MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - I - PAPER - 1 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Solve the following : (Any 5)

(i) A natural number is 'x'
y Its square is x^2
As per the given condition,
$x + x^2 = 30$
y $x^2 + x - 30 = 0$ **1**

(ii) The roots of the quadratic equation are 5 and – 7.
Let ~ = 5 and = –7
y ~ + = 5 + (– 7) = 5 – 7 = – 2 and ~. = 5 × – 7 = – 35 **½**
We know that,
$x^2 - (\sim +)x + \sim. = 0$
y $x^2 - (-2)x + (-35) = 0$
y $x^2 + 2x - 35 = 0$
y The required quadratic equation is $x^2 + 2x - 35 = 0$ **½**

(iii) If one of the root of the quadratic equation is $3 - 2\sqrt{5}$,
then the other root is $3 + 2\sqrt{5}$
y ~ = $3 - 2\sqrt{5}$ and = $3 + 2\sqrt{5}$
y ~ + = $3 - 2\sqrt{5} + 3 + 2\sqrt{5} = 6$ **½**
and ~. = $\left(3 - 2\sqrt{5}\right) \times \left(3 + 2\sqrt{5}\right)$
= $(3)^2 - \left(2\sqrt{5}\right)^2$
= $9 - 4 \times 5$
= $9 - 20$
= -11 **½**

(iv) $\bar{d} = \frac{pf_i d_i}{pf_i}$
$= \frac{6}{50}$
$= 0.12$ **½**

Mean $(\bar{x})$ = $A + \bar{d}$
= 57 + 0.12
= 57.12 ½

(v)

No. of packages	No. of days	
10 - 20	2	
20 - 30	8	
30 - 40	16	
40 - 50	24	ì f_1
50 - 60	30	ì f_m
60 - 70	20	ì f_2

½

Here the maximum frequency f_m = 30
The corresponding class 50 - 60 is the modal class.
L = 50, f_m = 30, f_1 = 24, f_2 = 20, h = 10 ½

(vi) Let the total loan disbursed be Rs. x crores
The measure of central angle for dairy sector is 40º.
∵ Dairy sector received Rs. 20 crores of the total loan i.e. x

$\therefore \frac{40}{360} \times x = 20$ ½

$\therefore x = \frac{20 \times 360}{40}$

$\therefore x = 180$

∴ Total loan disbursed is Rs. 180 crores. ½

A.2. Solve the following : (Any 4)

(i) $y^2 - 5y + 11 = 0$
Comparing with $ay^2 + by + c = 0$ we have a = 1, b = – 5, c = 11
$a = b^2 - 4ac$ ½
$= (-5)^2 - 4\ (1)\ (11)$
$= 25 - 44$
$= -19$ ½
$\therefore a < 0$
Hence roots of the quadratic equation are not real. **1**

(ii) Class width (h) = 10

No. of days (x_i)	**Class Mark (f_i)**	**No. of employees**	**$f_i x_i$**
0 - 10	5	5	25
10 - 20	15	7	105
20 - 30	25	11	275
30 - 40	35	4	140
40 - 50	45	3	135
Total		**30**	**680**

1

Mean $= \dfrac{\text{p } f_i x_i}{\text{p } f_i}$

y Mean $= \dfrac{680}{30}$

y Mean $= 22.67$ (approximately)

y Mean of medical leave is 22.67 days. **1**

(iii) If one of the root of the quadratic equation is $\sqrt{2} + \sqrt{3}$, then the other root is $\sqrt{2} - \sqrt{3}$

Let ~ = $\sqrt{2} + \sqrt{3}$ and = $\sqrt{2} - \sqrt{3}$ **½**

y ~ + = $\sqrt{2} + \sqrt{3} + \sqrt{2} - \sqrt{3} = 2\sqrt{2}$

and ~. = $\left(\sqrt{2} + \sqrt{3}\right) \times \left(\sqrt{2} - \sqrt{3}\right)$

= $\left(\sqrt{2}\right)^2 - \left(\sqrt{3}\right)^2$ **½**

= 2 – 3

= – 1

We know that,

$x^2 - (\sim +)x + \sim. = 0$

y $x^2 - 2\sqrt{2}\,x + (-1) = 0$

y $x^2 - 2\sqrt{2}\,x - 1 = 0$

y The required quadratic equation is $x^2 - 2\sqrt{2}\,x - 1 = 0$ **1**

(iv)

Weight (in gms)	No. of packets
200 - 201	12 ì f_1
201 - 202	26 ì f_m
202 - 203	20 ì f_2
203 - 204	9
204 - 205	2
205 - 206	1

Here the maximum frequency $f_m = 26$.

