = 0$

Dividing throughout by 2 we get,

$x^2 + 5x - 24 = 0$ **1**

y $x^2 - 3x + 8x - 24 = 0$

y $x(x - 3) + 8(x - 3) = 0$

y $(x - 3)(x + 8) = 0$

y $x - 3 = 0$ or $x + 8 = 0$

y $x = 3$ or $x = -8$ **1**

∵ The number is a natural number

y $x \div -8$

Hence x = 3

And x + 5 = 3 + 5 = 8

y The natural numbers are 3 and 8. **1**

(ii)

No. of rooms occupied	**Frequency (f_i) (No. of days)**	**Cumulative frequency less than type**
0 - 10	5	5
10 - 20	27	32 ← *c.f.*
20 - 30	17 ← *f*	49
30 - 40	11	60
40 - 50	9	69
50 - 60	1	70
Total	**70 ← N**	

1

Here total frequency = $\sum f_i = N = 70$

y $\frac{N}{2} = \frac{70}{2} = 35$

Cumulative frequency (less than type) which is just greater than 35 is 49. Therefore corresponding class 20 - 30 is median class.

L = 20, N = 70, *c.f.* = 32, f = 17, h = 10 **1**

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$

$$= 20 + \left(\frac{70}{2} - 32\right)\frac{10}{17}$$ **1**

$$= 20 + (35 - 32)\frac{10}{17}$$

$$= 20 + \frac{30}{17}$$

$$= 20 + 1.76$$

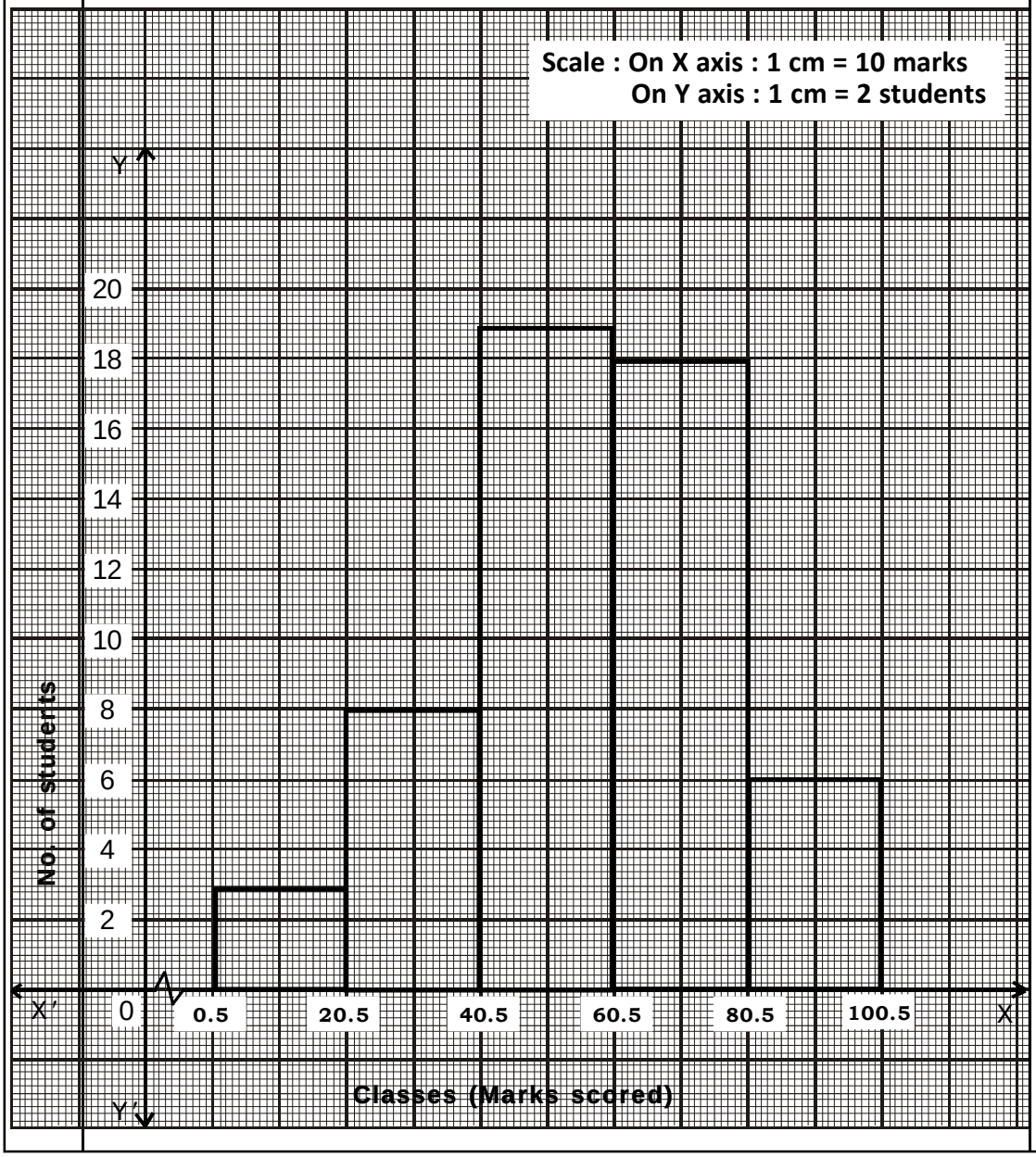
$$= 21.76$$

y Median of rooms occipied is 21.76 rooms. **1**

(iii)

Marks obtained	Continuous classes No. of students	Frequency
1 - 20	0.5 - 20.5	3
21 - 40	20.5 - 40.5	8
41 - 60	40.5 - 60.5	19
61 - 80	60.5 - 80.5	18
81 - 100	80.5 - 100.5	6

1



3

A.5. **Solve the following : (Any 2)**

