MT

2017 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 4

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) Slope of the line (m) = 4

y intercept of the line (c) = 0 ½

By slope intercept form,

The equation of the line is y = mx + c

y y = (4)x + (0) ½

y y = 4x

y The equation of the given line is y = 4x

(ii) If the terminal arm lies on the positive Y-axis, then angle made is 90° or – 270°. **1**

(iii) Equation of a line parallel to Y-axis and passing through point (3, 0) is x = 3. **1**

(iv) tan ¯ = $\sqrt{15}$

y 1 + tan² ¯ = sec² ¯ ½

y $1 + \left(\sqrt{15}\right)^2$ = sec² ¯

y 1 + 15 = sec² ¯

y sec² ¯ = 16

y sec ¯ = 4 [Taking square roots] ½

(v) y = – 5 (x + 3)

Comparing with the equation of a line in slope point form,

$y - y_1 = m (x - x_1)$ **1**

y m = – 5

y Slope of the line y = – 5 (x + 3) is – 5

(vi) ~ **+** = 90° [Given]

tan ~ = $\frac{3}{4}$ [Given]

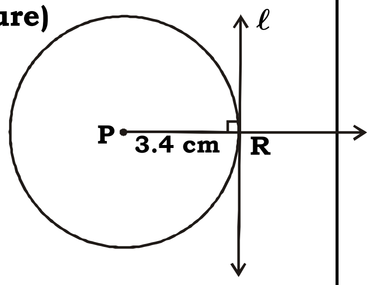
cot = tan ~ [∵ cot ¯ = tan (90 – ¯)] ½

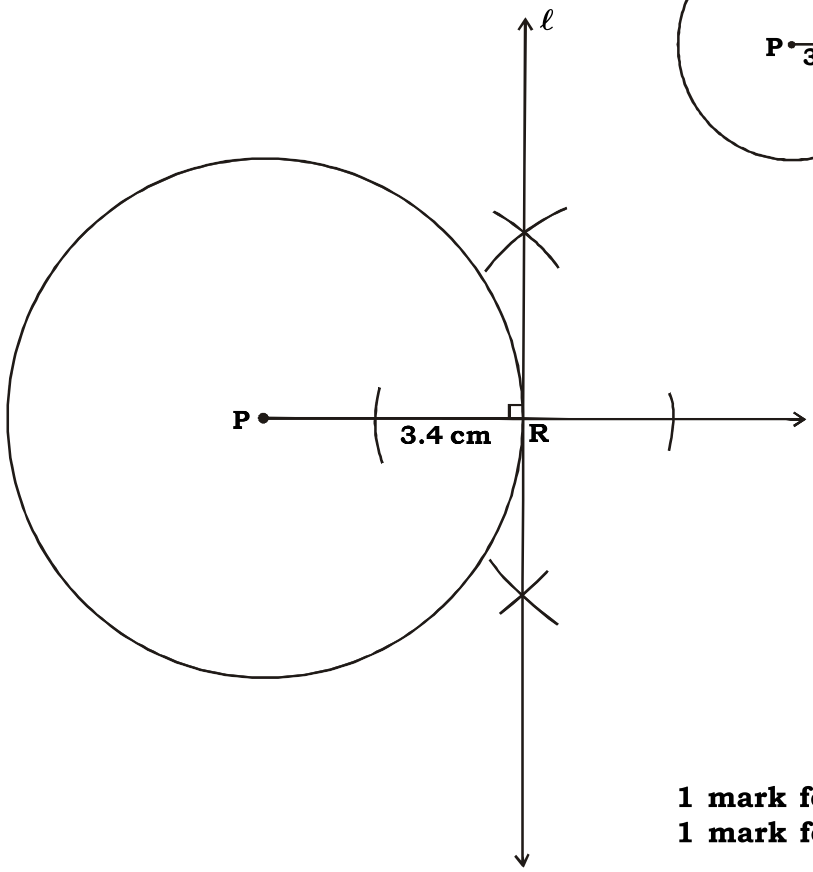
y $\boxed{\cot = \frac{3}{4}}$ ½

A.2. **Solve ANY FOUR of the following :**

(i)

