MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - I - PAPER - 6 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Solve the following : (Any 5)

(i) Let natural number be 'x'

y Its reciprocal is $\frac{1}{x}$

From the given condition,

$x + \frac{1}{x} = \frac{10}{3}$ **1**

(ii) The roots of the quadratic equation are 0 and – 6

Let ~ = 0 and = – 6

y ~ + = 0 + (– 6) = 0 – 6 = – 6 **½**

~ . = 0 × – 6 = 0

We know that,

$x^2 - (\sim +) x + \sim . = 0$

y $x^2 - (- 6) x + 0 = 0$

y $x^2 + 6x = 0$

y The required quadratic equation is $x^2 + 6x = 0$. **½**

(iii) If one of the root of the quadratic equation is $3 - 2\sqrt{3}$ then the other root is $3 + 2\sqrt{3}$

Let ~ = $3 - 2\sqrt{3}$ and = $3 + 2\sqrt{3}$

y ~ + = $3 - 2\sqrt{3} + 3 + 2\sqrt{3} = 6$ **½**

~ . = $\left(3 - 2\sqrt{3}\right) \times \left(3 + 2\sqrt{3}\right)$

= $(3)^2 - \left(2\sqrt{3}\right)^2$

= 9 – 12

= – 3 **½**

(iv) $pf_ix_i = 595$, $pf_i = 25$

Mean $\left(\bar{x}\right) = \frac{pf_ix_i}{pf_i}$

$= \frac{595}{25}$

= 23.8 **1**

(v)

No. of rooms occupied	Frequency (f_i) (No. of days)	Cumulative frequency less than type
0 - 10	5	5
10 - 20	27	32 ← *c.f.*
20 - 30	17 ← f	49
30 - 40	11	60
40 - 50	9	69
50 - 60	1	70
Total	**70 ← N**	

Here total frequency = $\sum f_i$ = N = 70

∴ $\frac{N}{2} = \frac{70}{2} = 35$

Cumulative frequency (less than type) which is just greater than 35 is 49. Therefore corresponding class 20 - 30 is median class.

L = 20, N = 70, *c.f.* = 32, f = 17, h = 10 **1**

(vi) (a) The item with maximum expenditure is labour. **½**

(b) The item with minimum expenditure is steel. **½**

A.2. Solve the following : (Any 4)

(i) $x^2 + 3\sqrt{2}x - 8 = 0$

Comparing with $ax^2 + bx + c = 0$ we have a = 1, b = $3\sqrt{2}$, c = – 8 **½**

$a = b^2 - 4ac$

$= (3\sqrt{2})^2 - 4\ (1)\ (-8)$

$= 9 \times 2 + 32$

$= 18 + 32$

$= 50$ **½**

∴ $a > 0$

Hence roots of the quadratic equation are real and unequal. **1**

(ii) Class width (h) = 10

Money (in Rs.) (x_i)	Class Mark (f_i)	No. of students	$f_i x_i$
0 - 10	5	5	25
10 - 20	15	7	105
20 - 30	25	5	125
30 - 40	35	2	70
40 - 50	45	6	270
Total		**25**	**595**

1

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

y Mean = $\frac{595}{25}$

y Mean = Rs. 23.8

y Mean of money collected is Rs. 23.8 **1**

(iii) If one of the root of the quadratic equation is $\sqrt{2} + \sqrt{3}$, then the other root is $\sqrt{2} - \sqrt{3}$

Let $\alpha = \sqrt{2} + \sqrt{3}$ and $\beta = \sqrt{2} - \sqrt{3}$

y $\alpha + \beta = \sqrt{2} + \sqrt{3} + \sqrt{2} - \sqrt{3} = 2\sqrt{2}$ **½**

and $\alpha\beta = \left(\sqrt{2} + \sqrt{3}\right) \times \left(\sqrt{2} - \sqrt{3}\right)$

$= \left(\sqrt{2}\right)^2 - \left(\sqrt{3}\right)^2$

$= 2 - 3$

$= -1$ **½**

We know that,

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

y $x^2 - 2\sqrt{2}\,x + (-1) = 0$

y $x^2 - 2\sqrt{2}\,x - 1 = 0$

y The required quadratic equation is $x^2 - 2\sqrt{2}\,x - 1 = 0$ **1**

(iv)

No. of packages	No. of days
10 - 20	2
20 - 30	8
30 - 40	16
40 - 50	24 ← f_1
50 - 60	30 ← f_m
60 - 70	20 ← f_2

