MT

2017 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 5

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i)
Slope of the line (m) = 0
y intercept of the line (c) = 2 ½
By slope intercept form,
The equation of the line is y = mx + c
y y = (0)x + 2
y y = 2
y **The equation of the given line is y = 2** ½

(ii)
sec ¯ = 2 [Given]
But, sec 60 = 2 ½
y sec ¯ = sec 60
y **¯ = 60°** ½

(iii)
Slope of the line (m) = 2
Its y-intercepts (c) = 5 ½
y Equation of the line by slope-intercept form,
y = mx + c
y **y = 2x + 5** ½

(iv)
3 sin ¯ – 4 cos ¯ = 0
y 3 sin¯ = 4 cos ¯ ½
y $\frac{\sin ¯}{\cos ¯} = \frac{4}{3}$
y **$\tan ¯ = \frac{4}{3}$** ½

(v)
3 (x + 3) = y – 1
y y – 1 = 3 (x + 3) ½
Comparing with the equation of a line in slope point form,
$y - y_1 = m (x - x_1)$
y m = 3
y **Slope of the line 3 (x + 3) = y – 1 is 3.** ½

(vi)

$\bar{\ } = -30^\circ$ [Given]

$\sin \bar{\ } = \sin(-30)$ ½

$= -\sin 30$

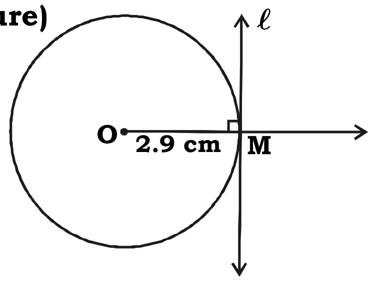
$= -\frac{1}{2}$

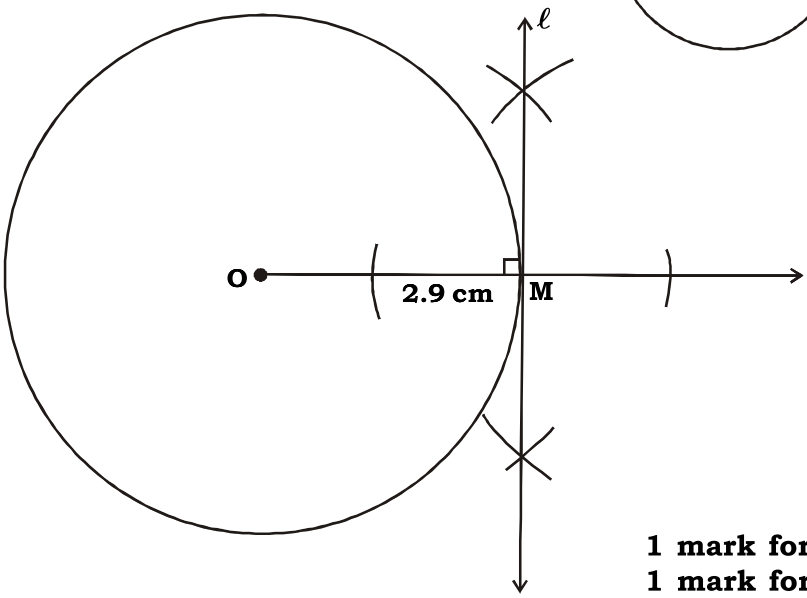
y $\boxed{\sin(-30) = -\frac{1}{2}}$ ½

A.2. Solve ANY FOUR of the following :

(i)

(Analytical figure)





1 mark for circle
1 mark for tangent

(ii) The terminal arm passes through P (1, – 1)

y x = 1 and y = – 1 ½

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{1+1}$

$= \sqrt{2}$

y $r = \sqrt{2}$ units ½

Let the angle be ¯

y $\sin ¯ = \frac{y}{r} = \frac{-1}{\sqrt{2}}$ $\quad$ $\text{cosec } ¯ = \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

$\cos ¯ = \frac{x}{r} = \frac{1}{\sqrt{2}}$ $\quad$ $\sec ¯ = \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2}$ **1**

$\tan ¯ = \frac{y}{x} = \frac{-1}{1} = -1$ $\quad$ $\cot ¯ = \frac{x}{y} = \frac{1}{-1} = -1$

