MT

2017 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 2

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1.	**Attempt ANY FIVE of the following :**	
(i)	Slope of the line (m) = – 4	
	y intercept of the line (c) = – 3	½
	By slope intercept form,	
	The equation of the line is y = mx + c	
	y y = (– 4)x + (– 3)	
	y y = – 4x – 3	½
	y The equation of the given line is y = – 4x – 3	
(ii)	$(1 + \tan^2 ¯)\ (1 - \sin ¯)\ (1 + \sin ¯)$	
	= $\sec^2 ¯\ (1 - \sin^2 ¯)$ [∵ $1 + \tan^2 ¯ = \sec^2 ¯$]	½
	= $\sec^2 ¯ \times \cos^2 ¯$ [$\sin^2 ¯ + \cos^2 ¯ = 1$ y $\cos^2 ¯ = 1 - \sin^2 ¯$]	
	= $\frac{1}{\cos^2 ¯} \times \cos^2 ¯$	½
	= 1	
(iii)	Equation of a line parallel to X-axis and passing through the point (– 2, 5) is y = 5.	**1**
(iv)	$\tan ¯ = \sqrt{15}$	½
	y $1 + \tan^2 ¯ = \sec^2 ¯$	
	y $1 + \left(\sqrt{15}\right)^2 = \sec^2 ¯$	
	y $1 + 15 = \sec^2 ¯$	
	y $\sec^2 ¯ = 16$	½
	y $\sec ¯ = 4$ [Taking square roots]	
(v)	y – 2 = 5 (x – 2)	
	Comparing with the equation of a line in slope point form,	
	$y - y_1 = m\ (x - x_1)$	½
	y m = 5	
	y Slope of the line y – 2 = 5 (x – 2) is 5	½

(vi) ∷‾ = – 30° [Given] ½

sin ‾ = sin (– 30)

= – sin 30

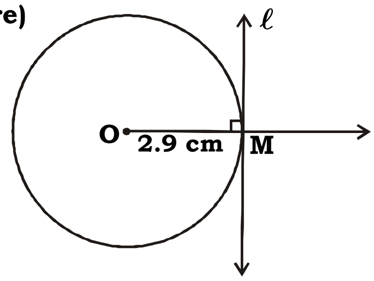
$= -\frac{1}{2}$

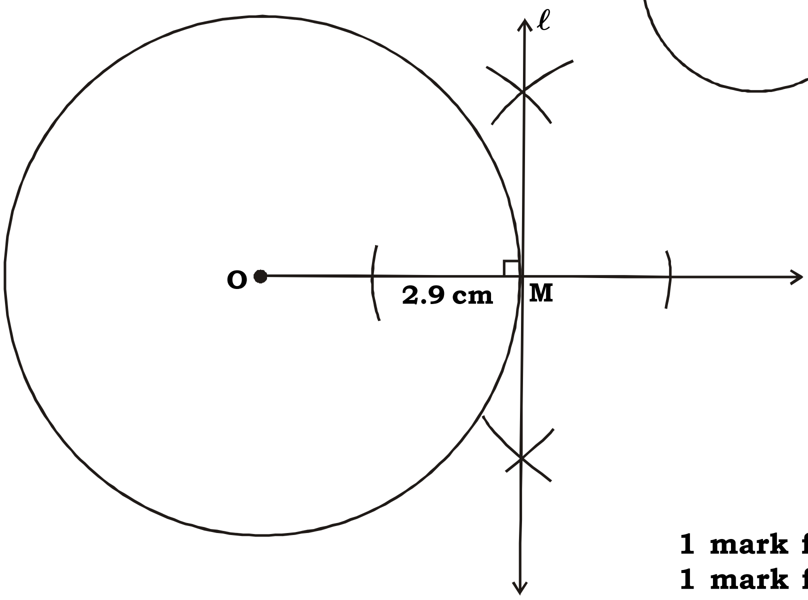
y $\boxed{\sin(-30) = -\frac{1}{2}}$ ½

A.2. Solve ANY FOUR of the following :

(i)

(Analytical figure)





1 mark for circle
1 mark for tangent

(ii) The terminal arm passes through P (– 24, – 7)

y x = – 24 and y = – 7

$r = \sqrt{x^2 + y^2}$ ½

$= \sqrt{(-24)^2 + (-7)^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

y r = 25 units ½

Let the angle be ‾

y $\sin - = \frac{y}{r} = \frac{-7}{25}$ $\quad$ $\text{cosec} - = \frac{r}{y} = \frac{-25}{7}$

