MT

2017 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 3

Time : 2 Hours | **Model Answer Paper** | **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) Slope of the line (m) = – 2
y intercept of the line (c) = 3 ½
By slope intercept form,
The equation of the line is y = mx + c
y y = (– 2)x + 3
y y = – 2x + 3 ½
y **The equation of the given line is y = – 2x + 3**

(ii) tan ~ = $\sqrt{3}$ [Given]
But, tan 60 = $\sqrt{3}$ ½
y tan ~ = tan 60
y **~ = 60** ½

(iii) Equation of a line parallel to Y-axis and passing through the point (3, 4) is x = 3. 1

(iv) ¯ + ~ = 90º [Given] ½
y ¯ = (90 – ~)
cosec ¯ = $\sqrt{2}$ [Given]
y sec ~ = cosec (90 – ~) [∵ sec ¯ = cosec (90 – ¯)]
y sec a = cosec ¯
y **sec ~ = $\sqrt{2}$** ½

(v) y + 3 = $\frac{1}{2}$ (x – 5)
Comparing with the equation of a line in slope point form,
$y - y_1 = m (x - x_1)$ ½
y m = $\frac{1}{2}$
y **Slope of the line y + 3 = $\frac{1}{2}$ (x – 5) is $\frac{1}{2}$** ½

(vi) ~ + = 90° [Given]

tan ~ = $\frac{3}{4}$ [Given]

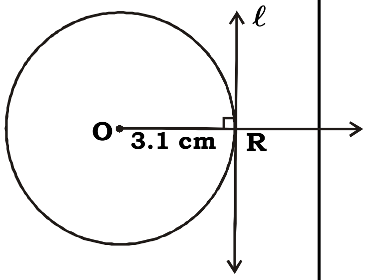
cot = tan ~ [∵ cot ¯ = tan (90 – ¯)] ½

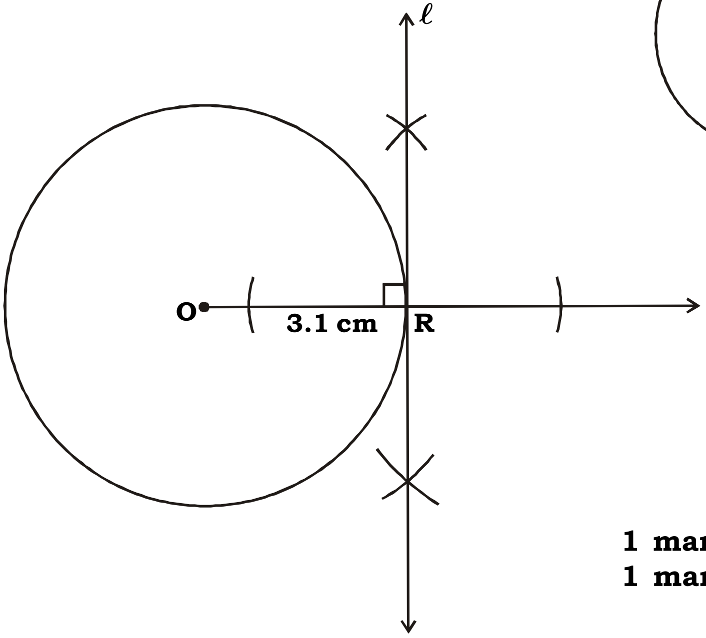
y $\boxed{\cot = \frac{3}{4}}$ ½

A.2. Solve ANY FOUR of the following :

(i)

(Analytical figure)





1 mark for circle
1 mark for tangent

(ii) The terminal arm passes through P $\left(-1, \sqrt{3}\right)$

y x = – 1 and y = $\sqrt{3}$

$r = \sqrt{x^2 + y^2}$ ½

$= \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2}$

$= \sqrt{1 + 3}$

$= \sqrt{4}$

y r = 2 units ½

Let the angle be ¯

y $\sin ¯ = \frac{y}{r} = \frac{\sqrt{3}}{2}$ $\text{cosec } ¯ = \frac{r}{y} = \frac{2}{\sqrt{3}}$