(iii) Let A ø (3, 4) ø (x_1, y_1) and m = 5

The equation of the line passing through A and having slope 5 by slope point form is, ½

$y - y_1 = m(x - x_1)$ ½

y $y - 4 = 5(x - 3)$

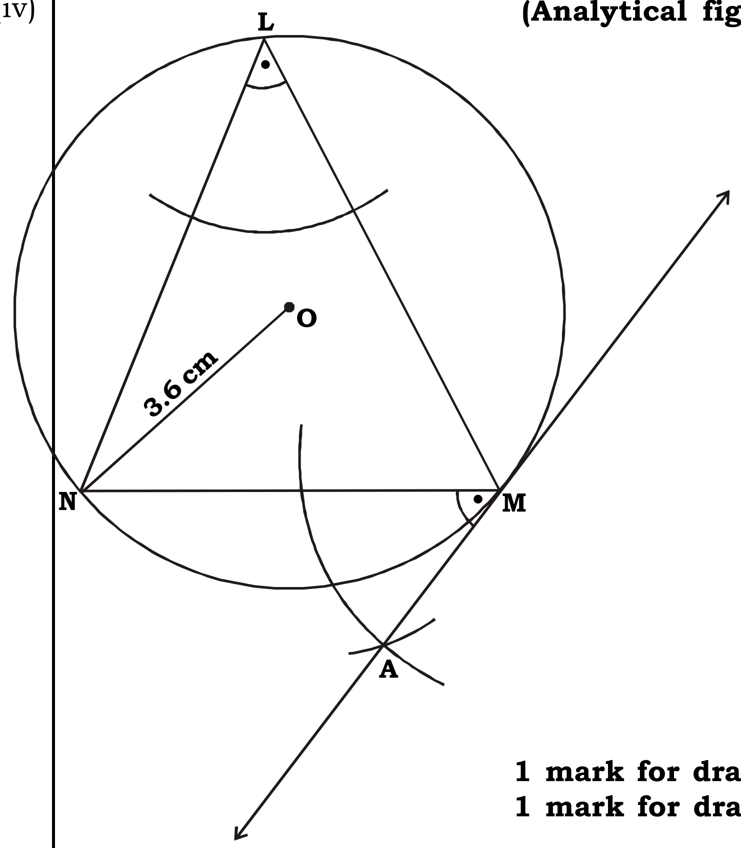
y $y - 4 = 5x - 15$

y $5x - y - 15 + 4 = 0$

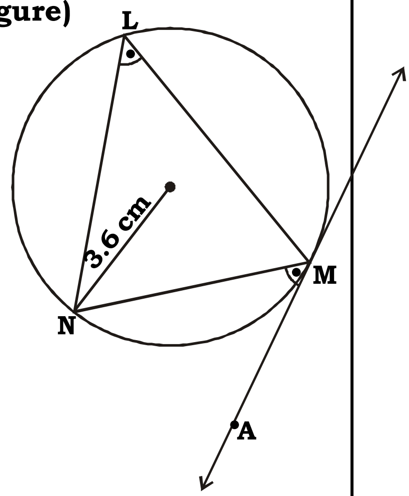
y $5x - y - 11 = 0$ **1**

y The equation of the line passing through the points (3, 4) and having slope 5 is 5x – y – 11 = 0.

(iv)



(Analytical figure)



1 mark for drawing circle
1 mark for drawing tangent

(v)

$$\tan A + \frac{1}{\tan A} = 2 \quad \text{[Given]}$$

y $\left(\tan A + \frac{1}{\tan A}\right)^2 = 4$ [Squaring both sides] ½

y $\tan^2 A + 2 \tan A \cdot \frac{1}{\tan A} + \frac{1}{\tan^2 A} = 4$ ½

y $\tan^2 A + 2 + \frac{1}{\tan^2 A} = 4$ ½

y $\tan^2 A + \frac{1}{\tan^2 A} = 4 - 2$ ½

y $\boldsymbol{\tan^2 A + \frac{1}{\tan^2 A} = 2}$

(vi)

Let, A ø (– 3, 5) ø (x_1, y_1)

B ø (4, – 7) ø (x_2, y_2)