$\cos - = \frac{x}{r} = \frac{-24}{25}$ $\quad$ $\sec - = \frac{r}{x} = \frac{-25}{24}$ **1**

$\tan - = \frac{y}{x} = \frac{-7}{-24} = \frac{7}{24}$ $\quad$ $\cot - = \frac{x}{y} = \frac{-24}{-7} = \frac{24}{7}$

(iii) Let A ø (2, 3) ø (x_1, y_1), B ø (4, 5) ø (x_2, y_2)

y The equation of line AB by two point form is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

y $$\frac{x - 2}{2 - 4} = \frac{y - 3}{3 - 5}$$ ½

y $$\frac{x - 2}{-2} = \frac{y - 3}{-2}$$ ½

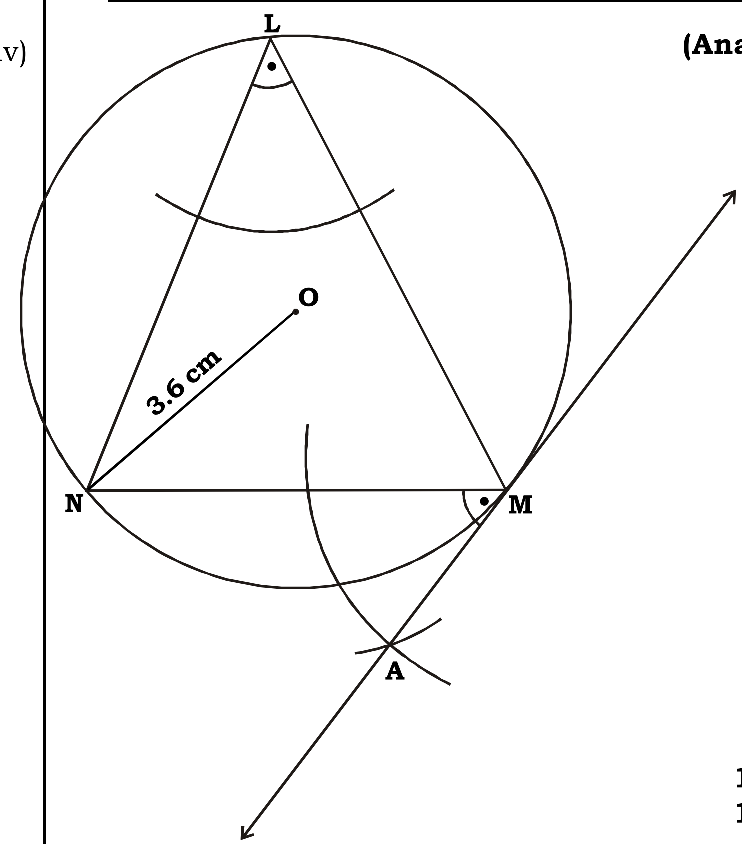
y $x - 2 = y - 3$

y $x - y - 2 + 3 = 0$

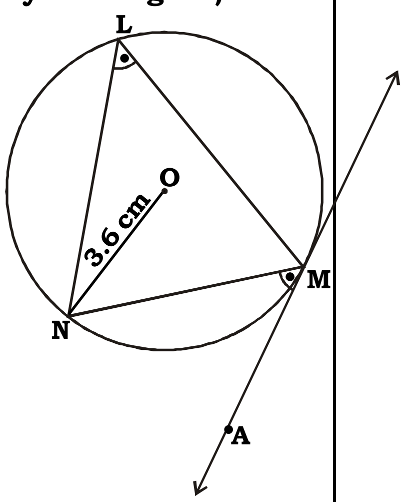
y $x - y + 1 = 0$ ½

y The equation of the line passing through the points (2, 3) and (4, 5) is $x - y + 1 = 0$.