Here the maximum frequency $f_m = 30$

The corresponding class 50 - 60 is the modal class. **½**

L = 50, $f_m = 30$, $f_1 = 24$, $f_2 = 20$, h = 10

Mode = $L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$

= $50 + \left(\frac{30 - 24}{2(30) - 24 - 20}\right) \times 10$ **½**

= $50 + \left(\frac{6}{60 - 44}\right) \times 10$

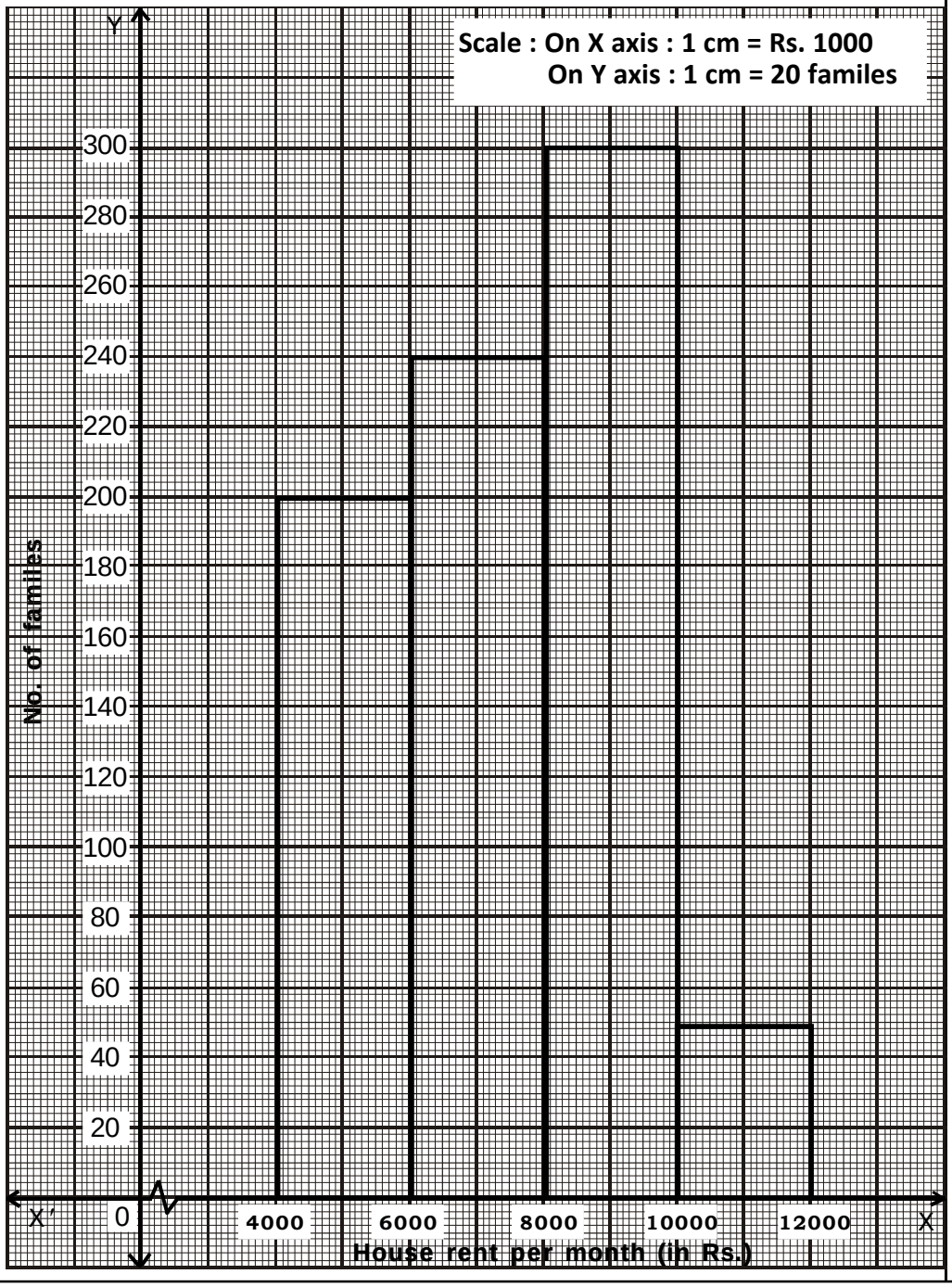
$= \quad 50 + \left(\frac{6}{16}\right) \times 10$

