MT

2017 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 6

Time : 2 Hours | **Model Answer Paper** | **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) Slope of the line (m) = 0
y intercept of the line (c) = – 3 ½
By slope intercept form,
The equation of the line is y = mx + c
y y = (0)x + (– 3)
y y = 0 – 3
y y = – 3
y The equation of the given line is y = – 3 ½

(ii) (1 + cot² ¯) (1 + cos ¯) (1 – cos ¯) ½
= cosec² ¯ (1 – cos² ¯) [∵ 1 + cot² ¯ = cosec² ¯]
= cosec² ¯ × sin² ¯ [sin² ¯ + cos² ¯ = 1, y sin² ¯ = 1 – cos² ¯]
= $\frac{1}{\sin^2 ¯} \times \sin^2 ¯$ ½
= 1

(iii) A (2, 5) and B (4, 1)
y Co-ordinates of the midpoint of line segment AB by midpoint formula is,
$\left(\frac{2+4}{2}, \frac{5+1}{2}\right)$
= $\left(\frac{6}{2}, \frac{6}{2}\right)$
= (3, 3)

(iv) ¯ + ~ = 90° [Given] ½
y ¯ = (90 – ~)
cosec ¯ = $\sqrt{2}$ [Given]
y sec ~ = cosec (90 – ~) [∵ sec ¯ = cosec (90 – ¯)]
y sec a = cosec ¯
y sec ~ = $\sqrt{2}$ ½

(v) $y - 5 = 2(x - 7)$

Comparing with the equation of a line in slope point form,

$y - y_1 = m(x - x_1)$ ½

y $m = 2$

y **Slope of the line $y - 5 = 2(x - 7)$ is 2** ½

(vi) ~ + = 90º [Given]

tan ~ = $\frac{3}{4}$ [Given] ½

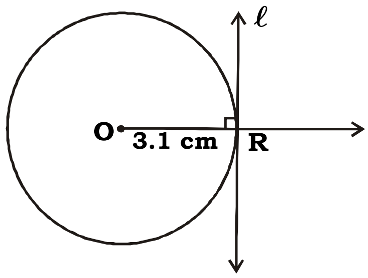
cot = tan ~ [∵ cot ¯ = tan (90 – ¯)]

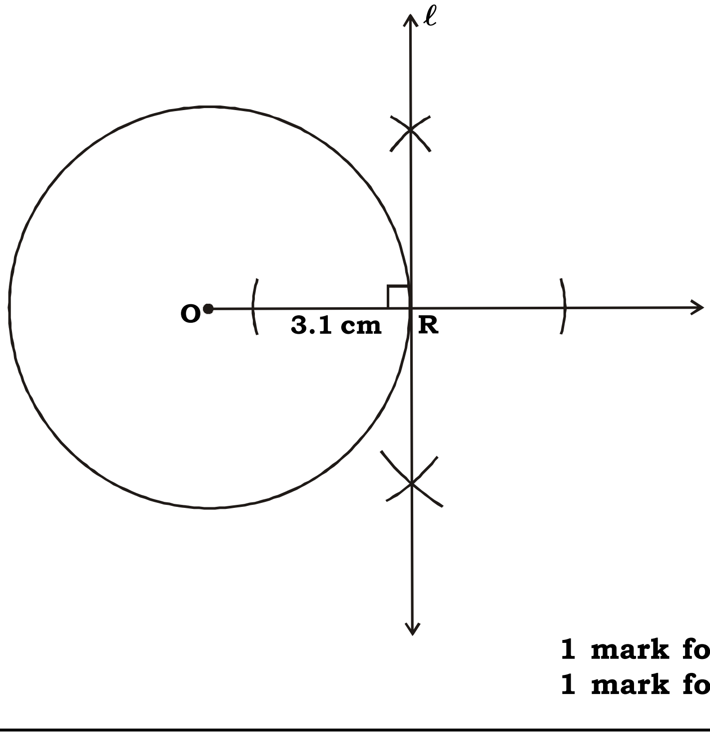
y cot = $\frac{3}{4}$ ½

A.2. Solve ANY FOUR of the following :

(i)

(Analytical figure)