(Analytical figure)





1 mark for circle
1 mark for tangent

(ii) The terminal arm passes through P (5, – 12)

y x = 5 and y = – 12

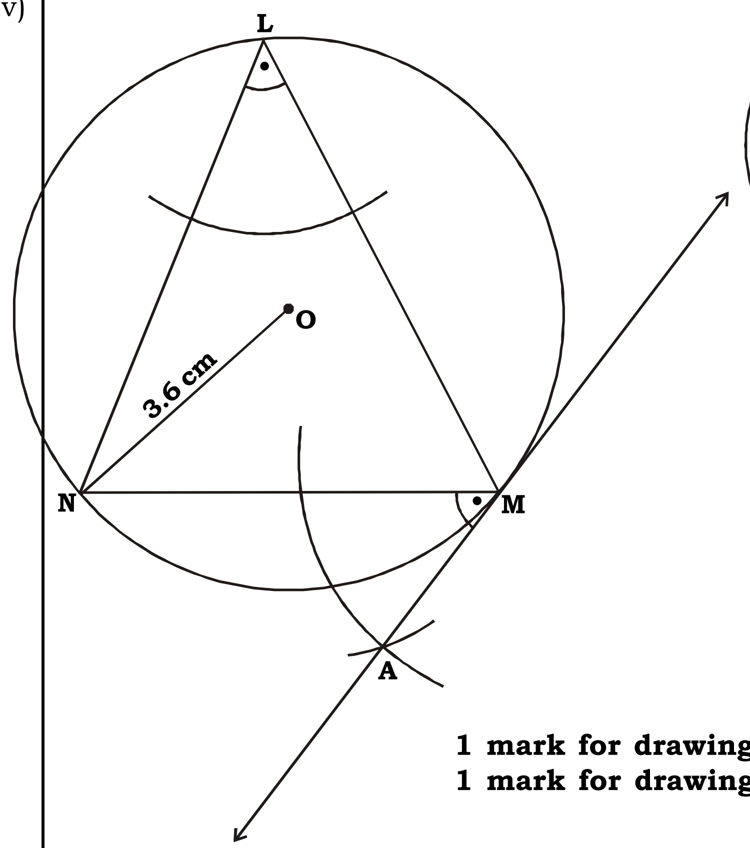
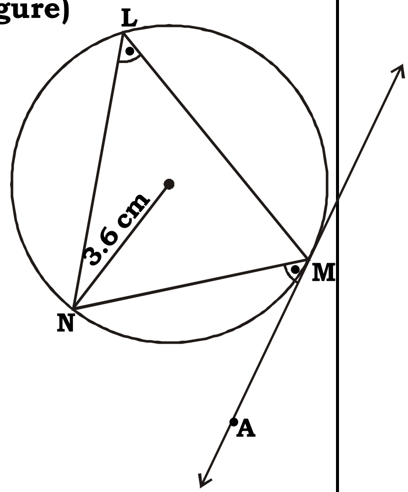
$r = \sqrt{x^2 + y^2}$ ½

$= \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

y $r = 13$ units ½

	Let the angle be ¯	
	y sin ¯ = $\frac{y}{r}$ = $\frac{-12}{13}$ cosec ¯ = $\frac{r}{y}$ = $\frac{-13}{12}$	
	cos ¯ = $\frac{x}{r}$ = $\frac{5}{13}$ sec ¯ = $\frac{r}{x}$ = $\frac{13}{5}$	1
	tan ¯ = $\frac{y}{x}$ = $\frac{-12}{5}$ cot ¯ = $\frac{x}{y}$ = $\frac{-5}{12}$	
(iii)	P ø (2, 4), Q ø (3, 6), R ø (8, 1), S ø (10, k)	
	Line PQ is parallel to line RS	½
	y Slope of line PQ = Slope of line RS	½
	y $\frac{6-4}{3-2} = \frac{k-1}{10-8}$	
	y $\frac{2}{1} = \frac{k-1}{2}$	
	y 4 = k – 1	1
	y k = 4 + 1	
	y k = 5	
	y **Value of k is 5.**	
(iv)	 (Analytical figure) 	
	1 mark for drawing circle **1 mark for drawing tangent**	