The corresponding class 201 - 202 is the modal class. **½**

L = 201, $f_m = 26$, $f_1 = 12$, $f_2 = 20$, h = 1

Mode $= L + \left(\dfrac{f_m - f_1}{2f_m - f_1 - f_2}\right) h$

$= 201 + \left(\dfrac{26 - 12}{2\,(26) - 12 - 20}\right) 1$ **½**

$= 201 + \left(\frac{14}{52 - 32}\right)$

$= 201 + \left(\frac{14}{20}\right)$

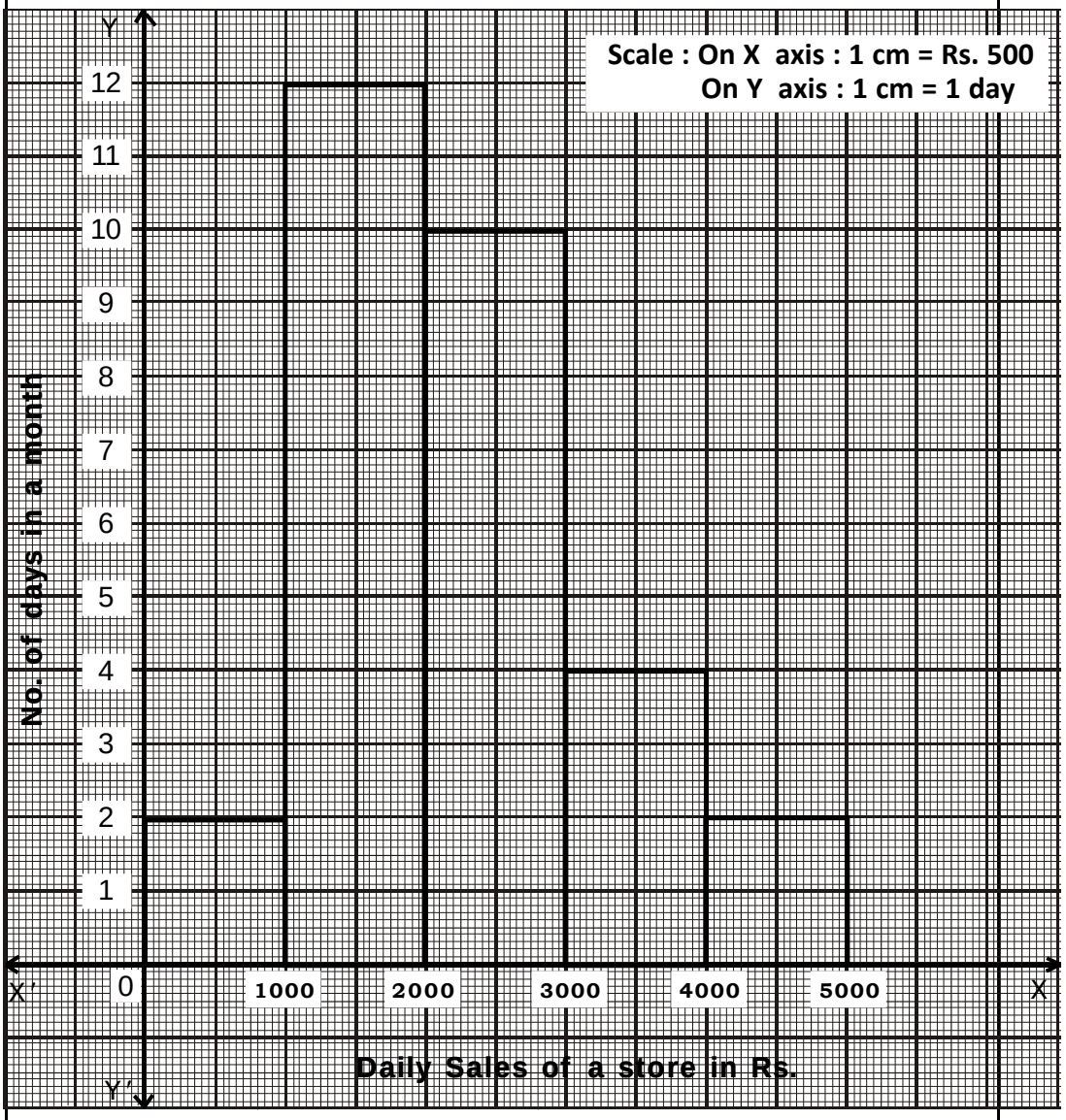
$= 201 + \left(\frac{7}{10}\right)$

$= 201 + 0.7$

$= 201.7$

y Mode of weight of coffee is 201.7 gms. **1**

(v)



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(vi) Total expenditure = Rs. 540000

Items	Measure of central angle	Expenditure (in Rs.)
Cement	75°	$\frac{75}{360} \times 540000 = 112500$
Bricks	50°	$\frac{50}{360} \times 540000 = 75000$
Labour	100°	$\frac{100}{360} \times 540000 = 150000$
Timber	90°	$\frac{90}{360} \times 540000 = 135000$
Steel	45°	$\frac{45}{360} \times 540000 = 67500$
Total	**360°**	**540000**

2

A.3. Solve the following : (Any 3)