$\cos ¯ = \frac{x}{r} = \frac{-1}{2}$ $\sec ¯ = \frac{r}{x} = \frac{-2}{1}$ **1**

$\tan ¯ = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$ $\cot ¯ = \frac{x}{y} = \frac{-1}{\sqrt{3}}$

(iii) Let A ø (3, 4) ø (x_1, y_1) and m = 5

The equation of the line passing through A and having slope 5 by slope point form is, ½

$y - y_1 = m(x - x_1)$ ½

y $y - 4 = 5(x - 3)$

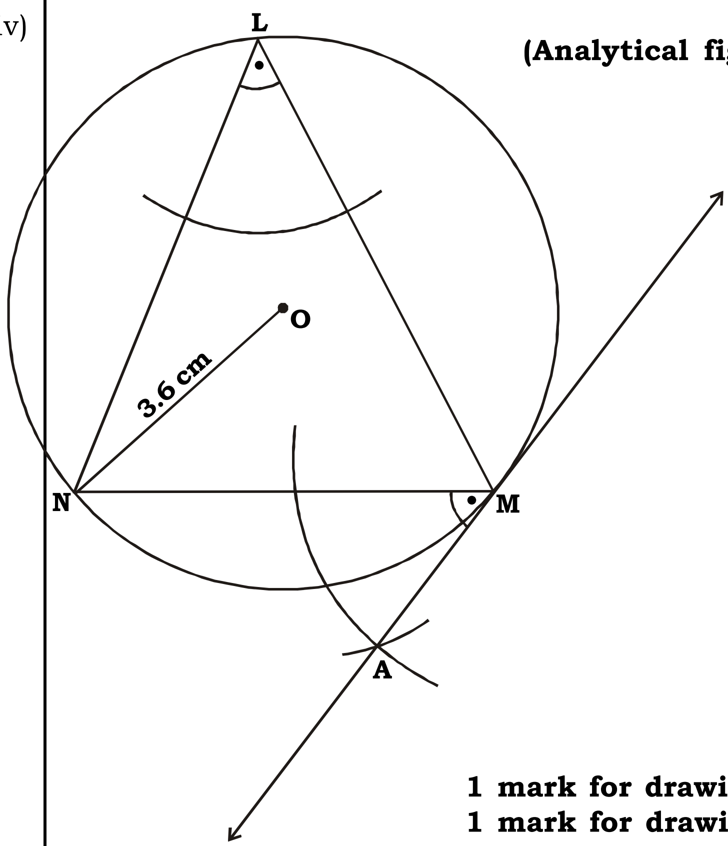
y $y - 4 = 5x - 15$ ½

y $5x - y - 15 + 4 = 0$

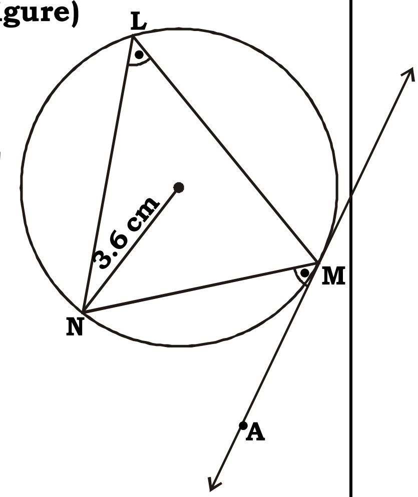
y $5x - y - 11 = 0$

y The equation of the line passing through the points (3, 4) and having slope 5 is 5x – y – 11 = 0. ½

(iv)



(Analytical figure)



1 mark for drawing circle
1 mark for drawing tangent

(v) L.H.S. $= \sec^2\theta + \operatorname{cosec}^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta} \quad \left[\because \sec\theta = \dfrac{1}{\cos\theta}, \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}\right]$ ½

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \,.\, \sin^2\theta}$ ½

$= \dfrac{1}{\cos^2\theta \,.\, \sin^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$ ½

$= \sec^2\theta \,.\, \operatorname{cosec}^2\theta$ ½

= R.H.S.

