MT

2017 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 1

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) Slope of the line (m) = 5

y intercept of the line (c) = – 1 ½

By slope intercept form,

The equation of the line is y = mx + c

∴ y = 5 (x) + (–1)

∴ y = 5x – 1

∴ The equation of the given line is y = 5x – 1 ½

(ii) $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= \cot^2\theta - \operatorname{cosec}^2\theta$ ½

$= -1$ [∵ $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ ½

∴ $\cot^2\theta - \operatorname{cosec}^2\theta = -1$]

(iii) Equation of a line parallel to X-axis and passing through the point (5, –7) is y = – 7

(iv) $3\sin\theta - 4\cos\theta = 0$

$3\sin\theta = 4\cos\theta$ ½

$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\tan\theta = \frac{4}{3}$ ½

(v) y – 5 = 2 (x – 7)

Comparing with the equation of a line in slope point form,

$y - y_1 = m(x - x_1)$ ½

∴ m = 2

∴ Slope of the line y – 5 = 2 (x – 7) is 2 ½

(vi) $\sim + = 90^\circ$ [Given]

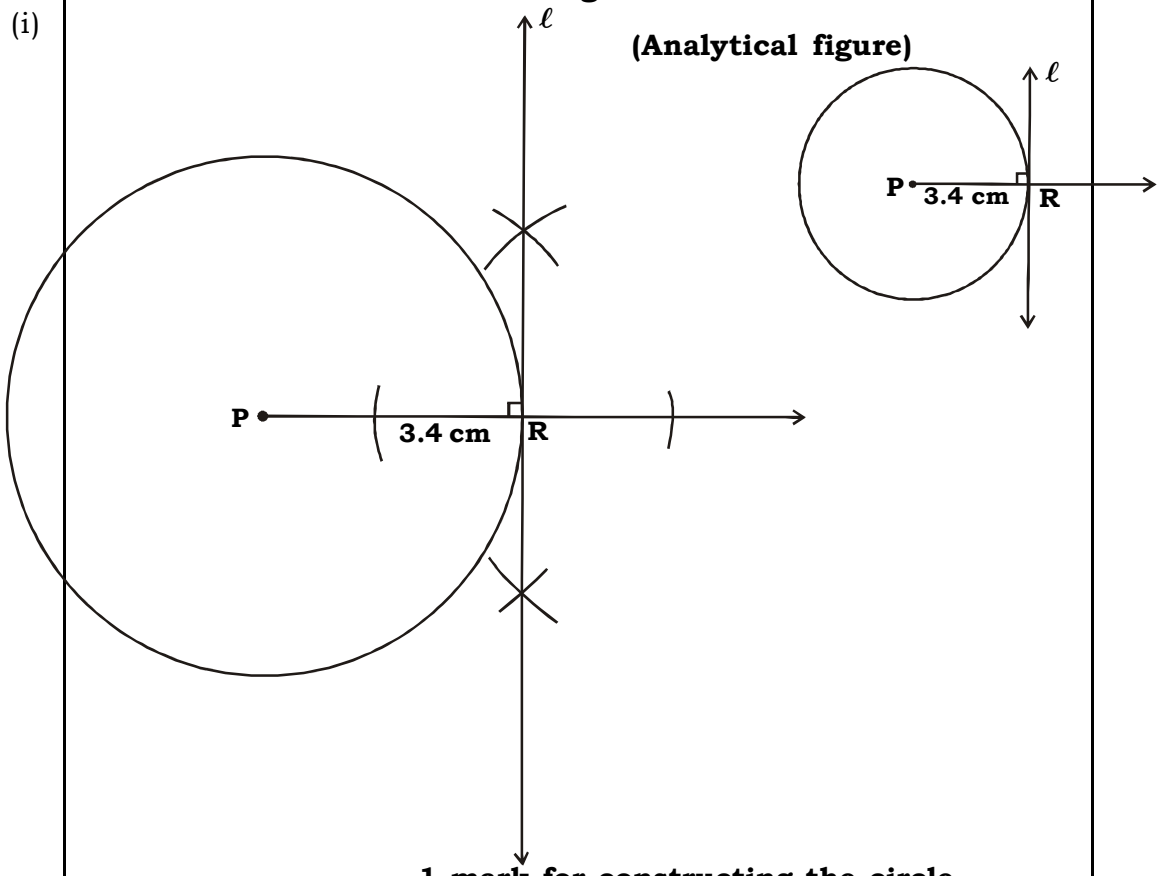
$\tan \sim = \frac{3}{4}$ [Given]