The line passes through points A and B ½

y The equation of the line by two point form is

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

y $\frac{x - (-3)}{-3 - 4} = \frac{y - 5}{5 - (-7)}$

y $\frac{x + 3}{-7} = \frac{y - 5}{12}$

y $12(x + 3) = -7(y - 5)$ ½

y $12x + 36 = -7y + 35$

y $7y = -12x + 35 - 36$

y $7y = -12x - 1$

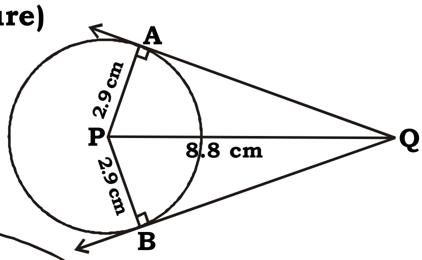
y $y = \frac{-12}{7}x - \frac{1}{7}$ [Dividing throughout by 7]

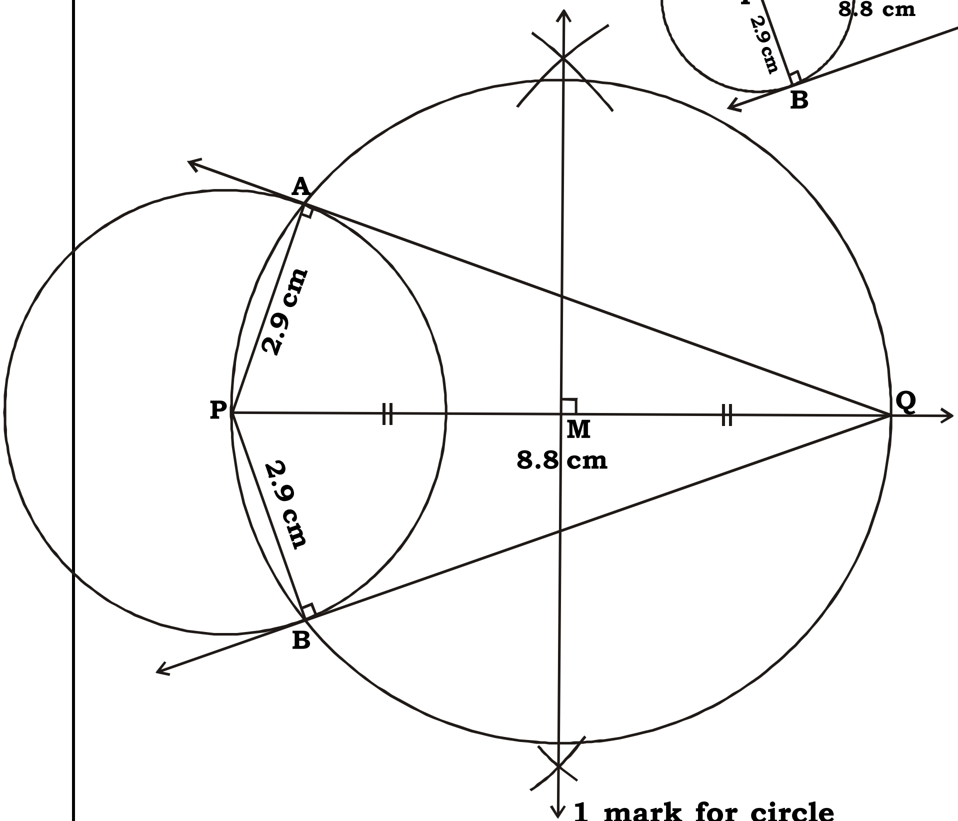
y $y = \frac{-12}{7}x - \frac{1}{7}$ is the equation of the line passing through (– 3, 5) and (4, – 7) ½

A.3. **Solve ANY THREE of the following :**

(i)

(Analytical figure)





1 mark for circle
1 mark for perpendicular bisector
1 mark for tangents

(ii)