1 mark for circle
1 mark for tangent

(ii) The terminal arm passes through P (– 2, – 3)

y x = – 2 and y = – 3

r $= \sqrt{x^2 + y^2}$ ½

$= \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

y r $= \sqrt{13}$ units ½

Let the angle be ¯

y sin ¯ $= \frac{y}{r} = \frac{-3}{\sqrt{13}}$ cosec ¯ $= \frac{r}{y} = \frac{\sqrt{13}}{-3} = \frac{-\sqrt{13}}{3}$

cos ¯ $= \frac{x}{r} = \frac{-2}{\sqrt{13}}$ sec ¯ $= \frac{r}{x} = \frac{\sqrt{13}}{-2} = \frac{-\sqrt{13}}{2}$ 1

tan ¯ $= \frac{y}{x} = \frac{-3}{-2} = \frac{3}{2}$ cot ¯ $= \frac{x}{y} = \frac{-2}{-3} = \frac{2}{3}$

(iii) P ø (2, 4), Q ø (3, 6), R ø (8, 1), S ø (10, k)

Line PQ is parallel to line RS ½

y Slope of line PQ = Slope of line RS

y $\frac{6 - 4}{3 - 2} = \frac{k - 1}{10 - 8}$ ½

y $\frac{2}{1} = \frac{k - 1}{2}$

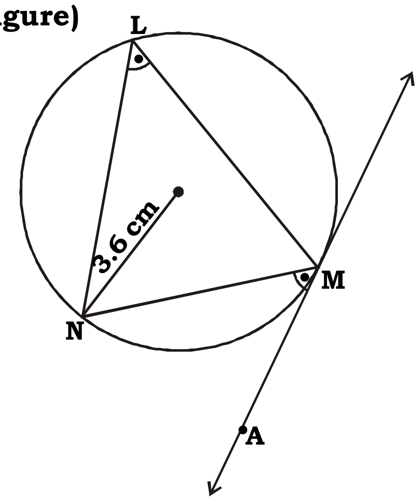
y 4 = k – 1

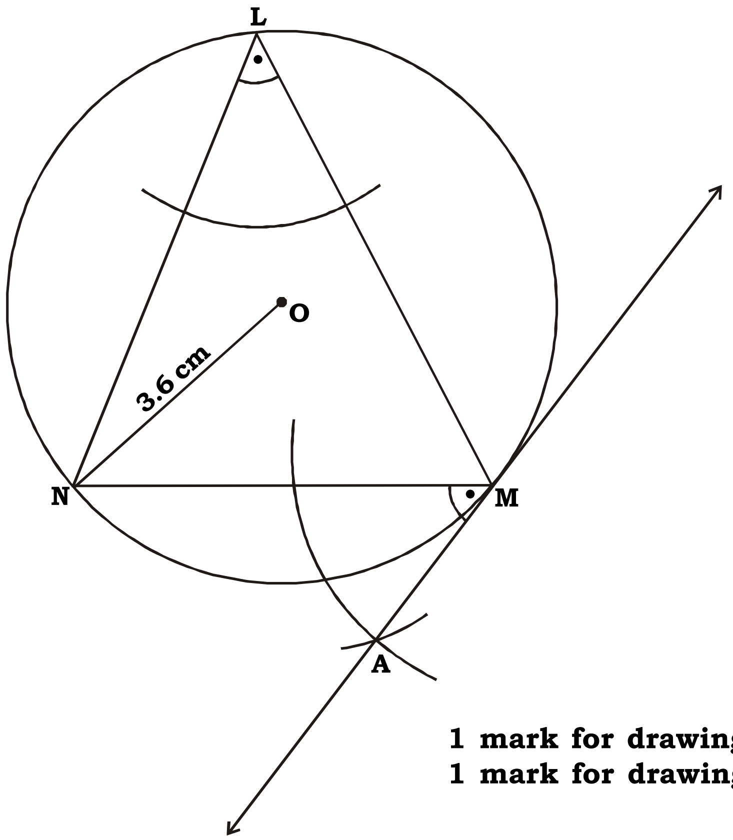
y k = 4 + 1

y k = 5 1

y **Value of k is 5.**

(iv) **(Analytical figure)**





1 mark for drawing circle
1 mark for drawing tangent

(v) L.H.S. $= \sec^2\theta + \operatorname{cosec}^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$ $\left[\because \sec\theta = \dfrac{1}{\cos\theta}, \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}\right]$ ½