$\cot = \tan \sim$ [$\because \cot \theta = \tan(90 - \theta)$] ½

y $\boxed{\cot = \frac{3}{4}}$ ½

A.2. Solve ANY FOUR of the following :

(i)



1 mark for constructing the circle
1 mark for constructing the tangent at R

(ii) The terminal arm passes through P (4, 3)

y x = 4 and y = 3

$r = \sqrt{x^2 + y^2}$ ½

$= \sqrt{(4)^2 + (3)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

y r = 5 units ½

Let the angle be θ

y $\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\quad \operatorname{cosec}\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{4}{5}$ $\quad \sec\theta = \frac{r}{x} = \frac{5}{4}$ **1**

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\quad \cot\theta = \frac{x}{y} = \frac{4}{3}$

(iii) Let O ø (0, 0) ø (x_1, y_1)

A ø (– 3, 5) ø (x_2, y_2)

y The equation of line OA by two point form is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

y $\frac{x - 0}{0 - (-3)} = \frac{y - 0}{0 - 5}$ ½

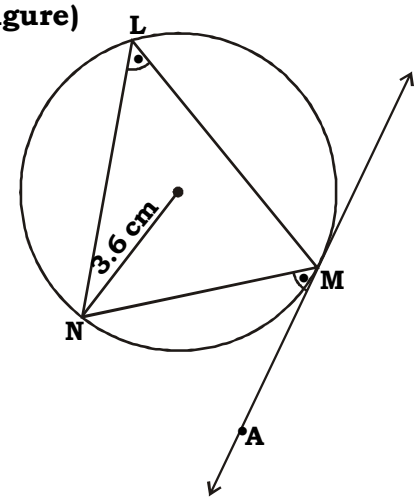
y $\frac{x}{3} = \frac{y}{-5}$ ½

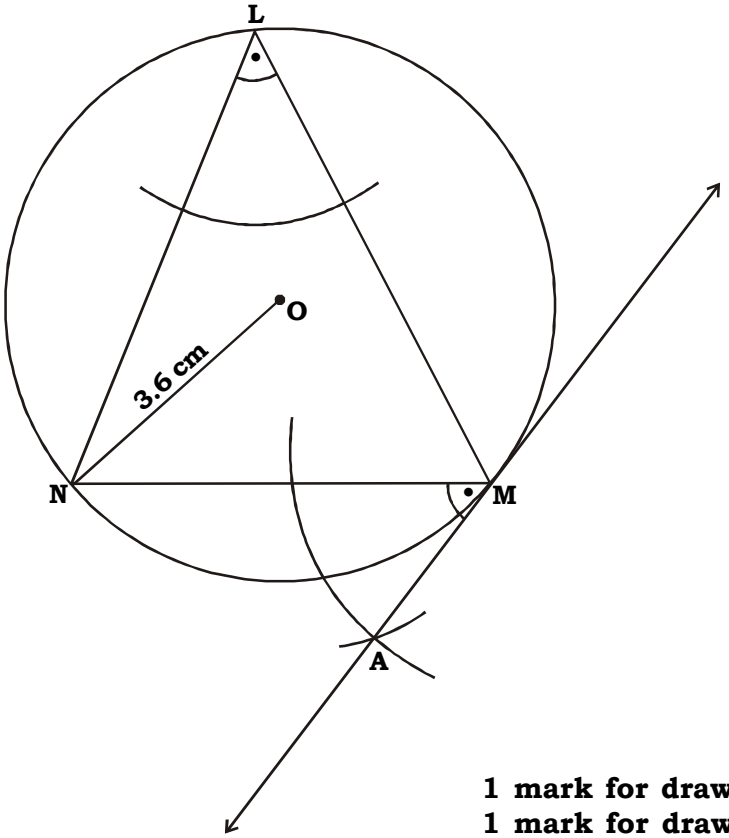
y $-5x = 3y$

y $5x + 3y = 0$

y The equation of the line passing through the origin and the point (– 3, 5) is $5x + 3y = 0$. ½