$\sqrt{3}\tan \text{–} = 3\sin \text{–}$

$\sqrt{3}\dfrac{\sin \text{–}}{\cos \text{–}} = 3\sin \text{–}$ ½

y $\dfrac{\sqrt{3}}{\cos \text{–}} = 3$

y $\cos \text{–} = \dfrac{\sqrt{3}}{3}$

y $\cos^2 \text{–} = \dfrac{3}{9}$

y $\cos^2 \text{–} = \dfrac{1}{3}$ ....(i) ½

$\sin^2$ ¯ + $\cos^2$ ¯ = 1

y $\sin^2$ ¯ = 1 – $\cos^2$ ¯ ½

= $1 - \frac{1}{3}$ [From (i)]

= $\frac{3-1}{3}$

y $\sin^2$ ¯ = $\frac{2}{3}$ ......(ii) ½

y $\sin^2$ ¯ – $\cos^2$ ¯ = $\frac{2}{3} - \frac{1}{3}$ [From (i) and (ii)]

= $\frac{2-1}{3}$

y **[** $\sin^2$ ¯ – $\cos^2$ ¯ = $\frac{1}{3}$ **]** **1**

(iii) Let, A ø (k, 3),

B ø (2, – 4),

C ø (– k + 1, – 2)

∵ Points A, B and C are collinear

Slope of line AB = Slope of line BC ½

y $\frac{-4-3}{2-k} = \frac{-2-(-4)}{(-k+1)-2}$ ½

y $\frac{-7}{2-k} = \frac{-2+4}{-k+1-2}$

y $\frac{-7}{2-k} = \frac{2}{-k-1}$ ½

y – 7 (– k – 1) = 2 (2 – k) ½

y 7k + 7 = 4 – 2k

y 7k + 2k = 4 – 7

y 9k = – 3

y k = $\frac{-3}{9}$ ½

y k = $\frac{-1}{3}$

y **[** The value of k is $\frac{-1}{3}$. **]** ½

(iv)

$$\text{L.H.S.} = \frac{\tan\theta}{\sec\theta + 1} + \frac{\sec\theta + 1}{\tan\theta}$$

$$= \frac{\tan^2\theta + (\sec\theta + 1)^2}{(\sec\theta + 1)\tan\theta}$$ ½

$$= \frac{\tan^2\theta + \sec^2\theta + 2\sec\theta + 1}{(\sec\theta + 1)\tan\theta}$$ ½

$$= \frac{\sec^2\theta + \sec^2\theta + 2\sec\theta}{(\sec\theta + 1)\tan\theta} \quad [\because 1 + \tan^2\theta = \sec^2\theta]$$ ½

$$= \frac{2\sec^2\theta + 2\sec\theta}{(\sec\theta + 1)\tan\theta}$$

$$= \frac{2\sec\theta(\sec\theta + 1)}{(\sec\theta + 1)\tan\theta}$$ ½

$$= \frac{2\sec\theta}{\tan\theta}$$

$$= 2\sec\theta \div \tan\theta$$ ½

$$= 2 \times \frac{1}{\cos\theta} \div \frac{\sin\theta}{\cos\theta}$$

$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} \quad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}, \sec\theta = \frac{1}{\cos\theta}\right]$$

$$= \frac{2}{\sin\theta}$$ ½

$$= 2\,\text{cosec}\,\theta$$

$$= \text{R.H.S.}$$

$$\therefore \mathbf{\frac{\tan\theta}{\sec\theta + 1} + \frac{\sec\theta + 1}{\tan\theta} = 2\,cosec\,\theta}$$

(v)

Let, $A \equiv (x, -2) \equiv (x_1, y_1)$

$B \equiv (8, -11) \equiv (x_2, y_2)$ ½

Slope of line AB $= \frac{-3}{4}$ [Given]

Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

$$\therefore \frac{-3}{4} = \frac{-11 - (-2)}{8 - x}$$

$$\therefore \frac{-3}{4} = \frac{-11 + 2}{8 - x}$$ ½

$$\therefore \frac{-3}{4} = \frac{-9}{8 - x}$$ ½

y $3(8 - x) = 9 \times 4$

y $24 - 3x = 36$

y $3x = 24 - 36$ ½

y $3x = -12$

y $x = \frac{-12}{3}$

y $x = -4$ ½

y **The value of x is – 4.**

A.4. Solve ANY TWO of the following :