(iv)

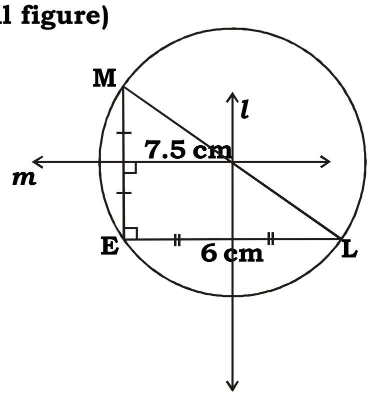


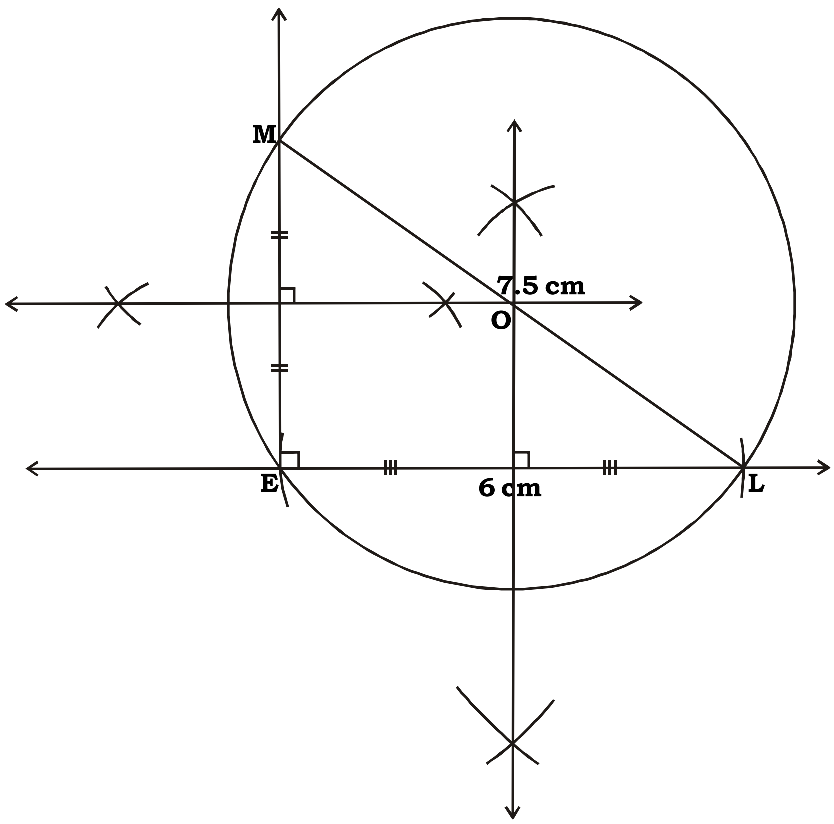
(Analytical figure)



1 mark for circle
1 mark for tangent

(v)	$\tan A + \dfrac{1}{\tan A} = 2$	½
	y $\left(\tan A + \dfrac{1}{\tan A}\right)^2 = 4$ [Squaring both sides]	
	y $\tan^2 A + 2 \tan A \cdot \dfrac{1}{\tan A} + \dfrac{1}{\tan^2 A} = 4$	½
	y $\tan^2 A + 2 + \dfrac{1}{\tan^2 A} = 4$	½
	y $\tan^2 A + \dfrac{1}{\tan^2 A} = 4 - 2$	
	y $\mathbf{\tan^2 A + \dfrac{1}{\tan^2 A} = 2}$	½
(vi)	Let A ø (3, 4) ø (x_1, y_1) and m = 5 The equation of the line passing through A and having slope 5 by slope point form is,	
	$y - y_1 = m (x - x_1)$	½
	y $y - 4 = 5 (x - 3)$	
	y $y - 4 = 5x - 15$	½
	y $5x - y - 15 + 4 = 0$	
	y $5x - y - 11 = 0$	**1**
	y The equation of the line passing through the points (3, 4) and having slope 5 is 5x – y – 11 = 0.	
A.3. (i)	**Solve ANY THREE of the following :** **(Analytical figure)**	