(i) Let the original speed of car be x km/hr.

∵ Distance covered is 240 km

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

y $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

y Time taken by car = $\left(\frac{240}{x}\right)$ hrs

New speed of car = (x + 20) km/hr **1**

y New time taken by car = $\left(\frac{240}{x+20}\right)$ hrs **1**

As per the given condition,

$$\frac{240}{x} - \frac{240}{x+20} = 2$$

y $$240\left[\frac{1}{x} - \frac{1}{x+20}\right] = 2$$

y $$\frac{x+20-x}{x(x+20)} = \frac{2}{240}$$

y $$\frac{20}{x^2+20x} = \frac{1}{120}$$

y $$20\,(120) = 1\,(x^2+20x)$$

y $$2400 = x^2+20x$$ **1**

y $$x^2+20x-2400 = 0$$

y $$x^2+60x-40x-2400 = 0$$

y $$x(x+60)-40(x+60) = 0$$

y $$(x+60)(x-40) = 0$$

y $$x+60=0 \quad \text{or} \quad x-40=0$$

y $$x=-60 \quad \text{or} \quad x=40$$ **1**

∵ The speed of car can never be negative.

y $x \div -60$

Hence x = 40

y The original speed of car is 40 km/hr. **1**

(ii) Class width (h) = 15, Assumed mean (A) = 52

Duration of call in sec.	**Class Mark (x_i)**	**$d_i = x_i - A$**	**$u_i = \frac{d_i}{h}$**	**No. of calls (f_i)**	**$f_i u_i$**
15 - 29	22	– 30	– 2	8	– 16
30 - 44	37	– 15	– 1	12	– 12
45 - 59	52 ← A	0	0	61	0
60 - 74	67	15	1	153	153
75 - 89	82	30	2	190	380
90 - 104	97	45	3	57	171
105 - 119	112	60	4	10	40
120 - 134	127	75	5	9	45
Total				**500**	**761**