(i) $\sec{}^{-} + \tan{}^{-} = p$ ½

y $\frac{1}{\cos{}^{-}} + \frac{\sin{}^{-}}{\cos{}^{-}} = p$

y $\frac{1 + \sin{}^{-}}{\cos{}^{-}} = p$ ½

y $\frac{(1 + \sin{}^{-})^2}{\cos^2{}^{-}} = p^2$

y $\frac{(1 + \sin{}^{-})^2}{1 - \sin^2{}^{-}} = p^2$ $\left[\begin{array}{l}\sin^2{}^{-} + \cos^2{}^{-} = 1 \\ \text{y } \cos^2{}^{-} = 1 - \sin^2{}^{-}\end{array}\right]$ **1**

y $\frac{(1 + \sin{}^{-})^2}{(1 + \sin{}^{-})(1 - \sin{}^{-})} = p^2$ ½

y $\frac{1 + \sin{}^{-}}{1 - \sin{}^{-}} = p^2$

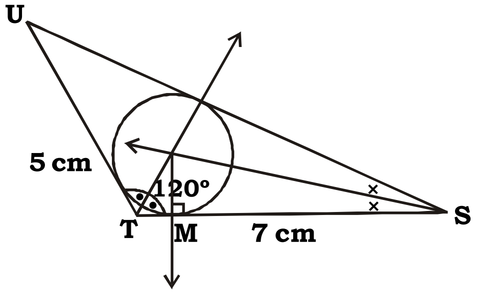
y $\frac{1 + \sin{}^{-} + 1 - \sin{}^{-}}{1 + \sin{}^{-} - 1 + \sin{}^{-}} = \frac{p^2 + 1}{p^2 - 1}$ [By Componendo-Dividendo] ½

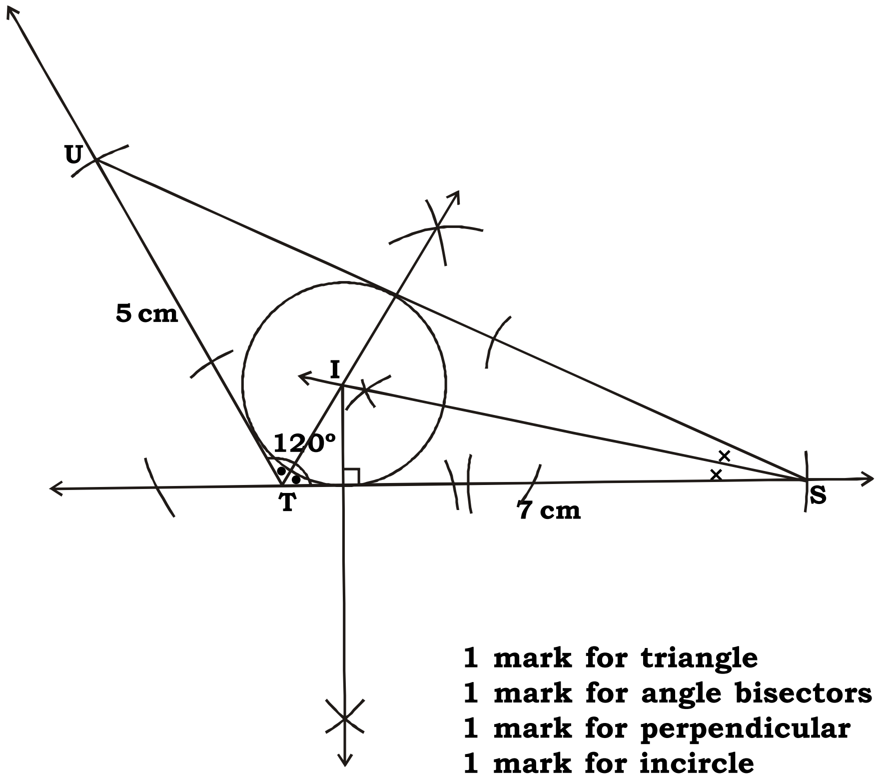
y $\frac{2}{2 \sin{}^{-}} = \frac{p^2 + 1}{p^2 - 1}$

y $\frac{1}{\sin{}^{-}} = \frac{p^2 + 1}{p^2 - 1}$ ½

y $\mathbf{\frac{p^2 - 1}{p^2 + 1} = \sin{}^{-}}$ [By Invertendo] ½

(ii) **(Analytical figure)**





1 mark for triangle
1 mark for angle bisectors
1 mark for perpendicular
1 mark for incircle