$= \quad 50 + \frac{15}{4}$

$= \quad 50 + 3.75$

$= \quad 53.75$ **1**

(v)



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(vi) Let total sales done be Rs. x
Sales done due to salesman A = Rs. 18000
Measure of central angle for salesman A = 90°
As per the given condition,

$\frac{90}{360} \times x = 18000$

$x = \frac{18000 \times 360}{90}$

$x = 18000 \times 4$

$x = 72000$

Total sales done is Rs. 72000 **1**

Salesman	**Measure of central angle**	**Sales done in Rs.**
A	90°	18000
B	120°	$\frac{120}{360} \times 72000 = 24000$
C	80°	$\frac{80}{360} \times 72000 = 16000$
D	70°	$\frac{70}{360} \times 72000 = 14000$
Total	**360°**	**72000** **1**

A.3. Solve the following : (Any 3)

(i) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = k - 12$, $b = 2(k - 12)$, $c = 2$ **½**

$a = b^2 - 4ac$

$= [2(k - 12)]^2 - 4(k - 12)(2)$ **½**

$= (2k - 24)^2 - 8(k - 12)$

$= 4k^2 - 96k + 576 - 8k + 96$

$= 4k^2 - 104k + 672$ **½**

∵ The roots of given equation are real and equal.

∴ a must be zero.

$4k^2 - 104k + 672 = 0$

$4(k^2 - 26k + 168) = 0$

$k^2 - 14k - 12k + 168 = 0$

$k(k - 14) - 12(k - 14) = 0$

$(k - 14)(k - 12) = 0$

$k - 14 = 0$ or $k - 12 = 0$

$k = 14$ or $k = 12$ **1**

If k = 12

= 12 – 12[$\because$ a = k – 12]

a = 0

But a $\neq$ 0

y k = 12 is not acceptable.

y k = 14 ½

(ii) Class width (h) = 3, Assumed mean (A) = 40

Diameter (in mm)	**Class Mark (x_i)**	**$d_i = x_i - A$**	**No. of screws (f_i)**	**$f_i d_i$**
33 - 35	34	– 6	10	– 60
36 - 38	37	– 3	19	– 57
39 - 41	40 ê÷A	0	23	0
42 - 44	43	3	21	63
45 - 47	46	6	27	162
Total			**100**	**108**

1

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$$

y $$\bar{d} = \frac{108}{100}$$

y $\bar{d} = 1.08$ **1**

Mean $(\bar{x})$ = $A + \bar{d}$

= 40 + 1.08

= 41.08

y Mean of diameter of screws is 41.08 mm. **1**

(iii) $$6m^2 + \frac{2}{m^2} = 7$$

Multiplying throughout by m^2, we get

$6m^4 + 2 = 7m^2$

y $6(m^2)^2 - 7m^2 + 2 = 0$

Substituting $m^2 = x$ we get,

y $6x^2 - 7x + 2 = 0$ **1**

y $6x^2 - 3x - 4x + 2 = 0$

y $3x(2x - 1) - 2(2x - 1) = 0$

y $(2x - 1)(3x - 2) = 0$

y $2x - 1 = 0$ or $3x - 2 = 0$

y $2x = 1$ or $3x = 2$

y $x = \frac{1}{2}$ or $x = \frac{2}{3}$ **1**

Resubstituting $x = m^2$ we get,

$$m^2 = \frac{1}{2} \quad \text{or} \quad m^2 = \frac{2}{3}$$

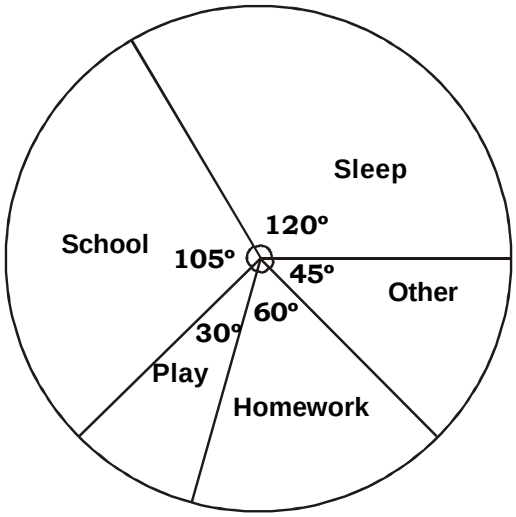
Taking square root on both the sides we get,

$$m = \pm \frac{1}{\sqrt{2}} \quad \text{or} \quad m = \pm \sqrt{\frac{2}{3}}$$ **1**