1 mark for triangle
1 mark for perpendicular bisectors
1 mark for circumcircle

(ii) $3 \sin \sim - 4 \cos \sim = 0$

y $3 \sin \sim = 4 \cos \sim$

y $\dfrac{\sin \sim}{\cos \sim} = \dfrac{4}{3}$ ½

y $\tan \sim = \dfrac{4}{3}$

$1 + \tan^2 \sim = \sec^2 \sim$

y $1 + \left(\dfrac{4}{3}\right)^2 = \sec^2 \sim$ ½

y $1 + \dfrac{16}{9} = \sec^2 \sim$

y $\dfrac{9 + 16}{9} = \sec^2 \sim$

y $\dfrac{25}{9} = \sec^2 \sim$

y $\sec \sim = \dfrac{5}{3}$ [Taking square roots] ½

$\cot \sim = \dfrac{1}{\tan \sim}$

$= \dfrac{1}{4/3}$

y $\cot \sim = \dfrac{3}{4}$ ½

$1 + \cot^2 \sim = \text{cosec}^2 \sim$ ½

y $1 + \left(\dfrac{3}{4}\right)^2 = \text{cosec}^2 \sim$

y $1 + \dfrac{9}{16} = \text{cosec}^2 \sim$

y $\dfrac{16+9}{16} = \text{cosec}^2 \sim$

y $\dfrac{25}{16} = \text{cosec}^2 \sim$

y $\boxed{\text{cosec} \sim = \dfrac{5}{4}}$ [Taking square roots] ½

(iii) Let, A ≡ (– 1, 1), B ≡ (– 9, 6), C ≡ (– 2, 14), D ≡ (6, 9)

Slope of a line $= \dfrac{y_2 - y_1}{x_2 - x_1}$ ½

Slope of line AB $= \dfrac{6-1}{-9-(-1)}$ ½

$= \dfrac{5}{-9+1}$

$= \dfrac{5}{-8}$

y Slope of line AB $= \dfrac{-5}{8}$ ½

Slope of line CD $= \dfrac{9-14}{6-(-2)}$

$= \dfrac{-5}{6+2}$

y Slope of line CD $= \dfrac{-5}{8}$ ½

$\therefore$ Slope of line AB and slope of line CD are equal.

$\therefore$ line AB || line CD

$\therefore$ **The line joining (– 1, 1) and (– 9, 6) is parallel to the line joining (– 2, 14) and (6, 9).** 1

(iv)

$$\sec\theta + \tan\theta = p$$

$$\therefore \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = p$$ ½

$$\therefore \frac{1 + \sin\theta}{\cos\theta} = p$$

$$\therefore \frac{(1 + \sin\theta)^2}{\cos^2\theta} = p^2$$ ½

$$\therefore \frac{(1 + \sin\theta)^2}{1 - \sin^2\theta} = p^2 \quad \left[\begin{array}{l}\sin^2\theta + \cos^2\theta = 1 \\ \therefore \cos^2\theta = 1 - \sin^2\theta\end{array}\right]$$ ½

$$\therefore \frac{(1 + \sin\theta)^2}{(1 + \sin\theta)(1 - \sin\theta)} = p^2$$

$$\therefore \frac{1 + \sin\theta}{1 - \sin\theta} = p^2$$ ½

$$\therefore \frac{1 + \sin\theta + 1 - \sin\theta}{1 + \sin\theta - 1 + \sin\theta} = \frac{p^2 + 1}{p^2 - 1}$$ [By Componendo-Dividendo] ½

$$\therefore \frac{2}{2\sin\theta} = \frac{p^2 + 1}{p^2 - 1}$$

$$\therefore \frac{1}{\sin\theta} = \frac{p^2 + 1}{p^2 - 1}$$

$$\therefore \mathbf{\frac{p^2 - 1}{p^2 + 1} = \sin\theta}$$ [By Invertendo] ½

(v) Let, A $\equiv$ (1, 2) $\equiv$ (x_1, y_1)

B $\equiv \left(\frac{1}{2}, 3\right) \equiv (x_2, y_2)$

C $\equiv$ (0, k) $\equiv$ (x_3, y_3)

$\because$ Points A, B and C are collinear

Slope of line AB = Slope of line BC ½

$\therefore \quad \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_3 - y_2}{x_3 - x_2}$ ½

$\therefore \quad \dfrac{3 - 2}{\frac{1}{2} - 1} = \dfrac{k - 3}{0 - \frac{1}{2}}$ ½

$\therefore \quad \dfrac{1}{-\frac{1}{2}} = \dfrac{k - 3}{\frac{-1}{2}}$ ½

$\therefore \quad 1 = k - 3$ ½

$\therefore \quad k = 1 + 3$

$\therefore \quad k = 4$ ½

$\therefore$ **The value of k is 4**

A.4. **Solve ANY TWO of the following :**

(i) L.H.S. $= \dfrac{\tan\theta}{\sec\theta + 1} + \dfrac{\sec\theta + 1}{\tan\theta}$