(v) sin ¯ + sin^2 ¯ = 1 [Given]

y sin ¯ = 1 – sin^2 ¯ ½

y sin ¯ = cos^2 ¯ $\left[\begin{array}{l} \sin^2 ¯ + \cos^2 ¯ = 1 \\ \because 1 - \sin^2 ¯ = \cos^2 ¯ \end{array}\right]$ ½

y sin^2 ¯ = cos^4 ¯ [Squaring both sides] ½

y 1 – cos^2 ¯ = cos^4 ¯ $\left[\begin{array}{l} \sin^2 ¯ + \cos^2 ¯ = 1 \\ \because 1 - \cos^2 ¯ = \sin^2 ¯ \end{array}\right]$ ½

y **cos^2 ¯ + cos^4 ¯ = 1**

(vi) Let,A ø (0, 5) ø (x_1, y_1)

B ø (5, 6) ø (x_2, y_2)

The line passes through points A and B ½

y The equation of the line by two point form is

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

y $$\frac{x - 0}{0 - 5} = \frac{y - 5}{5 - 6}$$

y $$\frac{x}{-5} = \frac{y - 5}{-1}$$ ½

y x = 5 (y – 5)

y x = 5y – 25

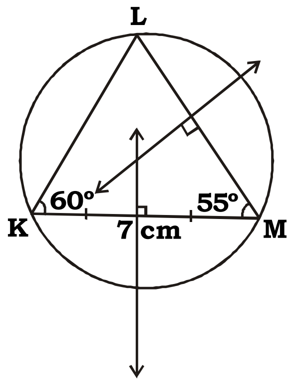
y 5y = x + 25

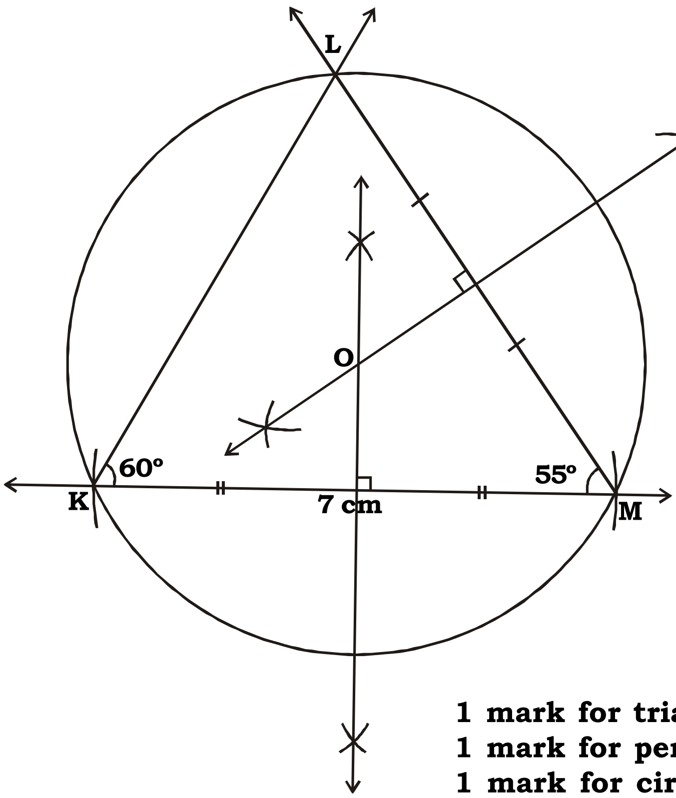
y $y = \frac{1}{5}x + 5$ [Dividing throughout by 5] ½

y $y = \frac{1}{5}x + 5$ is the equation of the line passing through (0, 5) and (5, 6)