(iv)

Activity	**No. of hrs.**	**Measure of central angle (°)**
Sleep	8	$\frac{8}{24} \times 360^\circ = 120^\circ$
School	7	$\frac{7}{24} \times 360^\circ = 105^\circ$
Play	2	$\frac{2}{24} \times 360^\circ = 30^\circ$
Home work	4	$\frac{4}{24} \times 360^\circ = 60^\circ$
Other	3	$\frac{3}{24} \times 360^\circ = 45^\circ$
Total	**24**	**360°**

1



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(iv) ~ and are the roots of a quadratic equation

$\because$ ~ + = 5 and $\sim^3 + {}^3 = 35$ [Given]

We know that,

$x^2 - (\sim +)\, x + \sim . = 0$ .....(i) **½**

Also, $\sim^3 + {}^3 = (\sim +)^3 - 3 \sim . (\sim +)$

y $35 = (5)^3 - 3 \sim . (5)$ [$\because$ ~ + = 5 and $\sim^3 + {}^3 = 35$] **½**

y $35 = 125 - 15\ \sim.$

y $15\ \sim. = 125 - 35$

y $15\ \sim. = 90$

y $\sim . = \frac{90}{15}$

y $\sim . = 6$ **1**

∵ $x^2 - (\sim +)x + \sim. = 0$ [From (i)]

y $x^2 - 5x + 6 = 0$

y The required quadratic equation is $x^2 - 5x + 6 = 0$. **1**

A.4. Solve the following : (Any 2)

(i) Let the breadth of the rectangle be 'x' cm.

∴ It's length is (x + 2) cm.

As per the given condition.

∵ Area of rectangle = Length × Breadth

∴ $24 = (x + 2) \times x$ **1**

∴ $24 = x^2 + 2x$

∴ $0 = x^2 + 2x - 24$

∴ $x^2 + 2x - 24 = 0$ **1**

∴ $x^2 - 4x + 6x - 24 = 0$

∴ $x(x - 4) + 6(x - 4) = 0$

∴ $(x - 4)(x + 6) = 0$

∴ $x - 4 = 0$ or $x + 6 = 0$

∴ $x = 4$ or $x = -6$ **1**

∵ The breadth of rectangle cannot be negative.

∴ $x \neq -6$

Hence x = 4 and x + 2 = 4 + 2 = 6

∴ The length of rectangle is 6 cm and its breadth is 4 cm. **1**

(ii)

Calories (No. of boys)	**Frequency (f_i) less than type**	**Cumulative frequency**
1000 - 1500	5	5
1500 - 2000	13	18
2000 - 2500	16	34 ← *c.f.*
2500 - 3000	18 ← *f*	52
3000 - 3500	27	79
3500 - 4000	10	89
4000 - 4500	4	93
Total	**93 ← N**	

1

Here total frequency = $p f_i = N = 93$

y $\frac{N}{2} = \frac{93}{2} = 46.5$

Cumulative frequency (less than type) which is just greater than 46.5 is 52. Therefore corresponding class 2500 - 3000 is median class.

L = 2500, N = 93, *c.f.* = 34, f = 18, h = 500 **1**

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$

$$= 2500 + \left(\frac{93}{2} - 34\right)\frac{500}{18}$$

$$= 2500 + (46.5 - 34)\ \frac{500}{18}$$ **1**

$$= 2500 + (12.5)\ \frac{500}{18}$$

$$= 2500 + \frac{6250}{18}$$

$$= 2500 + 347.22$$

$$= 2847.22$$

y Median of calories consumed by boys is 2847.22 calories. **1**

(iii)

Class marks	**Classes No. of students**	**Frequency**
5	0 - 10	0
15	10 - 20	2
25	20 - 30	16
35	30 - 40	36
45	40 - 50	27
55	50 - 60	11
65	60 - 70	8
75	70 - 80	0