$= \dfrac{\tan^2\theta + (\sec\theta + 1)^2}{(\sec\theta + 1)\tan\theta}$ ½

$= \dfrac{\tan^2\theta + \sec^2\theta + 2\sec\theta + 1}{(\sec\theta + 1)\tan\theta}$ ½

$= \dfrac{\sec^2\theta + \sec^2\theta + 2\sec\theta}{(\sec\theta + 1)\tan\theta}$ $[\because\ 1 + \tan^2\theta = \sec^2\theta]$ ½

$= \dfrac{2\sec^2\theta + 2\sec\theta}{(\sec\theta + 1)\tan\theta}$

$= \dfrac{2\sec\theta(\sec\theta + 1)}{(\sec\theta + 1)\tan\theta}$ ½

$= \dfrac{2\sec\theta}{\tan\theta}$

$= 2\sec\theta \div \tan\theta$ ½

$= 2 \times \dfrac{1}{\cos\theta} \div \dfrac{\sin\theta}{\cos\theta}$

$= 2 \times \dfrac{1}{\cos\theta} \times \dfrac{\cos\theta}{\sin\theta}$ $\left[\tan\theta = \dfrac{\sin\theta}{\cos\theta},\ \sec\theta = \dfrac{1}{\cos\theta}\right]$ ½

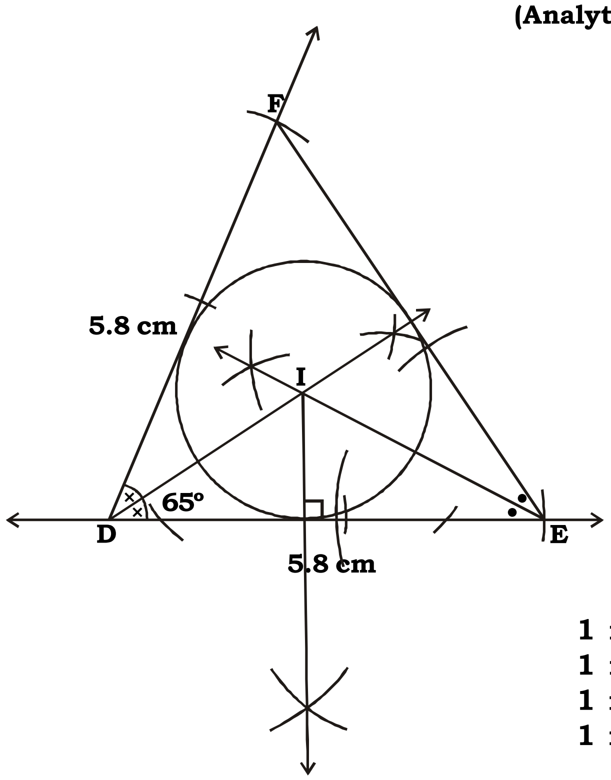
$= \dfrac{2}{\sin\theta}$ ½

$= 2\ \text{cosec}\ \theta$

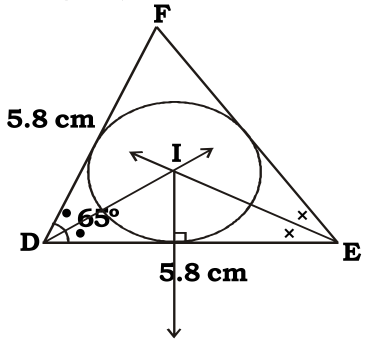
$=$ R.H.S. ½

$\therefore$ $\dfrac{\tan\theta}{\sec\theta + 1} + \dfrac{\sec\theta + 1}{\tan\theta} = 2\ \text{cosec}\ \theta$

(ii)



(Analytical figure)



1 mark for triangle
1 mark for angle bisectors
1 mark for perpendicular
1 mark for incircle

(iii)

$\sin^2\theta + \cos^2\theta = 1$

$\therefore$ $\cos^2\theta = 1 - \sin^2\theta$ ½

$\therefore$ $\cos\theta \cdot \cos\theta = (1 - \sin\theta)(1 + \sin\theta)$

$\therefore$ $\dfrac{\cos\theta}{1 - \sin\theta} = \dfrac{1 + \sin\theta}{\cos\theta}$ ½