A.3. Solve ANY THREE of the following :

(i) **(Analytical figure)**





1 mark for triangle
1 mark for perpendicular bisectors
1 mark for circumcircle

(ii) $\sec \sim = \frac{2}{\sqrt{3}}$

$\cos \sim = \frac{1}{\sec \sim}$

$= \frac{1}{2/\sqrt{3}}$

y $\cos \sim = \frac{\sqrt{3}}{2}$ ½

$\sin^2 \sim + \cos^2 \sim = 1$ ½

y $\sin^2 \sim + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$

y $\sin^2 \sim + \frac{3}{4} = 1$

y $\sin^2 \sim = 1 - \frac{3}{4}$

y $\sin^2 \sim = \frac{4-3}{4}$

y $\sin^2 \sim = \frac{1}{4}$

y $\sin \sim = \frac{1}{2}$ [Taking square roots] ½

$\operatorname{cosec} \sim = \frac{1}{\sin \sim}$

$= \frac{1}{1/2}$

$\operatorname{cosec} \sim = 2$

~ is in IV quadrant ½

y $\operatorname{cosec} \sim = -2$

$\frac{1 - \operatorname{cosec} \sim}{1 + \operatorname{cosec} \sim} = \frac{1-(-2)}{1+(-2)}$

y $\frac{1 - \operatorname{cosec} \sim}{1 + \operatorname{cosec} \sim} = \frac{1+2}{1-2}$

y $\boxed{\frac{1 - \operatorname{cosec} \sim}{1 + \operatorname{cosec} \sim} = -3}$ 1

(iii) Let, A ø $\left(\frac{2}{5}, \frac{1}{3}\right)$ ø (x_1, y_1)

B ø $\left(\frac{1}{2}, k\right)$ ø (x_2, y_2)

C ø $\left(\frac{4}{5}, 0\right)$ ø (x_3, y_3) 1

∵ Points A, B and C are collinear

Slope of line AB = Slope of line BC

y $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$ ½

y $\frac{k - \frac{1}{3}}{\frac{1}{2} - \frac{2}{5}} = \frac{0 - k}{\frac{4}{5} - \frac{1}{2}}$

y $\frac{\frac{3k-1}{3}}{\frac{1}{10}} = \frac{-k}{\frac{3}{10}}$ ½ y $\frac{3k-1}{3} \times 10 = -k \times \frac{10}{3}$ ½

y $3k - 1 = -k$

y $3k + k = 1$

y $4k = 1$

y $k = \frac{1}{4}$

y The value of k is $\frac{1}{4}$ ½

(iv) a cos ¯ + b sin ¯ = m

y (a cos ¯ + b sin ¯)^2 = m^2 …….(i) [Squaring both sides] ½

(a sin ¯ – b cos ¯) = n

y (a sin ¯ – b cos ¯)^2 = n^2 ……(ii) [Squaring both sides] ½

Adding (i) and (ii),

(a cos ¯ + b sin ¯)^2 + (a sin ¯ – b cos ¯)^2 = m^2 + n^2

y a^2 cos^2 ¯ + 2ab cos ¯ . sin ¯ + b^2 sin^2 ¯ + a^2 sin^2 ¯ – 2ab sin ¯ . cos ¯ + b^2 cos^2 ¯ = m^2 + n^2 **1**

y a^2 cos^2 ¯ + b^2 cos^2 ¯ + a^2 sin^2 ¯ + b^2 sin^2 ¯ = m^2 + n^2

y cos^2 ¯ (a^2 + b^2) + sin^2 ¯ (a^2 + b^2) = m^2 + n^2

y (a^2 + b^2) (cos^2 ¯ + sin^2 ¯) = m^2 + n^2 **1**

y **a^2 + b^2 = m^2 + n^2** [∵ sin^2 ¯ + cos^2 ¯ = 1]