$\therefore$ $\mathbf{\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \,.\, \operatorname{cosec}^2\theta}$

(vi) Let, $A \equiv (2, 3) \equiv (x_1, y_1)$

$B \equiv (4, 7) \equiv (x_2, y_2)$

The line passes through points A and B

$\therefore$ The equation of the line by two point form is

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

$\therefore \quad \dfrac{x - 2}{2 - 4} = \dfrac{y - 3}{3 - 7}$

$\therefore \quad \dfrac{x - 2}{-2} = \dfrac{y - 3}{-4}$ ½

$\therefore \quad 4(x - 2) = 2(y - 3)$

$\therefore \quad 4x - 8 = 2y - 6$ ½

$\therefore \quad 2y = 4x - 8 + 6$

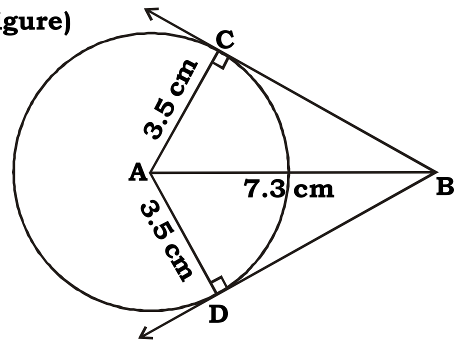
$\therefore \quad 2y = 4x - 2$

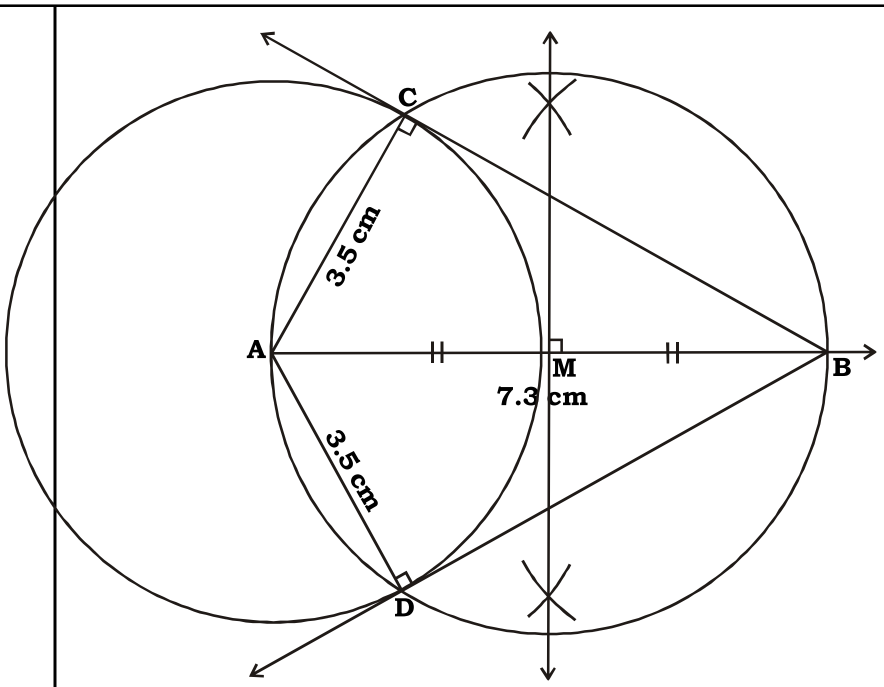
$\therefore \quad y = 2x - 1$ [Dividing throughout by 2] ½

$\therefore$ y = 2x – 1 is the equation of the line passing through (2, 3) and (4,7)

A.3. Solve ANY THREE of the following :

(i) **(Analytical figure)**





1 mark for circle
1 mark for perpendicular bisector
1 mark for tangents

(ii)