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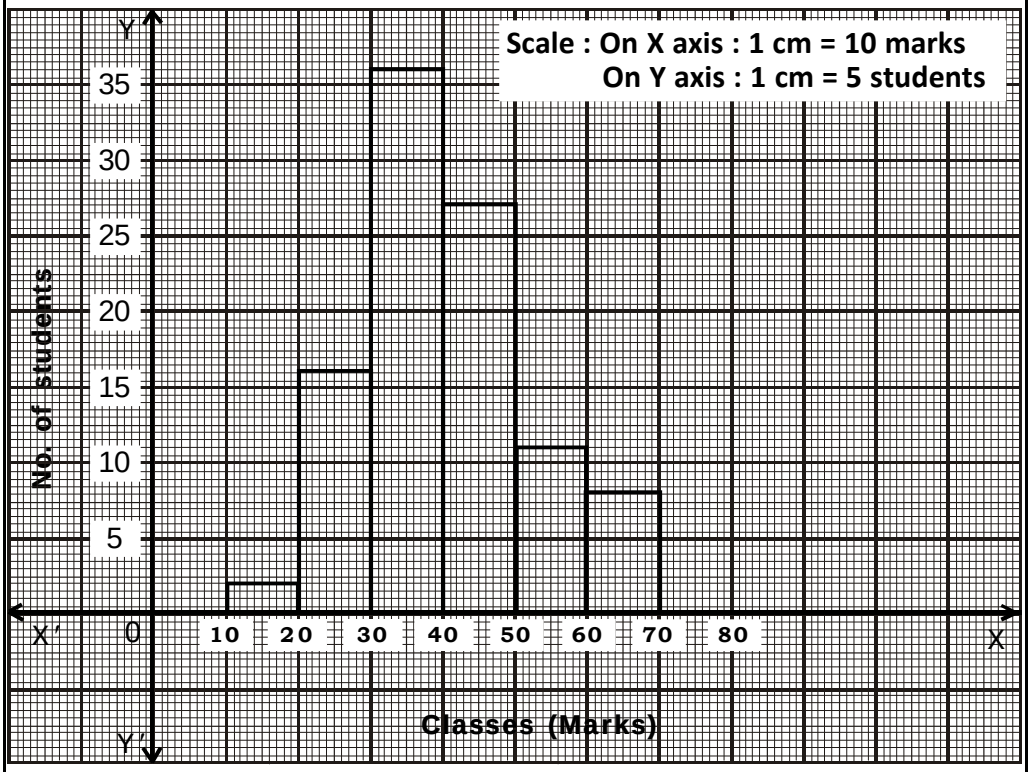

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A.5. Solve the following : (Any 2)

(i) Number of sides of one of the regular polygon = n

Number of sides of the other regular polygon = n^2

y Exterior angle of a polygon having 'n' sides = $\left(\frac{360}{n}\right)^\circ$

Exterior angle of a polygon having 'n^2' sides = $\left(\frac{360}{n^2}\right)^\circ$ **1**

As per the given condition,

$$\frac{360}{n} - \frac{360}{n^2} = 50$$ **1**

y $$360\left(\frac{1}{n} - \frac{1}{n^2}\right) = 50$$

y $$\frac{n-1}{n^2} = \frac{50}{360}$$

y $$\frac{n-1}{n^2} = \frac{5}{36}$$

y $$36\,(n-1) = 5\,(n^2)$$

y $$36n - 36 = 5n^2$$

y $0 = 5n^2 - 36n + 36$ **1**

y $5n^2 - 36n + 36 = 0$

y $5n^2 - 30n - 6n + 36 = 0$

y $5n(n - 6) - 6(n - 6) = 0$

y $(n - 6)(5n - 6) = 0$

y $n - 6 = 0$ or $5n - 6 = 0$

y $n = 6$ or $5n = 6$

y $n = 6$ or $n = \frac{6}{5}$ **1**

$\because$ 'n' is the number of sides of any polygon

y $n \div \frac{6}{5}$

Hence n = 6

y $n^2 = (6)^2 = 36$

y The number of sides of required regular polygons are 6 and 36 cm respectively. **1**

(ii) Class width (h) = 10, Assumed mean (A) = 44.5

Dividend (in %)	**Class Mark (x_i)**	**$d_i = x_i - A$**	**$u_i = \frac{d_i}{h}$**	**No. of companies (f_i)**	**$f_i u_i$**
10 - 19	14.5	– 30	– 3	5	– 15
20 - 29	24.5	– 20	– 2	15	– 30
30 - 39	34.5	– 10	– 1	28	– 28
40 - 49	44.5 ← A	0	0	42	0
50 - 59	54.5	10	1	15	15
60 - 69	64.5	20	2	12	24
70 - 79	74.5	30	3	3	9
Total				**120**	**– 25**