(v) Let, A ≡ (– 1, 1), B ≡ (– 9, 6), C ≡ (– 2, 14), D ≡ (6, 9)

Slope of a line $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

Slope of side AB $= \frac{6-1}{-9-(-1)}$

$= \frac{5}{-9+1}$

$= \frac{5}{-8}$

y Slope of line AB $= \frac{-5}{8}$ ½

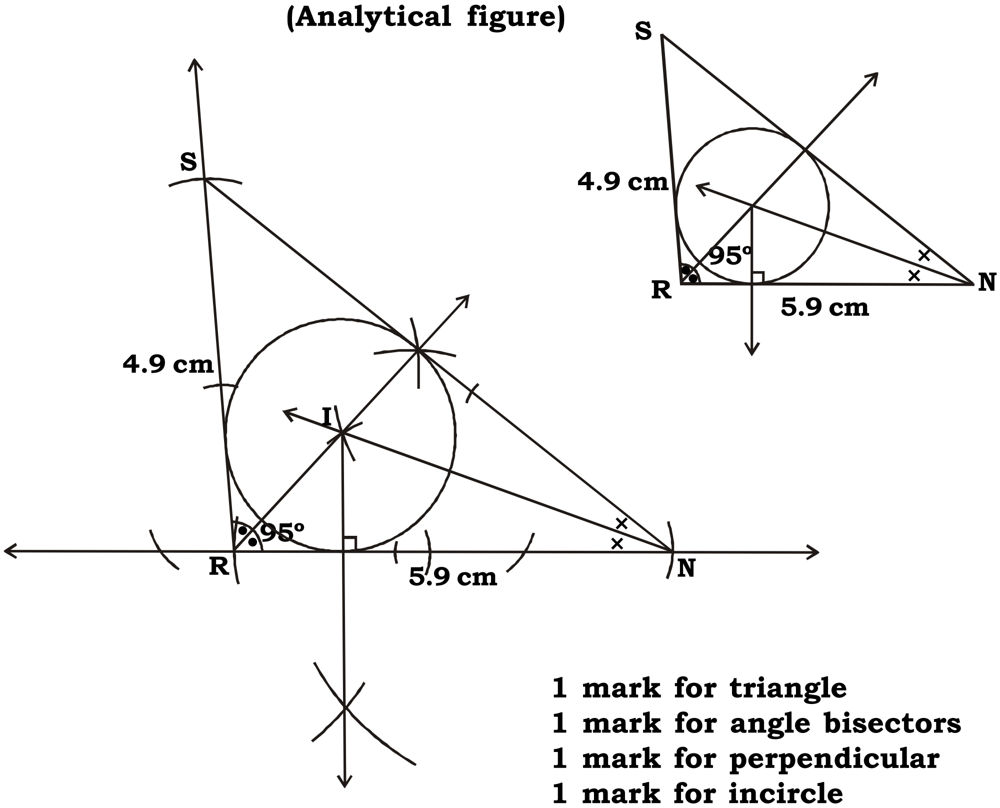
Slope of line CD $= \frac{9-14}{6-(-2)}$ ½