$\tan\theta = 1$

$\therefore \dfrac{\sin\theta}{\cos\theta} = 1$

$\therefore \sin\theta = \cos\theta$ ....(i) ½

$1 + \tan^2\theta = \sec^2\theta$

$\therefore 1 + (1)^2 = \sec^2\theta$

$\therefore 1 + 1 = \sec^2\theta$

$\therefore 2 = \sec^2\theta$

$\therefore \sec\theta = \sqrt{2}$ [Taking square roots] ½

$\cos\theta = \dfrac{1}{\sec\theta}$

$\therefore \cos\theta = \dfrac{1}{\sqrt{2}}$

$\therefore \sin\theta = \dfrac{1}{\sqrt{2}}$ [From (i)] ½

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$= \dfrac{1}{1/\sqrt{2}}$

$\therefore \operatorname{cosec}\theta = \sqrt{2}$ ½ y $\dfrac{\sin{}^{-} + \cos{}^{-}}{\sec{}^{-} + \operatorname{cosec}{}^{-}} = \dfrac{\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}}{\sqrt{2} + \sqrt{2}}$ ½

$= \dfrac{2/\sqrt{2}}{2\sqrt{2}}$

$= \dfrac{2}{2 \times \sqrt{2} \times \sqrt{2}}$

$= \dfrac{2}{2 \times 2}$

$= \dfrac{2}{4}$

y $\boxed{\dfrac{\sin{}^{-} + \cos{}^{-}}{\sec{}^{-} + \operatorname{cosec}{}^{-}} = \dfrac{1}{2}}$ ½

(iii) Let, A ø (1, 2) ø (x_1, y_1)

B ø $\left(\dfrac{1}{2}, 3\right)$ ø (x_2, y_2) ½

C ø(0, k) ø (x_3, y_3)

∵ Points A, B and C are collinear

Slope of line AB = Slope of line BC ½

y $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_3 - y_2}{x_3 - x_2}$ ½

y $\dfrac{3 - 2}{\frac{1}{2} - 1} = \dfrac{k - 3}{0 - \frac{1}{2}}$ ½

y $\dfrac{1}{-\frac{1}{2}} = \dfrac{k - 3}{\frac{-1}{2}}$ ½

y $1 = k - 3$

y $k = 1 + 3$

y $k = 4$ ½

y **The value of k is 4.**

(iv)	$3\tan^2 ¯ - 4\sqrt{3}\tan ¯ + 3 = 0$	
	y $\sqrt{3}\left(\sqrt{3}\tan^2 ¯ - 4\tan ¯ + \sqrt{3}\right) = 0$	½
	y $\sqrt{3}\tan^2 ¯ - 4\tan ¯ + \sqrt{3} = 0$	
	y $\sqrt{3}\tan^2 ¯ - 3\tan ¯ - \tan ¯ + \sqrt{3} = 0$	½
	y $\sqrt{3}\tan ¯ \left(\tan ¯ - \sqrt{3}\right) - 1\left(\tan ¯ - \sqrt{3}\right) = 0$	
	y $\left(\tan ¯ - \sqrt{3}\right)\left(\sqrt{3}\tan ¯ - 1\right) = 0$	½
	y $\tan ¯ - \sqrt{3} = 0$ OR $\sqrt{3}\tan ¯ - 1 = 0$	
	y $\tan ¯ = \sqrt{3}$ y $\sqrt{3}\tan ¯ = 1$	½
	But, $\tan 60 = \sqrt{3}$ y $\tan ¯ = \frac{1}{\sqrt{3}}$	
	y $\tan ¯ = \tan 60$ But, $\tan 30 = \frac{1}{\sqrt{3}}$	
	y ==============¯ = 60 y $\tan ¯ = \tan 30$	**1**
	y ==============¯ = 30	
(v)	Let, A ≡ (– 1, 1), B ≡ (– 9, 6), C ≡ (– 2, 14), D ≡ (6, 9)	
	Slope of a line $= \frac{y_2 - y_1}{x_2 - x_1}$	½
	Slope of side AB $= \frac{6 - 1}{-9 - (-1)}$	½
	$= \frac{5}{-9 + 1}$	
	$= \frac{5}{-8}$	
	y Slope of line AB $= \frac{-5}{8}$	½
	Slope of line CD $= \frac{9 - 14}{6 - (-2)}$	½
	$= \frac{-5}{6 + 2}$	
	y Slope of line CD $= \frac{-5}{8}$	½
	y Slope of line AB and slope of line CD are equal.	
	y line AB || line CD	
	y **The line joining (– 1, 1) and (– 9, 6) is parallel to the line joining (– 2, 14) and (6, 9).**	½