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \cdot \sin^2\theta}$ ½

$= \dfrac{1}{\cos^2\theta \cdot \sin^2\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$ ½

$= \sec^2\theta \cdot \operatorname{cosec}^2\theta$

$=$ R.H.S. ½

$\therefore$ $\mathbf{\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \cdot \operatorname{cosec}^2\theta}$

(vi) Let, $A \equiv (-2, -5) \equiv (x_1, y_1)$

$B \equiv (-4, -3) \equiv (x_2, y_2)$

The line passes through points A and B

$\therefore$ The equation of the line by two point form is

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

$\therefore$ $$\frac{x - (-2)}{-2 - (-4)} = \frac{y - (-5)}{-5 - (-3)}$$ ½ y $\frac{x + 2}{-2 + 4} = \frac{y + 5}{-5 + 3}$

y $\frac{x + 2}{2} = \frac{y + 5}{-2}$

y $-2(x + 2) = 2(y + 5)$

y $-2x - 4 = 2y + 10$

y $2y = -2x - 4 - 10$ ½

y $2y = -2x - 14$

y $y = -x - 7$ [Dividing throughout by 2] ½

y y = – x – 7 is the equation of the line passing through (– 2, – 5) and (– 4, – 3).

A.3. **Solve ANY THREE of the following :**

(i) **Analysis :**

In aPQR,

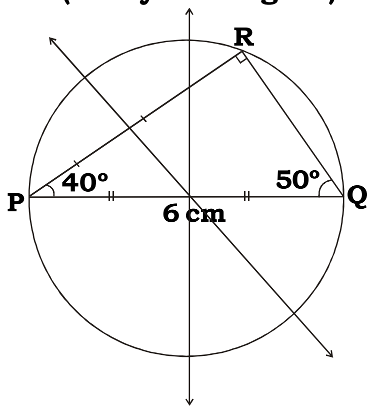
ĎP + ĎQ + ĎR = 180

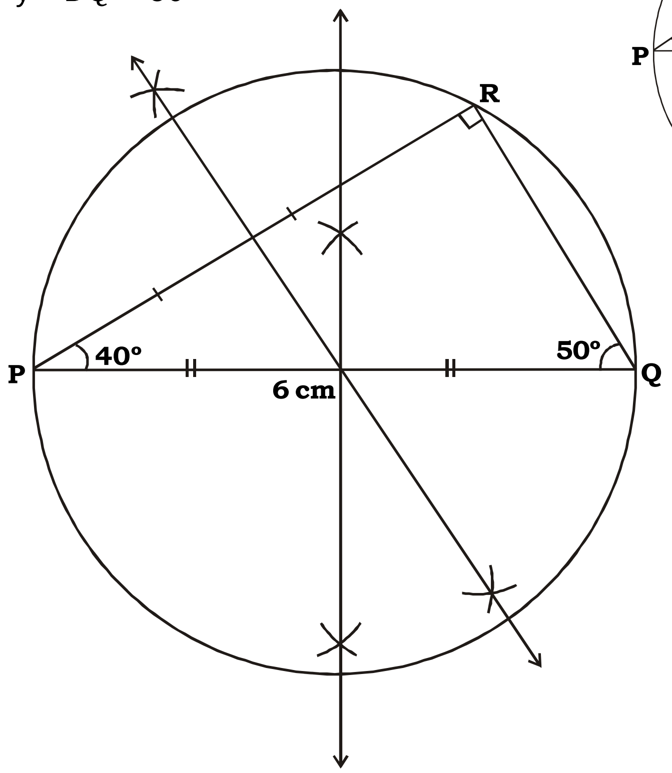
y 40 + ĎQ + 90 = 180

y ĎQ = 180 – 130

y ĎQ = 50°

(Analytical figure)





1 mark for drawing a PQR
1 mark for drawing perpendicular bisectors
1 mark for drawing circumcircle

(ii)