By theorem on equal ratios,

$\therefore$ $\dfrac{1 + \sin\theta - \cos\theta}{\cos\theta - (1 - \sin\theta)} = \dfrac{\cos\theta}{1 - \sin\theta} = \dfrac{1 + \sin\theta}{\cos\theta}$ ½

$\therefore$ $\dfrac{1 + \sin\theta - \cos\theta}{\cos\theta - (1 - \sin\theta)} = \dfrac{\cos\theta}{1 - \sin\theta}$

Dividing the numerator and denominator of R.H.S. by $\cos\theta$ ½

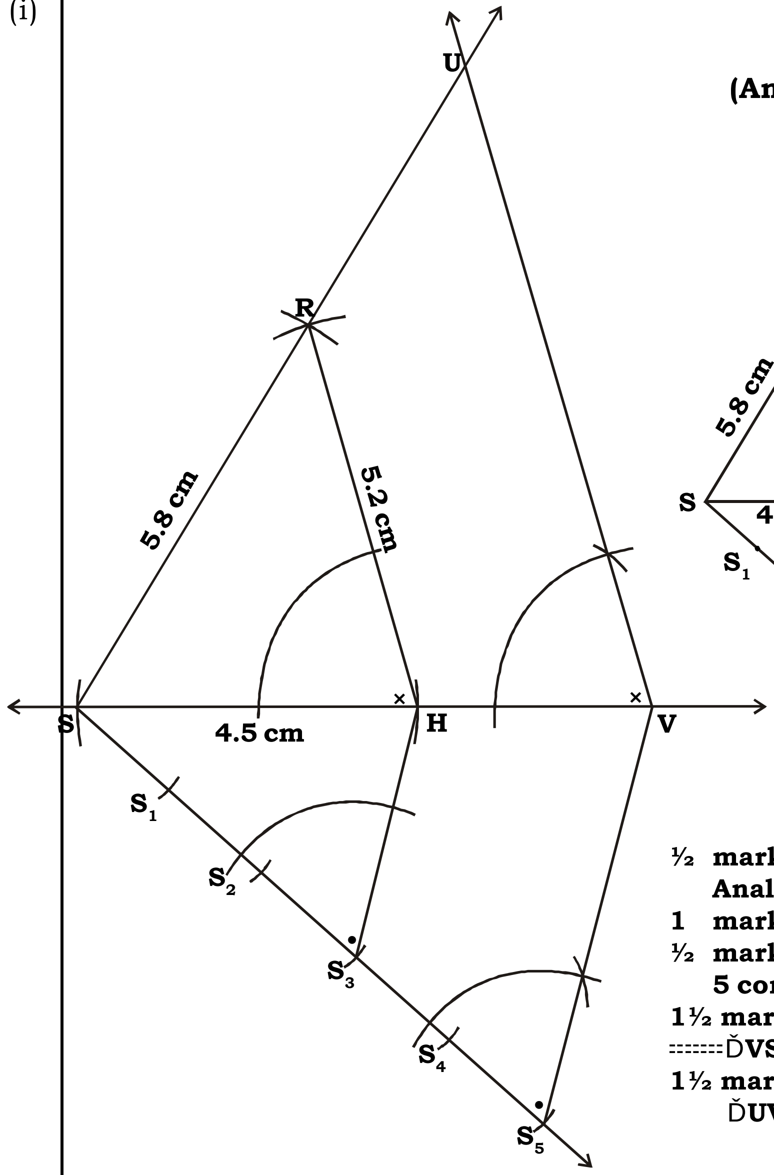
$$\therefore \quad \frac{1+\sin\theta-\cos\theta}{\cos\theta-1+\sin\theta} = \frac{\cos\theta/\cos\theta}{(1-\sin\theta)/\cos\theta}$$

$$\therefore \quad \frac{1+\sin\theta-\cos\theta}{\cos\theta-1+\sin\theta} = \frac{1}{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$ 1

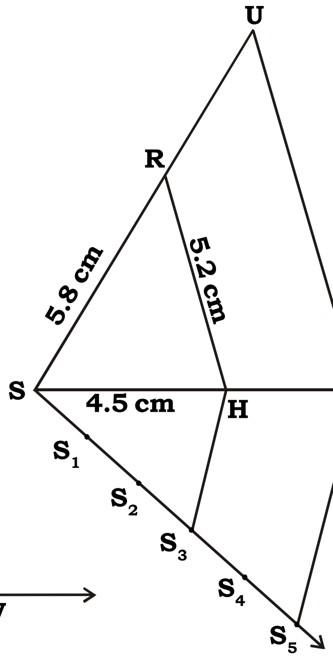
$$\therefore \quad \frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \frac{1}{\sec\theta-\tan\theta}$$ 1