A.4. Solve ANY TWO of the following :

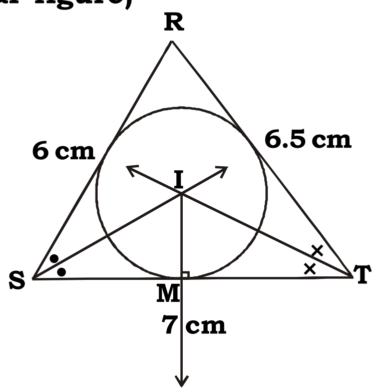
(i)

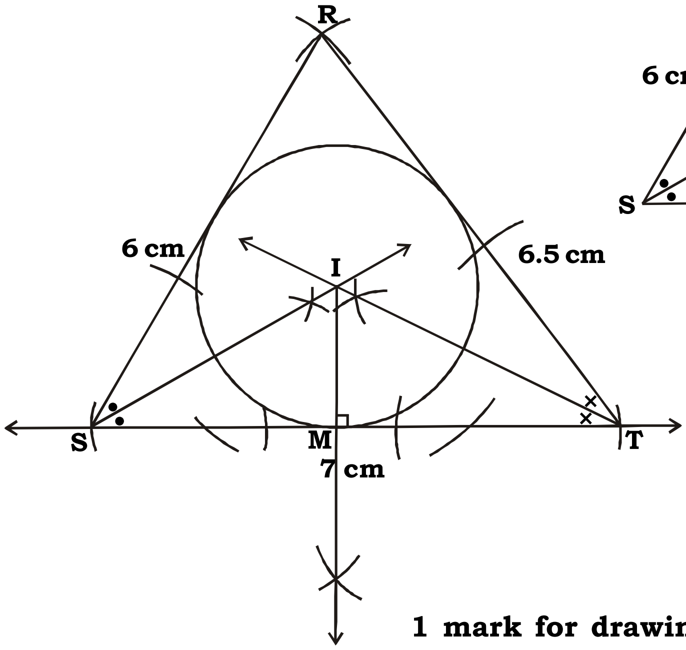
		Marks
L.H.S.	$= (1 + \tan\theta)^2 + (1 + \cot\theta)^2$	
	$= 1 + 2\tan\theta + \tan^2\theta + 1 + 2\cot\theta + \cot^2\theta$	½
	$= 1 + \tan^2\theta + 1 + \cot^2\theta + 2\tan\theta + 2\cot\theta$	½
	$= \sec^2\theta + \mathrm{cosec}^2\theta + 2(\tan\theta + \cot\theta)$ $[\because 1 + \tan^2\theta = \sec^2\theta,\ 1 + \cot^2\theta = \mathrm{cosec}^2\theta]$	
	$= \sec^2\theta + \mathrm{cosec}^2\theta + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$	½
	$= \sec^2\theta + \mathrm{cosec}^2\theta + 2\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \times \sin\theta}\right)$	½
	$= \sec^2\theta + \mathrm{cosec}^2\theta + 2 \times \frac{1}{\cos\theta \times \sin\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$	½
	$= \sec^2\theta + \mathrm{cosec}^2\theta + 2 \times \sec\theta \times \mathrm{cosec}\theta$	½
	$= (\sec\theta + \mathrm{cosec}\theta)^2$	1
	$=$ R.H.S.	