(iv) **(Analytical figure)**





1 mark for drawing circle
1 mark for drawing tangent

(v)

		$\sin^{-} + \sin^{2-} =$	1	[Given]	
y		$\sin^{-} =$	$1 - \sin^{2-}$		½
y		$\sin^{-} =$	$\cos^{2-}$	.....(i)	
y		$\sin^{2-} =$	$\cos^{4-}$		½
y		$1 - \cos^{2-} =$	$\cos^{4-}$		½
		$1 =$	$\cos^{2-} + \cos^{4-}$		
y		$\cos^{2-} + \cos^{4-} =$	1		½

(vi)

P ø (2, 4), Q ø (3, 6), R ø (8, 1), S ø (10, k)

Line PQ is parallel to line RS ½

y Slope of line PQ = Slope of line RS ½

y $\dfrac{6-4}{3-2} = \dfrac{k-1}{10-8}$

y $\dfrac{2}{1} = \dfrac{k-1}{2}$ ½

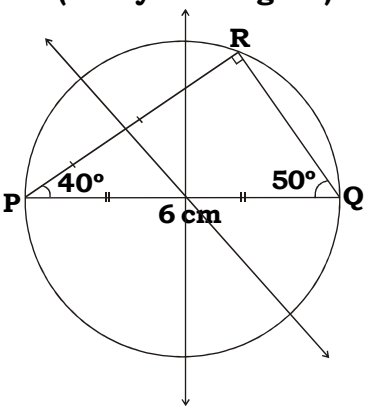
y 4 = k – 1
y k = 4 + 1
y k = 5 ½
y Value of k is 5.

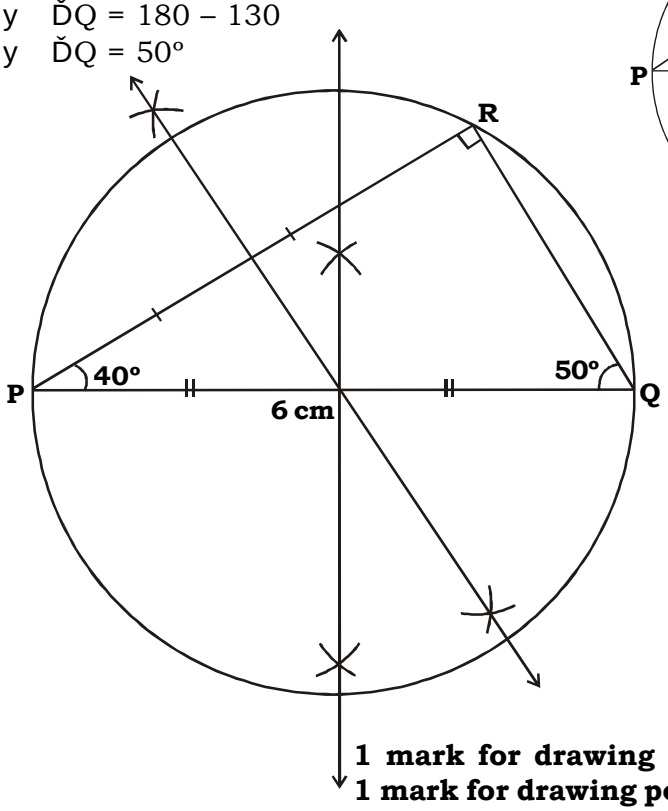
A.3. **Solve ANY THREE of the following :**

(i) **Analysis :**

In aPQR,
ĎP + ĎQ + ĎR = 180
y 40 + ĎQ + 90 = 180
y ĎQ = 180 – 130
y ĎQ = 50°

(Analytical figure)





1 mark for drawing aPQR
1 mark for drawing perpendicular bisectors
1 mark for drawing circumcircle

(ii) $\cot \theta = -\frac{7}{24}$

y $\tan \theta = -\frac{24}{7}$ $\left[\because \tan \theta = \frac{1}{\cot \theta}\right]$ ½