$= \frac{-5}{6+2}$

y Slope of line CD $= \frac{-5}{8}$ ½

	∴ Slope of line AB and slope of line CD are equal. ∴ line AB ∥ line CD ∴ **The line joining (– 1, 1) and (– 9, 6) is parallel to the line joining (– 2, 14) and (6, 9).**	1
A.4.	**Solve ANY TWO of the following :**	
(i)	L.H.S. $= \sqrt{\dfrac{\operatorname{cosec} x - 1}{\operatorname{cosec} x + 1}}$	
	$= \sqrt{\dfrac{(\operatorname{cosec} x - 1)(\operatorname{cosec} x - 1)}{(\operatorname{cosec} x + 1)(\operatorname{cosec} x - 1)}}$	½
	$= \sqrt{\dfrac{(\operatorname{cosec} x - 1)^2}{\operatorname{cosec}^2 x - 1}}$	½
	$= \sqrt{\dfrac{(\operatorname{cosec} x - 1)^2}{\cot^2 x}}$ $\left[\begin{array}{l} 1 + \cot^2 x = \operatorname{cosec}^2 x \\ \therefore \cot^2 x = \operatorname{cosec}^2 x - 1 \end{array}\right]$	½
	$= \dfrac{\operatorname{cosec} x - 1}{\cot x}$	
	$= \dfrac{\operatorname{cosec} x}{\cot x} - \dfrac{1}{\cot x}$	
	$= \dfrac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} - \tan x$	1
	$= \dfrac{1}{\cos x} - \tan x$ $= \sec x - \tan x$	
	$= \dfrac{(\sec x - \tan x)(\sec x + \tan x)}{(\sec x + \tan x)}$	½
	$= \dfrac{\sec^2 x - \tan^2 x}{\sec x + \tan x}$	
	$= \dfrac{1}{\sec x + \tan x}$ $\left[\begin{array}{l} 1 + \tan^2 x = \sec^2 x \\ \therefore \sec^2 x - \tan^2 x = 1 \end{array}\right]$	
	$=$ R.H.S.	1
	∴ $\sqrt{\dfrac{\textbf{cosec x – 1}}{\textbf{cosec x + 1}}} = \dfrac{\mathbf{1}}{\textbf{sec x + tan x}}$	

(ii) **(Analytical figure)**



1 mark for triangle
1 mark for angle bisectors
1 mark for perpendicular
1 mark for incircle

(iii) L.H.S. $= \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$= \dfrac{\sin\theta / \cos\theta}{1 - \cos\theta / \sin\theta} + \dfrac{\cos\theta / \sin\theta}{1 - \sin\theta / \cos\theta}$ ½

$= \dfrac{\sin\theta / \cos\theta}{\dfrac{\sin\theta - \cos\theta}{\sin\theta}} + \dfrac{\cos\theta / \sin\theta}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}}$ ½

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \dfrac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$ ½

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \dfrac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$

$= \dfrac{1}{\sin\theta - \cos\theta}\left[\dfrac{\sin^2\theta}{\cos\theta} - \dfrac{\cos^2\theta}{\sin\theta}\right]$ ½

$$= \frac{1}{\sin\theta - \cos\theta} \times \frac{\sin^3\theta - \cos^3\theta}{\cos\theta \cdot \sin\theta}$$

$$= \frac{1}{\sin\theta - \cos\theta} \times \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta \cdot \cos\theta + \cos^2\theta)}{\cos\theta \cdot \sin\theta}$$ ½

$$= \frac{\sin^2\theta + \sin\theta \, . \cos\theta + \cos^2\theta}{\cos\theta \, . \sin\theta}$$

$$= \frac{1 + \sin\theta \, . \cos\theta}{\cos\theta \, . \sin\theta}$$ $[\because \sin^2\theta + \cos^2\theta = 1]$ ½

$$= \frac{1}{\cos\theta \, . \sin\theta} + \frac{\sin\theta \, . \cos\theta}{\cos\theta \, . \sin\theta}$$ ½

$$= \frac{1}{\cos\theta} \times \frac{1}{\sin\theta} + 1$$

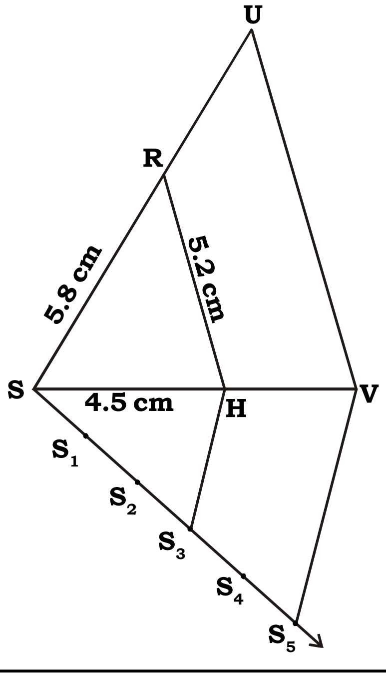
$$= \sec\theta \, . \operatorname{cosec}\theta + 1$$ $\left[\because \sec\theta = \frac{1}{\cos\theta}, \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$ ½