$\therefore$ $\mathbf{(1 + \tan\theta)^2 + (1 + \cot\theta)^2 = (\sec\theta + \mathrm{cosec}\theta)^2}$

(ii)

(Analytical figure)





1 mark for drawing triangle
1 mark for drawing angle bisectors
1 mark for drawing perpendicular
1 mark for incircle

(iii) L.H.S. $= \dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\cos^2\theta}{1-\dfrac{\sin\theta}{\cos\theta}} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$ ½

$= \dfrac{\cos^2\theta}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$ ½

$= \dfrac{\cos^3\theta}{\cos\theta-\sin\theta} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$ ½

$= \dfrac{\cos^3\theta}{\cos\theta-\sin\theta} - \dfrac{\sin^3\theta}{\cos\theta-\sin\theta}$ ½

$= \dfrac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

$= \dfrac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta\,.\,\sin\theta+\sin^2\theta)}{(\cos\theta-\sin\theta)}$ 1

$= \cos^2\theta+\sin^2\theta+\sin\theta\,.\,\cos\theta$

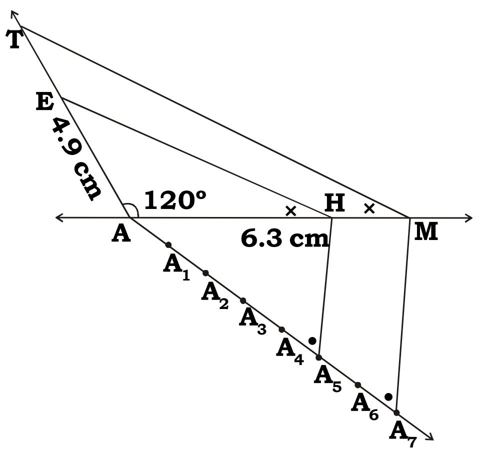
$= 1+\sin\theta\,.\,\cos\theta$ $[\because \sin^2\theta+\cos^2\theta=1]$ 1

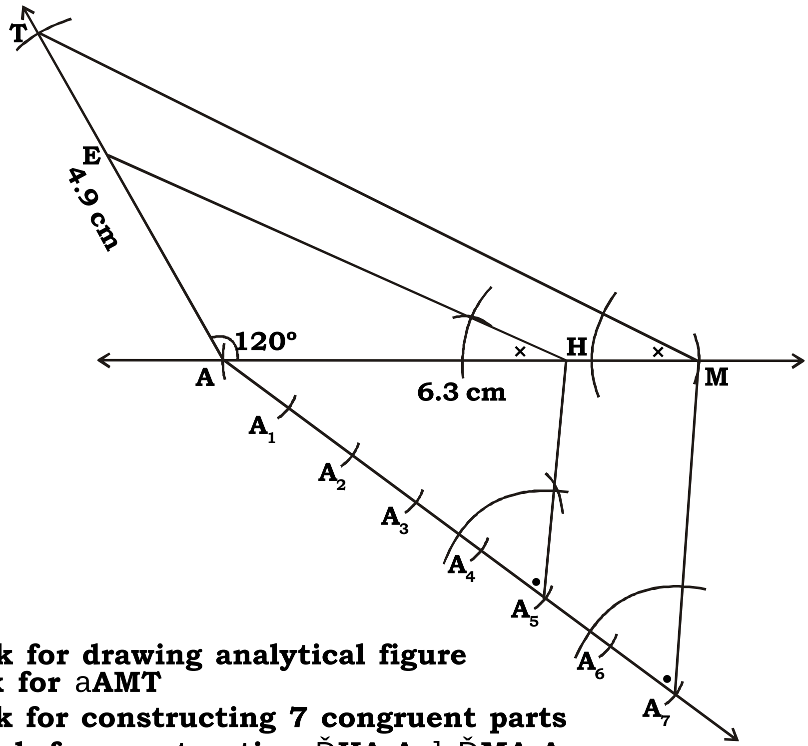
= R.H.S.