(i) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = k - 12$, $b = 2(k - 12)$, $c = 2$

$a = b^2 - 4ac$ ½

$= [2(k - 12)]^2 - 4(k - 12)(2)$

$= (2k - 24)^2 - 8(k - 12)$

$= 4k^2 - 96k + 576 - 8k + 96$

$= 4k^2 - 104k + 672$ ½

∵ The roots of given equation are real and equal.

∴ a must be zero. ½

$4k^2 - 104k + 672 = 0$

$4(k^2 - 26k + 168) = 0$

$k^2 - 14k - 12k + 168 = 0$

$k(k - 14) - 12(k - 14) = 0$ ½

$(k - 14)(k - 12) = 0$

$k - 14 = 0$ or $k - 12 = 0$

$k = 14$ or $k = 12$

If $k = 12$ ½

$a = 12 - 12$ $[\because a = k - 12]$

$a = 0$

But $a \neq 0$

∴ $k = 12$ is not acceptable.

∴ $\boxed{k = 14}$ ½

(ii) Class width (h) = 3, Assumed mean (A) = 72.5

Distance (in km)	**Class Mark (x_i)**	**$d_i = x_i - A$**	**No. of mopeds**	**$f_i d_i$ (f_i)**
62 - 65	63.5	– 9	5	– 45
65 - 68	66.5	– 6	8	– 48
68 - 71	69.5	– 3	12	– 36
71 - 74	72.5 ê A	0	28	0
74 - 77	75.5	3	35	105
77 - 80	78.5	6	10	60
80 - 83	81.5	9	2	18
Total			**100**	**54**