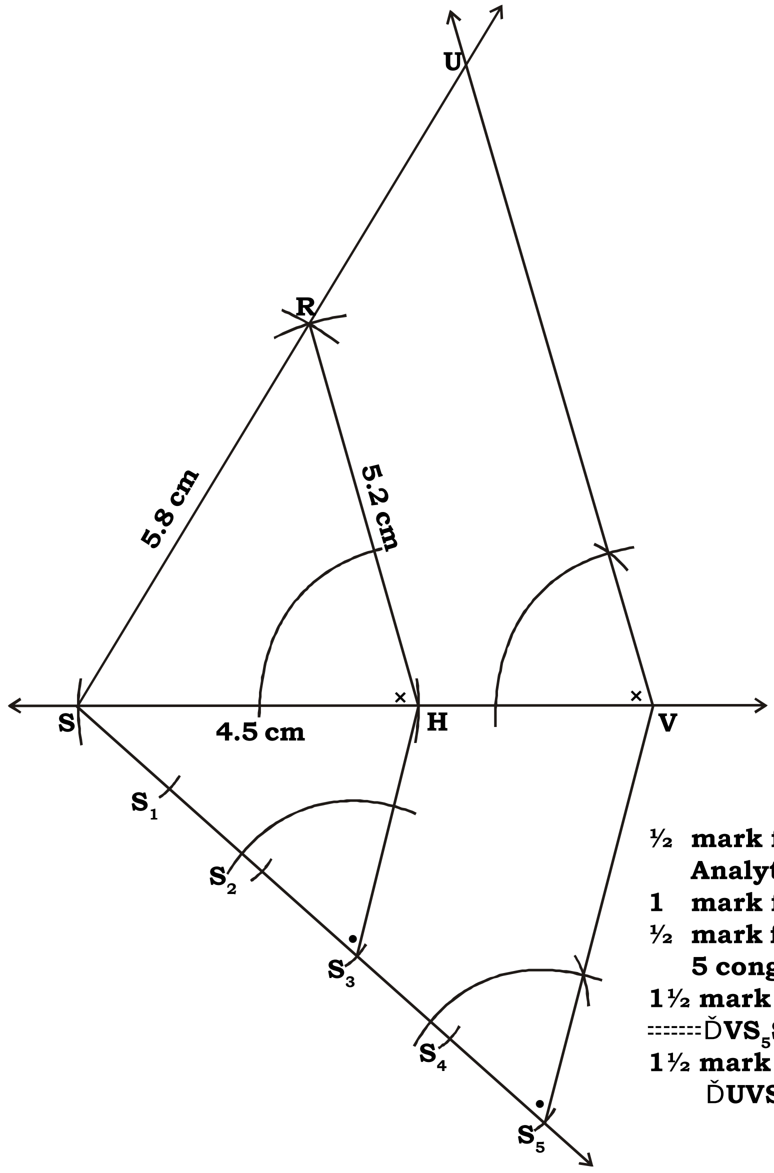
= R.H.S.

$$\therefore \frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \, . \operatorname{cosec}\theta.$$