$\therefore \; \dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta} = 1+\sin\theta\,.\,\cos\theta$

A.5. Solve ANY TWO of the following :

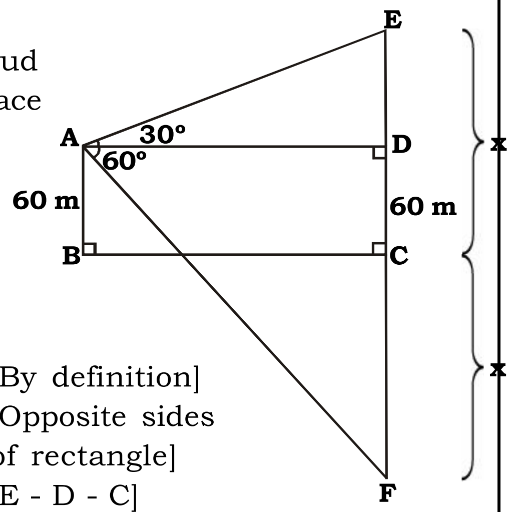
(i) **(Analytical figure)**





½ mark for drawing analytical figure
1 mark for △AMT
½ mark for constructing 7 congruent parts
1½ mark for constructing ∠HA_5A ≅ ∠MA_7A
1½ mark for constructing ∠EHA ≅ ∠TMA

(ii) Let E be the position of the cloud and let BC represent the surface of the lake.
Let A be the point of observer and let F be the reflection of the cloud



½

∴ EC = CF ½
Let EC = CF = x m
□ABCD is a rectangle [By definition]
∴ AB = CD = 60 m [Opposite sides of rectangle] ½
∴ EC = ED + DC [E - D - C] ½
∴ x = ED + 60
∴ ED = (x – 60)m
Also,
DF = DC + CF [D - C - F] ½
∴ DF = (60 + x)
∴ DF = (x + 60) m

In right angled aADE,

$\tan 30^\circ = \dfrac{ED}{AD}$ [By definition]

y $\dfrac{1}{\sqrt{3}} = \dfrac{x - 60}{AD}$

y $AD = \sqrt{3}\,(x - 60)\,m$ ½

In right angled aADF,

$\tan 60^\circ = \dfrac{DF}{AD}$ [By definition]

y $\sqrt{3} = \dfrac{x + 60}{\sqrt{3}\,(x - 60)}$ 1

y $3\,(x - 60) = x + 60$

y $3x - 180 = x + 60$

y $3x - x = 60 + 180$

y $2x = 240$

y $x = 120$ 1

y **The height of the cloud above the lake is 120 m.**

(iii) A ø (5, 4), B ø (– 3, – 2), C ø (1, – 8)

seg AD is the median of seg BC

y D is midpoint of seg BC

y D ø $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

ø $\left(\dfrac{-3 + 1}{2}, \dfrac{-2 + (-8)}{2}\right)$

ø $\left(\dfrac{-2}{2}, \dfrac{-2 - 8}{2}\right)$

ø $\left(-1, \dfrac{-10}{2}\right)$ 1

ø (– 1, – 5)

By two point form,

The equation of median AD

$\dfrac{x - x_1}{x_1 - x_2} = \dfrac{y - y_1}{y_1 - y_2}$ ½

y $\dfrac{x - 5}{5 - (-1)} = \dfrac{y - 4}{4 - (-5)}$ ½

y $\dfrac{x - 5}{5 + 1} = \dfrac{y - 4}{4 + 5}$

y $\dfrac{x - 5}{6} = \dfrac{y - 4}{9}$

y $9\,(x - 5) = 6\,(y - 4)$

y $9x - 45 = 6y - 24$
y $9x - 6y - 45 + 24 = 0$
y $9x - 6y - 21 = 0$
y $3x - 2y - 7 = 0$ [Dividing throughout by 3] **1**

y The equation of median AD is $3x - 2y - 7 = 0$

Slope of line AC $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-8 - 4}{1 - 5}$

$= \frac{-12}{-4}$ ½

$= 3$

∵ Slope of parallel lines are equal
Slope of the line parallel to line AC is 3
The line passes through B (– 3, – 2)

y The equation of the line parallel to line AC passing through point B by the slope point form is

$y - y_1 = m(x - x_1)$ ½
y $y - (-2) = 3[x - (-3)]$
y $y + 2 = 3(x + 3)$
y $y + 2 = 3x + 9$
y $3x - y + 9 - 2 = 0$
y $3x - y + 7 = 0$ **1**

y The equation of the line parallel to AC passing through point B is $3x - y + 7 = 0$

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