(iii)

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \sin^2\theta$

$\therefore \cos\theta \cdot \cos\theta = (1 - \sin\theta)(1 + \sin\theta)$ ½

$\therefore \dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$ ½

By theorem on equal ratios,

$\therefore \dfrac{1+\sin\theta-\cos\theta}{\cos\theta-(1-\sin\theta)} = \dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

$\therefore \dfrac{1+\sin\theta-\cos\theta}{\cos\theta-(1-\sin\theta)} = \dfrac{\cos\theta}{1-\sin\theta}$ 1

Dividing the numerator and denominator of R.H.S. by $\cos\theta$

$\therefore \dfrac{1+\sin\theta-\cos\theta}{\cos\theta-1+\sin\theta} = \dfrac{\cos\theta/\cos\theta}{(1-\sin\theta)/\cos\theta}$ 1

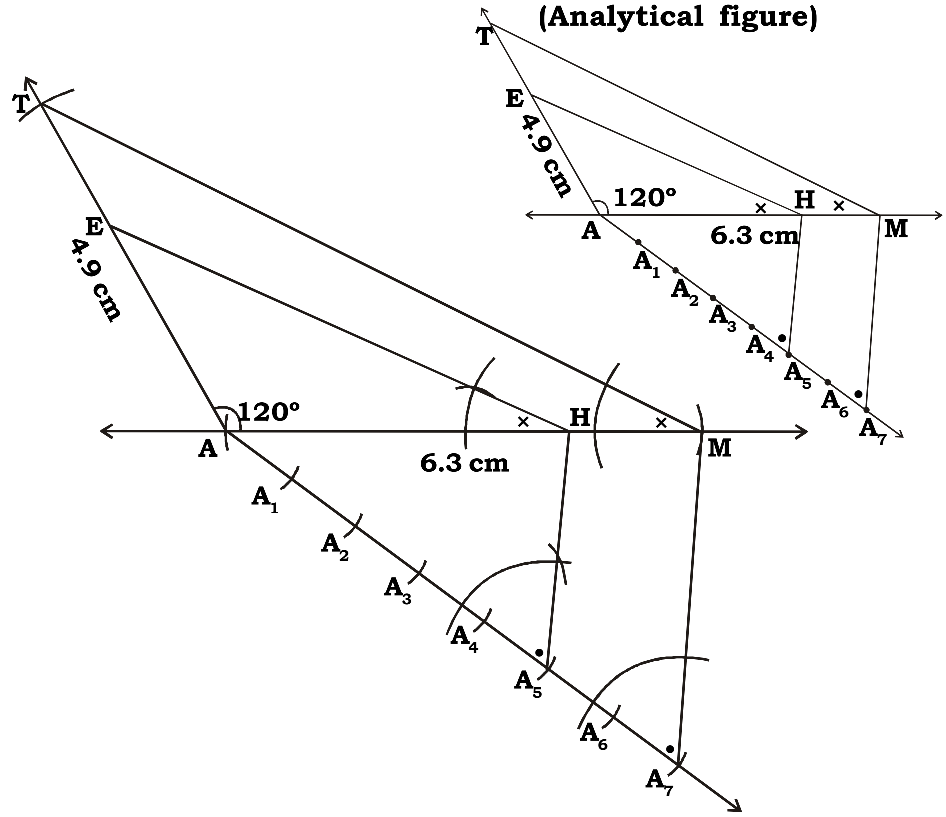
$\therefore \dfrac{1+\sin\theta-\cos\theta}{\cos\theta-1+\sin\theta} = \dfrac{1}{\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}}$ ½

$\therefore \boldsymbol{\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \dfrac{1}{\sec\theta-\tan\theta}}$ ½