$3 \sin \sim - 4 \cos \sim = 0$

y $3 \sin \sim = 4 \cos \sim$

y $\dfrac{\sin \sim}{\cos \sim} = \dfrac{4}{3}$

y $\tan \sim = \dfrac{4}{3}$ **½**

$1 + \tan^2 \sim = \sec^2 \sim$

y $1 + \left(\dfrac{4}{3}\right)^2 = \sec^2 \sim$

y $1 + \dfrac{16}{9} = \sec^2 \sim$

y $\dfrac{9 + 16}{9} = \sec^2 \sim$

y $\dfrac{25}{9} = \sec^2 \sim$

y $\sec \sim = \dfrac{5}{3}$ [Taking square roots]

$\cot \sim = \dfrac{1}{\tan \sim}$

$= \dfrac{1}{4/3}$ **1**

y $\cot \sim = \dfrac{3}{4}$ **½**

$1 + \cot^2 \sim = \text{cosec}^2 \sim$

y $1 + \left(\dfrac{3}{4}\right)^2 = \text{cosec}^2 \sim$

y $1 + \dfrac{9}{16} = \text{cosec}^2 \sim$

y $\dfrac{16 + 9}{16} = \text{cosec}^2 \sim$

y $\dfrac{25}{16} = \text{cosec}^2 \sim$ **1**

y $\boxed{\text{cosec} \sim = \dfrac{5}{4}}$ [Taking square roots]

(iii)

Let, A ø $\left(\dfrac{2}{5}, \dfrac{1}{3}\right)$ ø (x_1, y_1)

B ø $\left(\dfrac{1}{2}, k\right)$ ø (x_2, y_2) **½**

C ø $\left(\dfrac{4}{5}, 0\right)$ ø (x_3, y_3)

	$\because$ Points A, B and C are collinear	
	Slope of line AB = Slope of line BC	½
	y $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_3 - y_2}{x_3 - x_2}$	½
	y $\dfrac{k - \frac{1}{3}}{\frac{1}{2} - \frac{2}{5}} = \dfrac{0 - k}{\frac{4}{5} - \frac{1}{2}}$	½
	y $\dfrac{\frac{3k - 1}{3}}{\frac{1}{10}} = \dfrac{\frac{-k}{3}}{\frac{3}{10}}$	
	y $\dfrac{3k - 1}{3} \times 10 = -k \times \dfrac{10}{3}$	½
	y $3k - 1 = -k$	
	y $3k + k = 1$	
	y $4k = 1$	
	y $k = \dfrac{1}{4}$	½
	y **The value of k is $\dfrac{1}{4}$**	
(iv)	L.H.S. $= \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$	
	$= \sqrt{\dfrac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}}$	½
	$= \sqrt{\dfrac{(1 - \cos A)^2}{1 - \cos^2 A}}$	½
	$= \sqrt{\dfrac{(1 - \cos A)^2}{\sin^2 A}}$ $\left[\begin{array}{l}\sin^2 A + \cos^2 A = 1 \\ \text{y } \sin^2 A = 1 - \cos^2 A\end{array}\right]$	1
	$= \dfrac{1 - \cos A}{\sin A}$	
	$= \dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A}$	½
	$=$ cosec A – cot A $\left[\because \text{cosec} - = \dfrac{1}{\sin -}, \cot - = \dfrac{\cos -}{\sin -}\right]$	
	$=$ R.H.S.	½
	y $\boldsymbol{\sqrt{\dfrac{1 - \cos A}{1 + \cos A}} = \text{cosec } A - \cot A}$	

(v) Let A ø (k, – 3) ø (x_1, y_1)
B ø (4, 5) ø (x_2, y_2)

Slope of line AB $= \frac{1}{2}$ [Given]

Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

y $\frac{1}{2} = \frac{5 - (-3)}{4 - k}$ ½

y $\frac{1}{2} = \frac{5 + 3}{4 - k}$ ½

y $\frac{1}{2} = \frac{8}{4 - k}$ ½

y $4 - k = 16$
y $- k = 16 - 4$ ½
y $- k = 12$
y $k = - 12$ ½

y **The value of k is – 12.**

A.4. Solve ANY TWO of the following :

(i) tan ¯ + sin ¯ = m

tan ¯ – sin ¯ = n

$m^2 - n^2$ = (tan ¯ + sin ¯)^2 – (tan ¯ – sin ¯)^2

= tan^2 ¯ + 2 tan ¯.sin ¯ + sin^2 ¯ – [tan^2 ¯ – 2 tan ¯ sin ¯ + sin^2 ¯]