2

$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$$

$$\bar{u} = \frac{761}{500}$$

$$\bar{u} = 1.522$$ **1**

$$\text{Mean } (\bar{x}) = A + h\bar{u}$$

$$= 52 + 15\,(1.522)$$ **1**

$$= 52 + 22.83$$

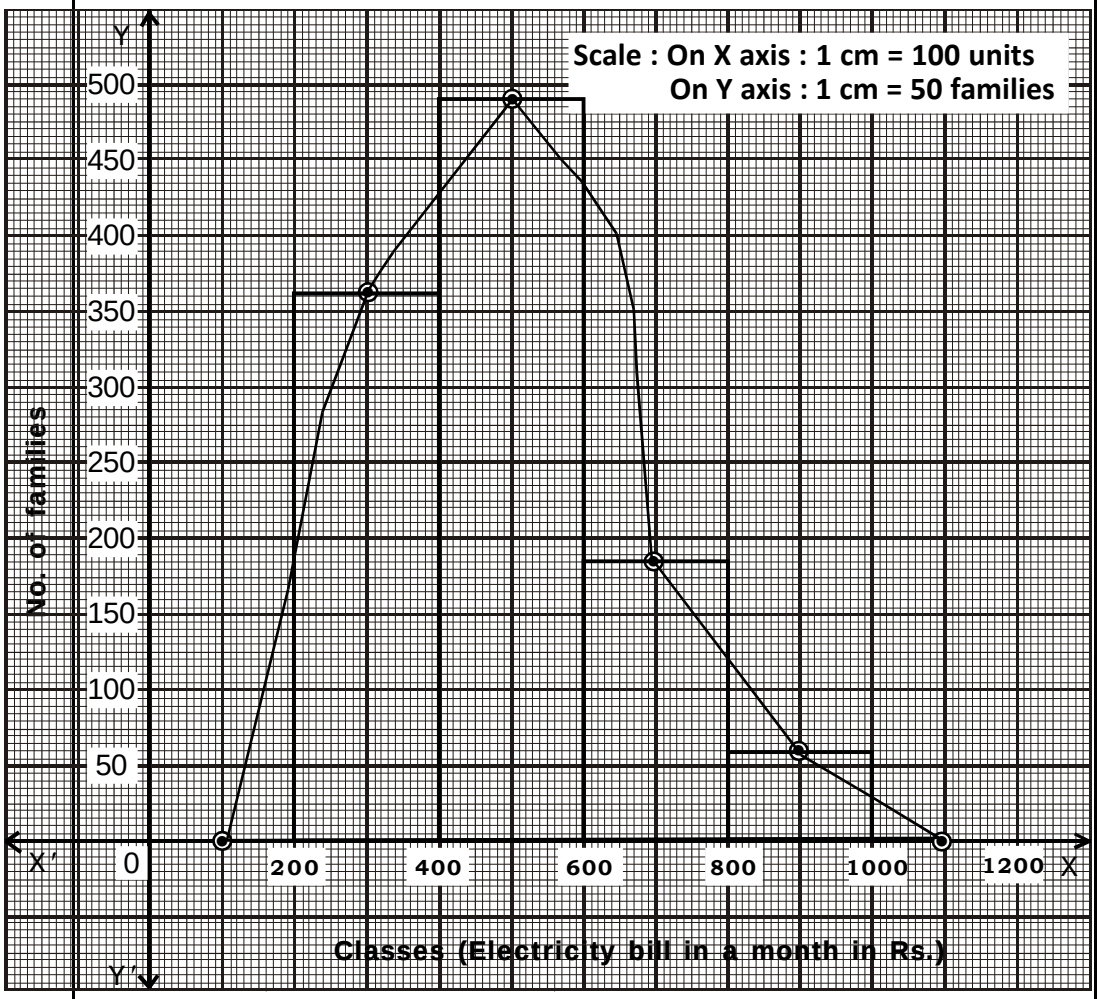
$$= 74.83$$

Mean of call duration is 74.83 second. **1**

(iii)

Electricity bill in a month in Rs.	**Class mark**	**No. of families**
200 - 400	300	362
400 - 600	500	490
600 - 800	700	185
800 - 1000	900	63

1


Scale : On X axis : 1 cm = 100 units
On Y axis : 1 cm = 50 families
Y
500
450
400
350
300
250
200
150
100
50
0
Y′
X′
X
200
400
600
800
1000
1200
No. of families
Classes (Electricity bill in a month in Rs.)

4

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