$1 + \tan^2 \theta = \sec^2 \theta$ ½

y $1 + \left(-\frac{24}{7}\right)^2 = \sec^2 \theta$

y $1 + \frac{576}{49} = \sec^2 -$

y $\frac{49 + 576}{49} = \sec^2 -$

y $\frac{625}{49} = \sec^2 -$

y $\boxed{\sec - = \frac{25}{7}}$ ½

y $\cos - = \frac{7}{25}$ $\left[\because \cos - = \frac{1}{\sec -}\right]$ ½

$\frac{\sin -}{\cos -} = \tan -$ ½

y $\sin - = \tan - \times \cos -$

y $\sin - = \frac{-24}{7} \times \frac{7}{25}$

y $\boxed{\sin - = \frac{-24}{25}}$ ½

(iii) V ø (– 7, 8) ø (x_1, y_1), W ø (– 5, 2) ø (x_2, y_2), U ø (3, 6) ø (x_3, y_3)

Slope of line VW $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

$= \frac{2 - 8}{-5 - (-7)}$ ½

$= \frac{-6}{-5 + 7}$

$= \frac{-6}{2}$

$= -3$ ½

Slope of line WU $= \frac{y_3 - y_2}{x_3 - x_2}$ ½

$= \frac{6 - 2}{3 - (-5)}$ ½

$= \frac{4}{3 + 5}$

$= \frac{4}{8}$

$= \frac{1}{2}$ ½

y Slope of line VW and slope of line WU are not equal.

y **Points V, W and U are not collinear.**

(iv)

$8 \sin x - \cos x = 4$

y $8 \sin x - 4 = \cos x$ ..... (i) ½

y $\sin^2 x + \cos^2 x = 1$

y $\sin^2 x + (8 \sin x - 4)^2 = 1$ [From (i)] ½

y $\sin^2 x + 64 \sin^2 x - 64 \sin x + 16 = 1$

y $\sin^2 x + 64 \sin^2 x - 64 \sin x + 16 - 1 = 0$

y $65 \sin^2 x - 64 \sin x + 15 = 0$ ½

y $64 \sin^2 x + \sin^2 x - 64 \sin x + 16 - 1 = 0$

y $65 \sin^2 x - 64 \sin x + 15 = 0$

y $65 \sin^2 x - 39 \sin x - 25 \sin x + 15 = 0$ ½

y $13 \sin x (5 \sin x - 3) - 5 (5 \sin x - 3) = 0$

y $(5 \sin x - 3)(13 \sin x - 5) = 0$

y $5 \sin x - 3 = 0$ or $13 \sin x - 5 = 0$

y $5 \sin x = 3$ or $13 \sin x = 5$

y $\boxed{\sin x = \frac{3}{5}}$ or $\boxed{\sin x = \frac{5}{13}}$ **1**

(v) Let, A ≡ (– 1, 1), B ≡ (– 9, 6), C ≡ (– 2, 14), D ≡ (6, 9)

Slope of a line $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

Slope of line AB $= \frac{6 - 1}{-9 - (-1)}$

$= \frac{5}{-9 + 1}$

$= \frac{5}{-8}$

y Slope of line AB $= \frac{-5}{8}$ ½

Slope of line CD $= \frac{9 - 14}{6 - (-2)}$ ½

$= \frac{-5}{6 + 2}$

y Slope of line CD $= \frac{-5}{8}$ ½

y Slope of line AB and slope of line CD are equal.

y line AB || line CD

y **The line joining (– 1, 1) and (– 9, 6) is parallel to the line joining (– 2, 14) and (6, 9).** **1**

A.4. Solve ANY TWO of the following :

(i) $x = 3\,\text{cosec}\,\theta + 4\cot\theta$ .....(i)

$y = 4\,\text{cosec}\,\theta - 3\cot\theta$ .....(ii)

Multiplying (i) by 4,

$4x = 12\,\text{cosec}\,\theta + 16\cot\theta$ .....(iii) ½

Multiplying (ii) by 3,

$3y = 12\,\text{cosec}\,\theta - 9\cot\theta$ .....(iv) ½

Subtracting (iv) from (iii),

$4x - 3y = 12\,\text{cosec}\,\theta + 16\cot\theta - (12\,\text{cosec}\,\theta - 9\cot\theta)$