= tan^2 ¯ + 2 tan ¯.sin ¯ + sin^2 ¯ – tan^2 ¯ + 2 tan ¯. sin ¯ – sin^2 ¯ 1

= 4 tan ¯.sin ¯ ....(i)

$4\sqrt{mn}$ = $4\sqrt{(\tan ¯ + \sin ¯)(\tan ¯ - \sin ¯)}$

= $4\sqrt{\tan^2 ¯ - \sin^2 ¯}$ ½

= $4\sqrt{\frac{\sin^2 ¯}{\cos^2 ¯} - \sin^2 ¯}$ ½

= $4\sqrt{\sin^2 ¯\left(\frac{1}{\cos^2 ¯} - 1\right)}$ ½

$= 4\sqrt{\sin^2\theta\left(\sec^2\theta - 1\right)}$ ½

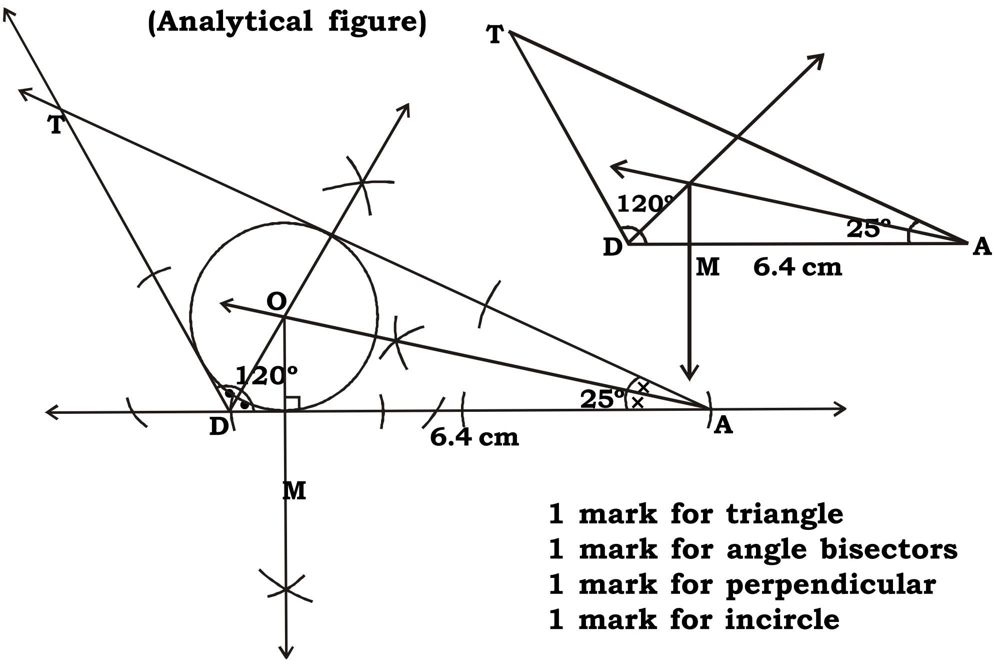
$= 4\sqrt{\tan^2\theta \,.\, \sin^2\theta}$ $\left[\begin{array}{l} 1 + \tan^2\theta = \sec^2\theta \\ \therefore \tan^2\theta = \sec^2\theta - 1 \end{array}\right]$

$= 4 \times \sin\theta \times \tan\theta$ .....(ii) 1

From (i) and (ii),

$\mathbf{m^2 - n^2 = 4\sqrt{mn}}$

(ii) **(Analytical figure)**



1 mark for triangle
1 mark for angle bisectors
1 mark for perpendicular
1 mark for incircle

(iii) L.H.S. $= \dfrac{\cos^2\theta}{1 - \tan\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta}$

$= \dfrac{\cos^2\theta}{1 - \dfrac{\sin\theta}{\cos\theta}} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta}$ ½

$= \dfrac{\cos^2\theta}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta}$ ½

$= \dfrac{\cos^3\theta}{\cos\theta - \sin\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta}$ ½

$= \dfrac{\cos^3\theta}{\cos\theta - \sin\theta} - \dfrac{\sin^3\theta}{\cos\theta - \sin\theta}$ ½

$= \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$ ½

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \cos\theta \, . \, \sin\theta + \sin^2\theta)}{(\cos\theta - \sin\theta)}$$ **1**