A.5. **Solve ANY TWO of the following :**

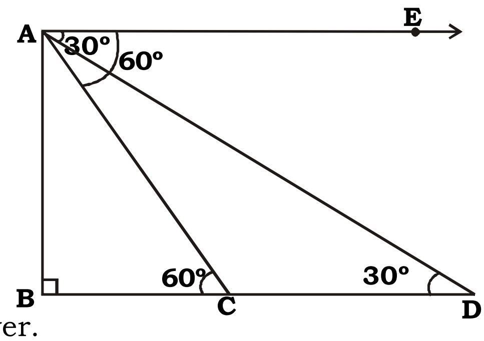
(i) **(Analytical figure)**





½ mark for drawing Analytical figure
1 mark for aSHR
½ mark for constructing 5 congruent parts
1½ mark for constructing ĎVS$_5$S] HS$_3$S
1½ mark for constructing ĎUVS] ĎRHS

(ii)



Seg AB represents a tower.
Let A be a position of an observer.
D and C are the initial and final position of the car. ½
ĎEAD and ĎEAC are angles of depression

m ĎEAD = m ĎADB = 30° } [Converse of alternate angles ½
m ĎEAC = m ĎACB = 60° } test]

The car took 6 sec. to travel from D to C

Let the speed of car be x units/seconds

Distance = speed × time

y CD = x × 6

y CD = 6x units ......(i) ½

In right angled aABC

$\tan 60 = \frac{AB}{BC}$ [By definition]

y $\sqrt{3} = \frac{AB}{BC}$

y $AB = \sqrt{3}\,BC$ .....(ii) 1

In right angled aABD,

$\tan 30 = \frac{AB}{BD}$

y $\frac{1}{\sqrt{3}} = \frac{AB}{BD}$

y $AB = \frac{BD}{\sqrt{3}}$ .....(iii)

y $\sqrt{3}\,BC = \frac{BD}{\sqrt{3}}$ [From (ii) and (iii)] 1

y 3BC = BD

y 3BC = BC + CD [∵ B - C - D] ½

y 3BC – BC = CD

y 2BC = CD

y 2BC = 6x [From (i)]

y $BC = \frac{6x}{2}$

y BC = 3x

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$ ½

$= \frac{BC}{x}$

$= \frac{3x}{x}$

= 3 seconds

y **The time taken by the car to reach the foot of the tower is 3 seconds.** ½

(iii)

seg AD is the median of seg BC

y D is midpoint of seg BC

y D ø $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

ø $\left(\frac{-3 + 1}{2}, \frac{-2 + (-8)}{2}\right)$

ø $\left(\frac{-2}{2}, \frac{-2 - 8}{2}\right)$ **1**

ø $\left(-1, \frac{-10}{2}\right)$

ø (– 1, – 5)

By two point form,

The equation of median AD

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ **½**

y $$\frac{x - 5}{5 - (-1)} = \frac{y - 4}{4 - (-5)}$$ **½**

y $$\frac{x - 5}{5 + 1} = \frac{y - 4}{4 + 5}$$

y $$\frac{x - 5}{6} = \frac{y - 4}{9}$$

y $9(x - 5) = 6(y - 4)$

y $9x - 45 = 6y - 24$

y $9x - 6y - 45 + 24 = 0$

y $9x - 6y - 21 = 0$

y $3x - 2y - 7 = 0$ [Dividing throughout by 3] **1**

y The equation of median AD is $3x - 2y - 7 = 0$

Slope of line AC $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-8 - 4}{1 - 5}$

$= \frac{-12}{-4}$

$= 3$ **½**

$\because$	Slope of parallel lines are equal Slope of the line parallel to line AC is 3 The line passes through B (– 3, – 2)	
y	The equation of the line parallel to line AC passing through point B by the slope point form is	½
	$y - y_1 = m(x - x_1)$	
y	$y - (-2) = 3[x - (-3)]$	
y	$y + 2 = 3(x + 3)$	
y	$y + 2 = 3x + 9$	
y	$3x - y + 9 - 2 = 0$	
y	$3x - y + 7 = 0$	1
y	The equation of the line parallel to AC passing through point B is $3x - y + 7 = 0$	

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