$4x - 3y = 12\,\text{cosec}\,\theta + 16\cot\theta - 12\,\text{cosec}\,\theta + 9\cot\theta$

$4x - 3y = 25\cot\theta$

$\cot\theta = \frac{4x - 3y}{25}$ **1**

Substituting $\cot\theta = \frac{4x - 3y}{25}$ in equation (i)

$x = 3\,\text{cosec}\,\theta + 4\left(\frac{4x - 3y}{25}\right)$

$x = 3\,\text{cosec}\,\theta + \frac{16x - 12y}{25}$ ½

$x - \frac{16x - 12y}{25} = 3\,\text{cosec}\,\theta$

$\frac{25x - 16x + 12y}{25} = 3\,\text{cosec}\,\theta$

$\frac{9x + 12y}{25} = 3\,\text{cosec}\,\theta$

$\frac{3(3x + 4y)}{3 \times 25} = \text{cosec}\,\theta$

$\text{cosec}\,\theta = \frac{3x + 4y}{25}$ ½

We know,

$\text{cosec}^2\theta - \cot^2\theta = 1$ ½

$\left(\frac{3x + 4y}{25}\right)^2 - \left(\frac{4x - 3y}{25}\right)^2 = 1$

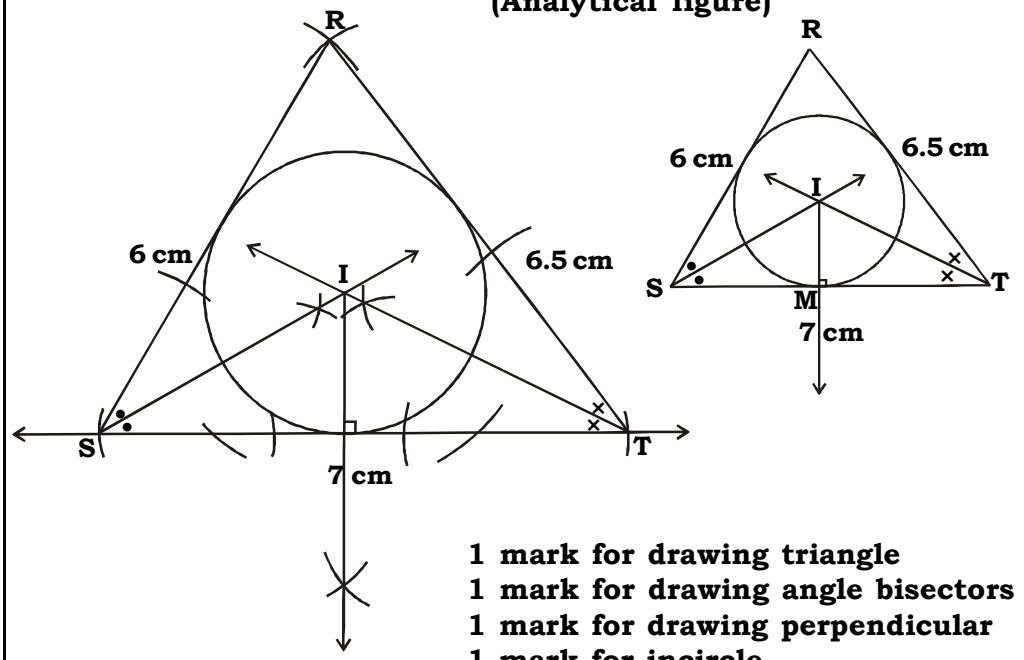
$\frac{(3x + 4y)^2}{625} - \frac{(4x - 3y)^2}{625} = 1$

Multiplying throughout by 625,

$\mathbf{(3x + 4y)^2 - (4x - 3y)^2 = 625}$ ½

(ii)

(Analytical figure)



1 mark for drawing triangle
1 mark for drawing angle bisectors
1 mark for drawing perpendicular
1 mark for incircle

(iii)