$= \cos^2\theta + \sin^2\theta + \sin\theta \, . \, \cos\theta$

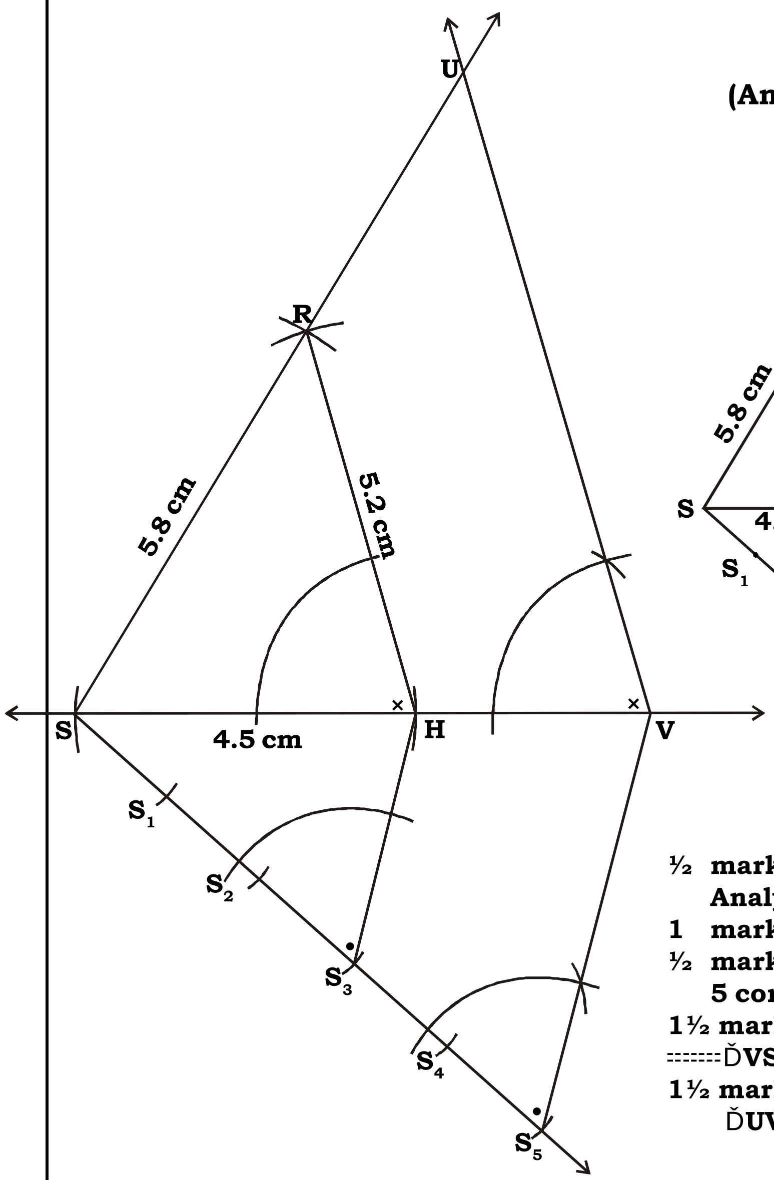
$= 1 + \sin\theta \, . \, \cos\theta$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$=$ R.H.S. **½**

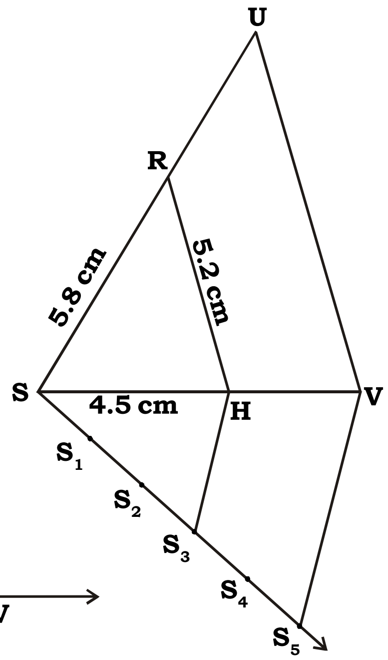
y $\dfrac{\cos^2\theta}{1 - \tan\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta \, . \, \cos\theta$

A.5. **Solve ANY TWO of the following :**

(i)