A.5. **Solve ANY TWO of the following :**

(i)



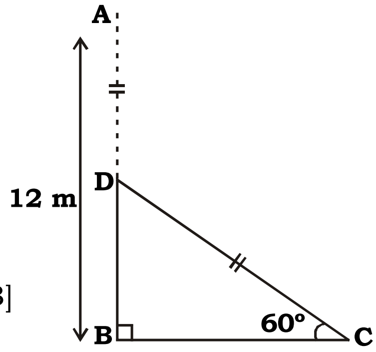
(Analytical figure)



½ mark for drawing Analytical figure
1 mark for ∠SHR
½ mark for constructing 5 congruent parts
1½ mark for constructing ∠$VS_5S \cong$ ∠HS_3S
1½ mark for constructing ∠UVS ≅ ∠RHS

(ii)

seg AB represents the tree
AB = 12 m
The tree breaks at point D
seg AD is the broken part of tree
which then takes the position of DC ½



y AD = DC
m ĎDCB = 60°
Let DB = x m
y AD + DB = AB [∵ A - D - B] ½
y AD + x = 12
y AD = (12 – x) m
y DC = (12 – x) m
In right angled aDBC,

$\sin 60° = \frac{DB}{DC}$ [By definition] ½

y $\frac{\sqrt{3}}{2} = \frac{x}{12 - x}$

y $\sqrt{3}\,(12 - x) = 2x$

y $12\sqrt{3} - \sqrt{3}\,x = 2x$

y $12\sqrt{3} = 2x + \sqrt{3}\,x$

y $x\left(2 + \sqrt{3}\right) = 12\sqrt{3}$ ½

y $x = \frac{12\sqrt{3}}{2 + \sqrt{3}}$

y $DB = \frac{12\sqrt{3}}{2 + \sqrt{3}}$ m

y $DB = \frac{12\sqrt{3}\left(2 - \sqrt{3}\right)}{\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)}$ 1

y $DB = \frac{24\sqrt{3} - 12\,(3)}{(2)^2 - \left(\sqrt{3}\right)^2}$

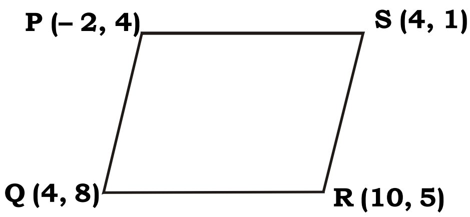
y $DB = \frac{24\sqrt{3} - 36}{4 - 3}$ 1

y $DB = \frac{24\,(1.73) - 36}{1}$

y DB = 41.52 – 36
y DB = 5.52 m 1

y The height at which the tree is broken from the bottom by the wind is 5.52 m.

(iii) P ≡ (– 2, 4), Q ≡ (4, 8), R ≡ (10, 5), S ≡ (4, 1)



Slope of a line $= \dfrac{y_2 - y_1}{x_2 - x_1}$ ½

Slope of line PQ $= \dfrac{8 - 4}{4 - (-2)}$

$= \dfrac{4}{4 + 2}$

$= \dfrac{4}{6}$ ½

y Slope of line PQ $= \dfrac{2}{3}$

Slope of line RS $= \dfrac{1 - 5}{4 - 10}$ ½

$= \dfrac{-4}{-6}$

y Slope of line RS $= \dfrac{2}{3}$ ½

y Slope of line PQ = Slope of line RS

y line PQ || line RS ....(i)

Slope of line QR $= \dfrac{5 - 8}{10 - 4}$

$= \dfrac{-3}{6}$ ½

y Slope of line QR $= \dfrac{-1}{2}$

Slope of line PS $= \dfrac{1 - 4}{4 - (-2)}$ ½

$= \dfrac{-3}{4 + 2}$

$= \dfrac{-3}{6}$ ½

y Slope of line PS $= \dfrac{-1}{2}$

y Slope of line QR = Slope of line PS

y line QR || line PS ....(ii) ½

In ☐PQRS,

side PQ || side RS [From (i)]

side QR || side PS [From (ii)]

y **☐PQRS is a parallelogram** [By definition] **1**

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