L.H.S. $= \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$= \dfrac{\sin\theta / \cos\theta}{1 - \cos\theta / \sin\theta} + \dfrac{\cos\theta / \sin\theta}{1 - \sin\theta / \cos\theta}$ ½

$= \dfrac{\sin\theta / \cos\theta}{\dfrac{\sin\theta - \cos\theta}{\sin\theta}} + \dfrac{\cos\theta / \sin\theta}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \dfrac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$ ½

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \dfrac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$ ½

$= \dfrac{1}{\sin\theta - \cos\theta}\left[\dfrac{\sin^2\theta}{\cos\theta} - \dfrac{\cos^2\theta}{\sin\theta}\right]$

$= \dfrac{1}{\sin\theta - \cos\theta} \times \dfrac{\sin^3\theta - \cos^3\theta}{\cos\theta \cdot \sin\theta}$ ½

$= \dfrac{1}{\sin\theta - \cos\theta} \times \dfrac{(\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta \cdot \cos\theta + \cos^2\theta)}{\cos\theta \cdot \sin\theta}$

$$= \frac{\sin^2\theta + \sin\theta \cdot \cos\theta + \cos^2\theta}{\cos\theta \cdot \sin\theta}$$

$$= \frac{1 + \sin\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1] \qquad \text{½}$$

$$= \frac{1}{\cos\theta \cdot \sin\theta} + \frac{\sin\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta} \qquad \text{½}$$

$$= \frac{1}{\cos\theta} \times \frac{1}{\sin\theta} + 1$$

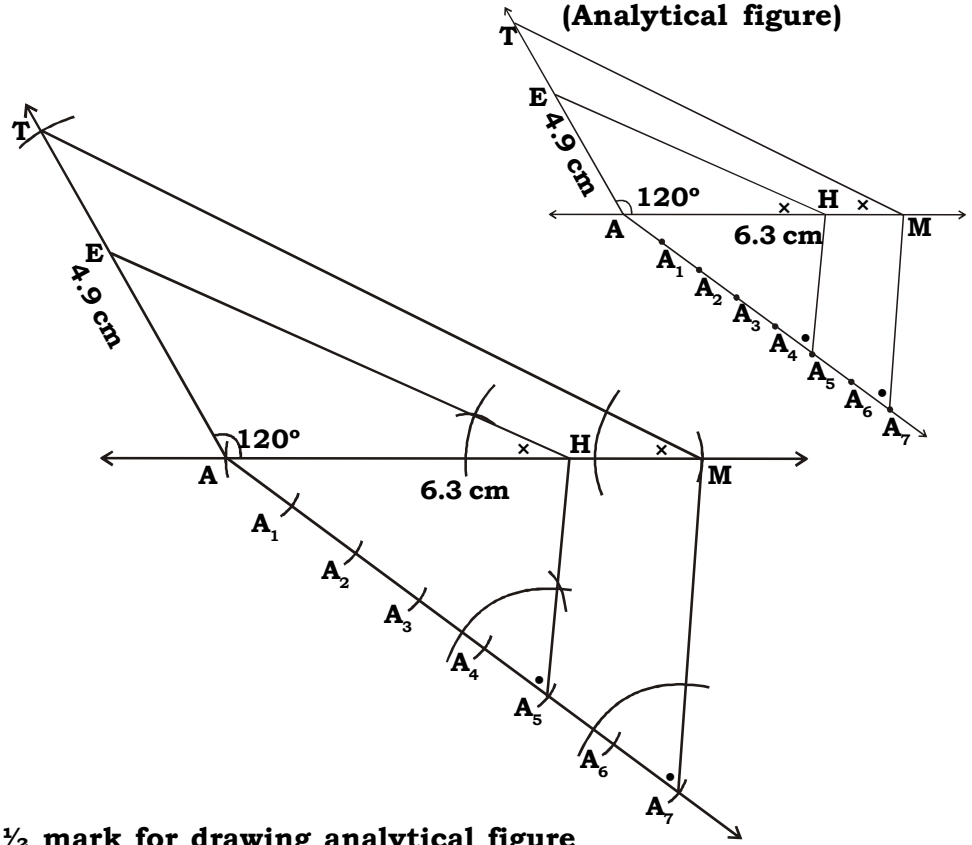
$$= \sec\theta \cdot \operatorname{cosec}\theta + 1 \left[\because \sec\theta = \frac{1}{\cos\theta}, \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right] \qquad \text{½}$$