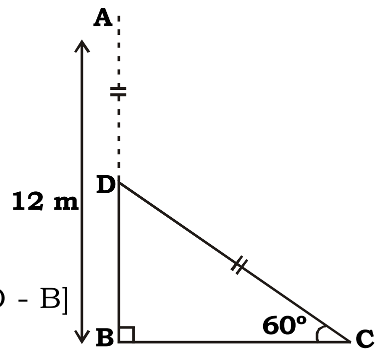


(Analytical figure)



½ mark for drawing Analytical figure
1 mark for aSHR
½ mark for constructing 5 congruent parts
1½ mark for constructing ĎVS$_5$S] HS$_3$S
1½ mark for constructing ĎUVS] ĎRHS

(ii)	seg AB represents the tree	
	AB = 12 m	
	The tree breaks at point D	
	seg AD is the broken part of tree	½
	which then takes the position of DC	
	y AD = DC	
	m ĎDCB = 60°	
	Let DB = x m	
	y AD + DB = AB [∵ A - D - B]	½
	y AD + x = 12	
	y AD = (12 – x) m	
	y DC = (12 – x) m	
	In right angled aDBC,	
	$\sin 60^\circ = \frac{DB}{DC}$ [By definition]	1
	y $\frac{\sqrt{3}}{2} = \frac{x}{12 - x}$	
	y $\sqrt{3}\left(12 - x\right) = 2x$	
	y $12\sqrt{3} - \sqrt{3}x = 2x$	
	y $12\sqrt{3} = 2x + \sqrt{3}x$	
	y $x\left(2 + \sqrt{3}\right) = 12\sqrt{3}$	½
	y $x = \frac{12\sqrt{3}}{2 + \sqrt{3}}$	
	y $DB = \frac{12\sqrt{3}}{2 + \sqrt{3}}$ m	
	y $DB = \frac{12\sqrt{3}\left(2 - \sqrt{3}\right)}{\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)}$	1
	y $DB = \frac{24\sqrt{3} - 12\ (3)}{(2)^2 - \left(\sqrt{3}\right)^2}$	½
	y $DB = \frac{24\sqrt{3} - 36}{4 - 3}$	
	y $DB = \frac{24\ (1.73) - 36}{1}$	½
	y DB = 41.52 – 36	
	y DB = 5.52 m	½
	y The height at which the tree is broken from the bottom by the wind is 5.52 m.	



(iii) seg AD is the median of seg BC

y D is midpoint of seg BC

y D ø $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

ø $\left(\frac{-3 + 1}{2}, \frac{-2 + (-8)}{2}\right)$

ø $\left(\frac{-2}{2}, \frac{-2 - 8}{2}\right)$

ø $\left(-1, \frac{-10}{2}\right)$

ø $(-1, -5)$ **1**

By two point form,

The equation of median AD

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ **½**

y $\frac{x - 5}{5 - (-1)} = \frac{y - 4}{4 - (-5)}$ **½**

y $\frac{x - 5}{5 + 1} = \frac{y - 4}{4 + 5}$

y $\frac{x - 5}{6} = \frac{y - 4}{9}$

y $9(x - 5) = 6(y - 4)$

y $9x - 45 = 6y - 24$

y $9x - 6y - 45 + 24 = 0$

y $9x - 6y - 21 = 0$

y $3x - 2y - 7 = 0$ [Dividing throughout by 3] **1**

y The equation of median AD is $3x - 2y - 7 = 0$

Slope of line AC $= \frac{y_2 - y_1}{x_2 - x_1}$ **½**

$= \frac{-8 - 4}{1 - 5}$

$= \frac{-12}{-4}$

$= 3$

∵ Slope of parallel lines are equal

Slope of the line parallel to line AC is 3

The line passes through B (– 3, – 2) ½

∴ The equation of the line parallel to line AC passing through point B by the slope point form is

$$y - y_1 = m (x - x_1)$$

∴ $y - (-2) = 3 [x - (-3)]$

∴ $y + 2 = 3 (x + 3)$

∴ $y + 2 = 3x + 9$ 1

∴ $3x - y + 9 - 2 = 0$

∴ $3x - y + 7 = 0$

∴ The equation of the line parallel to AC passing through point B is $3x - y + 7 = 0$

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