2

$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$$

y $\bar{u} = \frac{-25}{120}$

y $\bar{u} = -0.208$ **1**

Mean $(\bar{x}) = A + h\bar{u}$ **1**

$= 44.5 + 10(-0.208)$

$= 44.5 - 2.08$

$= 42.42$

y Mean of dividend is 42.42%. **1**

(iii)

No. of words typed per minute	Continuous Classes	Frequency No. of typists
30 - 39	29.5 - 39.5	2
40 - 49	39.5 - 49.5	8
50 - 59	49.5 - 59.5	15
60 - 69	59.5 - 69.5	12
70 - 79	69.5 - 79.5	3

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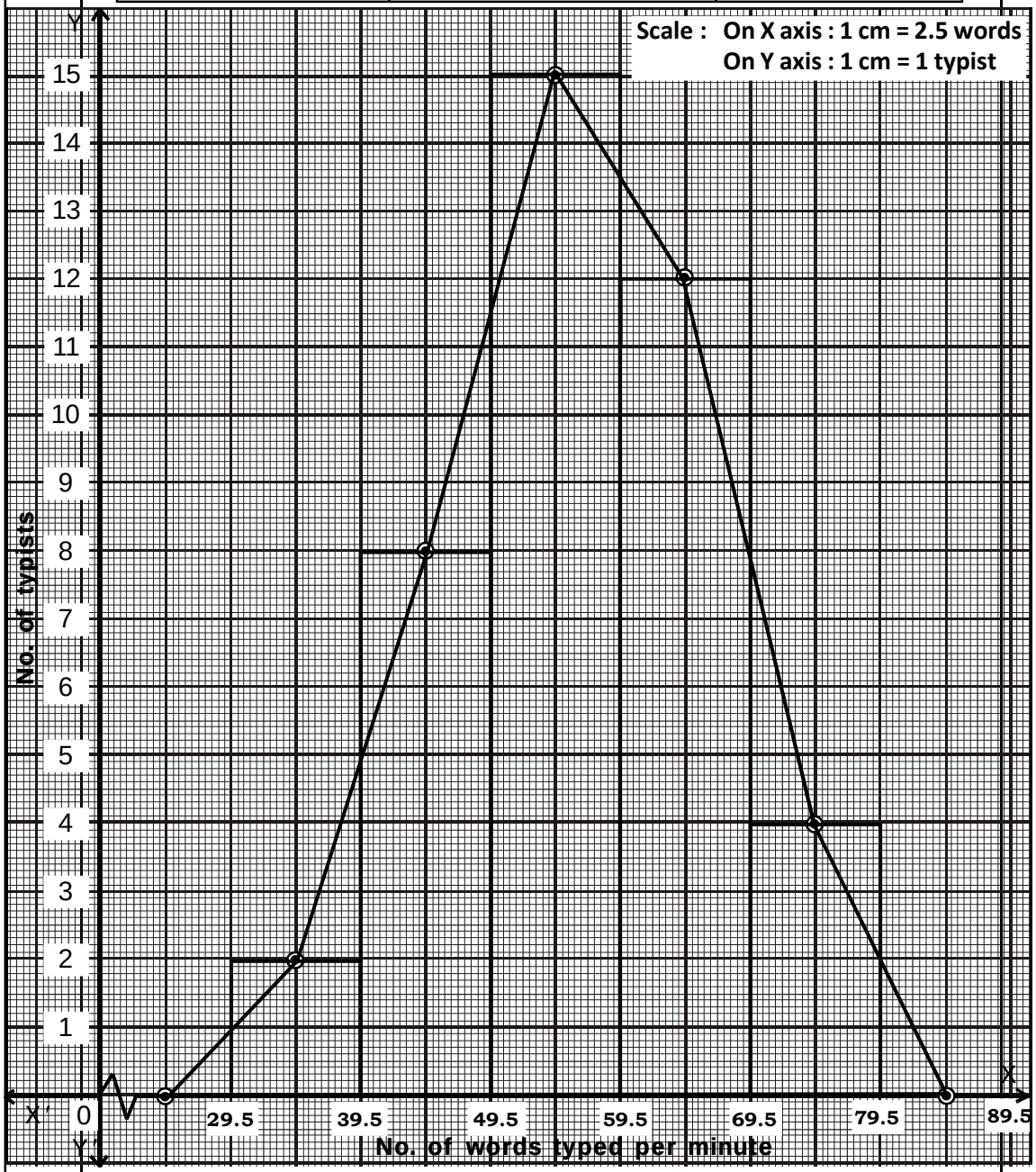

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