A.5. **Solve ANY TWO of the following :**

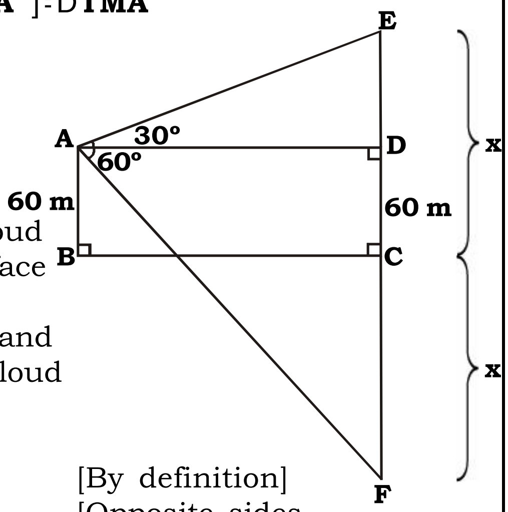
(i)

(Analytical figure)



½ mark for drawing analytical figure
1 mark for ∠AMT
½ mark for constructing 7 congruent parts
1½ mark for constructing $\angle HA_5A \cong \angle MA_7A$
1½ mark for constructing $\angle EHA \cong \angle TMA$

(ii)



Let E be the position of the cloud and let BC represent the surface of the lake. ½

Let A be the point of observer and let F be the reflection of the cloud

∴ EC = CF

Let EC = CF = x m

□ABCD is a rectangle [By definition] ½

∴ AB = CD = 60 m [Opposite sides of rectangle] y EC = ED + DC [E - D - C]

y x = ED + 60

y ED = (x – 60)m ½

Also,

DF = DC + CF [D - C - F] ½

y DF = (60 + x)

y DF = (x + 60) m

In right angled aADE,

$\tan 30° = \frac{ED}{AD}$ [By definition]

y $\frac{1}{\sqrt{3}} = \frac{x - 60}{AD}$

y $AD = \sqrt{3}\,(x - 60)\,m$ **1**

In right angled aADF,

$\tan 60° = \frac{DF}{AD}$ [By definition]

y $\sqrt{3} = \frac{x + 60}{\sqrt{3}\,(x - 60)}$ **1**

y $3\,(x - 60) = x + 60$

y $3x - 180 = x + 60$

y $3x - x = 60 + 180$

y $2x = 240$

y $x = 120$ **1**

y **The height of the cloud above the lake is 120 m.**

(iii) Let point P be the point of intersection of lines 4x + 3y + 2 = 0 and 6x + 5y + 6 = 0

$4x + 3y + 2 = 0$

y $4x + 3y = -2$ ……(i)

Multiplying throughout by 3 we get,

y $12x + 9y = -6$ …..(ii) ½

$6x + 5y + 6 = 0$

y $6x + 5y = -6$ …..(iii) ½

Multiplying throughout by – 2 we get,

$-12x - 10y = 12$ ……(iv)

Adding (ii) and (iv),

$$\begin{array}{r} 12x + 9y = -6 \\ -12x - 10y = 12 \\ \hline -y = 6 \end{array}$$

y $y = -6$ ½

Substituting y = – 6 in equation (i),

$4x + 3(-6) = -2$

y $4x - 18 = -2$

y $4x = -2 + 18$

y $4x = 16$

y $x = \frac{16}{4}$

y $x = 4$

y P ø (4, – 6) ½

Let Q be the point of intersection of lines $4x - 3y - 17 = 0$ and $2x + 3y + 5 = 0$

$4x - 3y - 17 = 0$

y $4x - 3y = 17$ .....(v) ½

$2x + 3y + 5 = 0$

y $2x + 3y = -5$ .....(vi) ½

Adding equation (v) and (vi),

$4x - 3y = 17$

$2x + 3y = -5$

y $6x = 12$

y $x = 2$ ½

x = 2 in equation (v),

$4 \times 2 - 3y = 17$

y $-3y = 17 - 8$

y $-3y = 9$

y $y = -3$

y Q ø (2, – 3) ½

The equation of line PQ by two point from,

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$ ½

y $\frac{y - (-6)}{-6 - (-3)} = \frac{x - 4}{4 - 2}$

y $\frac{y + 6}{-6 + 3} = \frac{x - 4}{2}$

y $\frac{y + 6}{-3} = \frac{x - 4}{2}$

y $2(y + 6) = -3(x - 4)$

y $2y + 12 = -3x + 12$ ½

y $3x + 2y + 12 - 12 = 0$

y $3x + 2y = 0$

y **The required equation of line is 3x + 2y = 0.**

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