= R.H.S.

y $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \cdot \operatorname{cosec}\theta$. ½

A.5. Solve ANY TWO of the following :

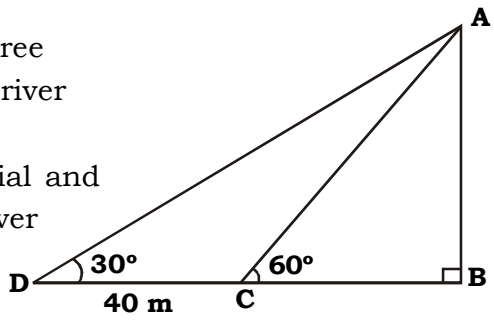
(i)



½ mark for drawing analytical figure
1 mark for aAMT
½ mark for constructing 7 congruent parts
1½ mark for constructing ĎHA_5A]: ĎMA_7A
1½ mark for constructing ĎEHA]: ĎTMA

(ii)

Let seg AB represent the tree
seg BC represent width of river
Let BC = x m
C and D represent the initial and final positions of the observer
DC = 40 m
ĎACB and ĎADB are the angles of elevation
m ĎACB = 60° and m ĎADB = 30°



1

In right angled aACB,

$\tan 60^\circ = \frac{AB}{BC}$ [By definition] ½

$\therefore \sqrt{3} = \frac{AB}{x}$

$\therefore AB = \sqrt{3}\,x$ m .....(i) ½

In right angled aADB,

$\tan 30^\circ = \frac{AB}{DB}$ [By definition] ½

$\therefore \frac{1}{\sqrt{3}} = \frac{AB}{40 + x}$

$\therefore AB = \frac{40 + x}{\sqrt{3}}$ m .....(ii) ½

From (i) and (ii) we get,

$\sqrt{3}\,x = \frac{40 + x}{\sqrt{3}}$ ½

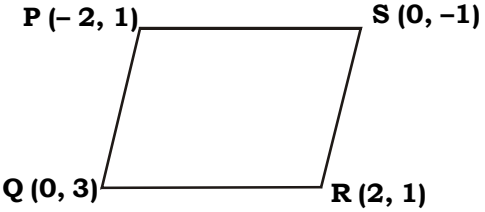
$\therefore 3x = 40 + x$
$\therefore 3x - x = 40$
$\therefore 2x = 40$ ½
$\therefore x = 20$
$\therefore BC = 20$ m
$\therefore AB = 20\sqrt{3}$ m [From (i)]
$\therefore AB = 20 \times 1.73$
$\therefore AB = 34.6$ m

$\therefore$ Height of tree is 34.6 m and width of river is 20 m. 1

(iii) Let, P ø (– 2, 1), Q ø (0, 3), R ø (2, 1), S ø (0, – 1)

Slope of a line = $\frac{y_2 - y_1}{x_2 - x_1}$ ½

Slope of line PQ = $\frac{3 - 1}{0 - (-2)}$



Z $\frac{2}{0 + 2}$

Z $\frac{2}{2}$

y Slope of line PQ = 1 ½

Slope of line RS = $\frac{-1 - 1}{0 - 2}$

= $\frac{-2}{-2}$

y Slope of line RS = 1

y Slope of line PQ = Slope of line RS ½

y line PQ || line RS .....(i) **1**

Slope of line QR = $\frac{1 - 3}{2 - 0}$

= $\frac{-2}{2}$

y Slope of line QR = – 1 ½

Slope of line PS = $\frac{-1 - 1}{0 - (-2)}$

= $\frac{-2}{0 + 2}$

= $\frac{-2}{2}$

y Slope of line PS = – 1 ½

y Slope of line QR = Slope of line PS

y line QR || line PS .....(ii) **1**

In □PQRS,

side PQ || side RS [From (i)]

side QR || side PS [From (ii)]

y □PQRS is a parallelogram [By definition]

y **The points (–2, 1), (0, 3), (2, 1) and (0, – 1) are the vertices of parallelogram.** ½

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