1

$\bar{d} = \frac{\text{p } f_i d_i}{\text{p } f_i}$

y $\bar{d} = \frac{54}{100}$

y $\bar{d} = 0.54$ **1**

Mean $(\bar{x}) = A + \bar{d}$

$= 72.5 + 0.54$

$= 73.04$

y Mean of distance covered by moped is 73.04 km/lit. **1**

(iii) $x^4 - 3x^2 + 2 = 0$

y $(x^2)^2 - 3x^2 + 2 = 0$

Substituting $x^2 = m$ we get,

$m^2 - 3m + 2 = 0$

y $m^2 - 2m - m + 2 = 0$ **1**

y $m(m - 2) - 1(m - 2) = 0$

y $(m - 2)(m - 1) = 0$

y $m - 2 = 0$ or $m - 1 = 0$

y $m = 2$ or $m = 1$ **1**

Resubstituting $m = x^2$ we get,

$x^2 = 2$ or $x^2 = 1$

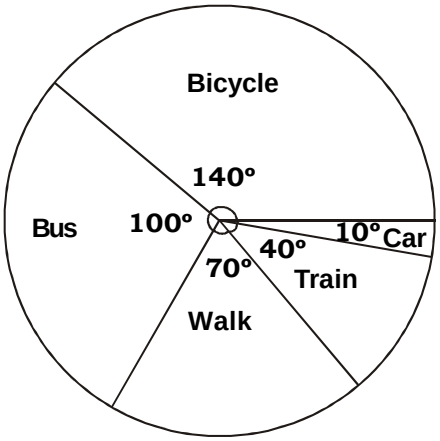
Taking square roots throughout,

$x = \pm\sqrt{2}$ or $x = \pm 1$ **1**

(iv)

Mode of transport	No. of Students	Measure of central angle (θ)
Bicycle	140	$\frac{140}{360} \times 360^\circ = 140^\circ$
Bus	100	$\frac{100}{360} \times 360^\circ = 100^\circ$
Walk	70	$\frac{70}{360} \times 360^\circ = 70^\circ$
Train	40	$\frac{40}{360} \times 360^\circ = 40^\circ$
Car	10	$\frac{10}{360} \times 360^\circ = 10^\circ$
Total	**360**	**360°**

1



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(v) Let ~ and be the roots of a quadratic equation.

∵ ~ + = 3 and ~³ + ³ = 63 [Given] ½

We know that,

$x^2 - (\sim +)x + \sim. = 0$ .....(i)

Also, ~³ + ³ = (~ +)³ – 3 ~. (~ +)

y 63 = (3)³ – 3 ~. (3) [∵ ~ + = 3 and ~³ + ³ = 63] ½

y 63 = 27 – 9 ~.

y 9 ~. = 27 – 63

y 9 ~. = – 36

y $\sim. = \frac{-36}{9}$

y ~. = – 4 1

y $x^2 - (\sim +)x + \sim.$ = 0 [From (i)]

y $x^2 - 3x + (-4)$ = 0 [∵ ~ + = 3 and ~. = – 4] ½

y $x^2 - 3x - 4$ = 0

y The required quadratic equation is $x^2 - 3x - 4 = 0$. ½

A.4. **Solve the following : (Any 2)**

(i) Let the third consecutive odd natural number be x, x + 2 and x + 4

As per the given condition,

$x(x + 4) = 4(x + 2) + 1$ ½

y $x^2 + 4x = 4x + 8 + 1$ **1**

y $x^2 = 9$ ½

Taking square root on both the sides we get,

$x = \pm 3$

∵ x is a natural number $x \div -3$ **1**

Hence, $x = 3$

$x + 2 = 3 + 2 = 5$ and $x + 4 = 3 + 4 = 7$ ½

y The 3 consecutive odd natural numbers are 3, 5 and 7 respectively. ½

(ii)

Size of form (in acres)	**Frequency (f_i) (No. of farms)**	**Cumulative frequency less than type**
5 - 15	7	7
15 - 25	12	19
25 - 35	17	36 ← *c.f.*
35 - 45	25 ← f	61
45 - 55	31	92
55 - 65	5	97
65 - 75	3	100
Total	**100 ← N**	

1

Here total frequency = $\sum f_i = N = 100$

y $\frac{N}{2} = \frac{100}{2} = 50$

Cumulative frequency (less than type) which is just greater than 50 is 61. Therefore corresponding class 35 - 45 is median class.

L = 35, N = 100, *c.f.* = 36, f = 25, h = 10

Median = $L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$ **1**

$= 35 + \left(\frac{100}{2} - 36\right)\frac{10}{25}$

$= 35 + (50 - 36)\frac{10}{25}$ **1**

$= 35 + (14)\frac{10}{25}$

$= 35 + \frac{140}{25}$

$= 35 + 5.6$

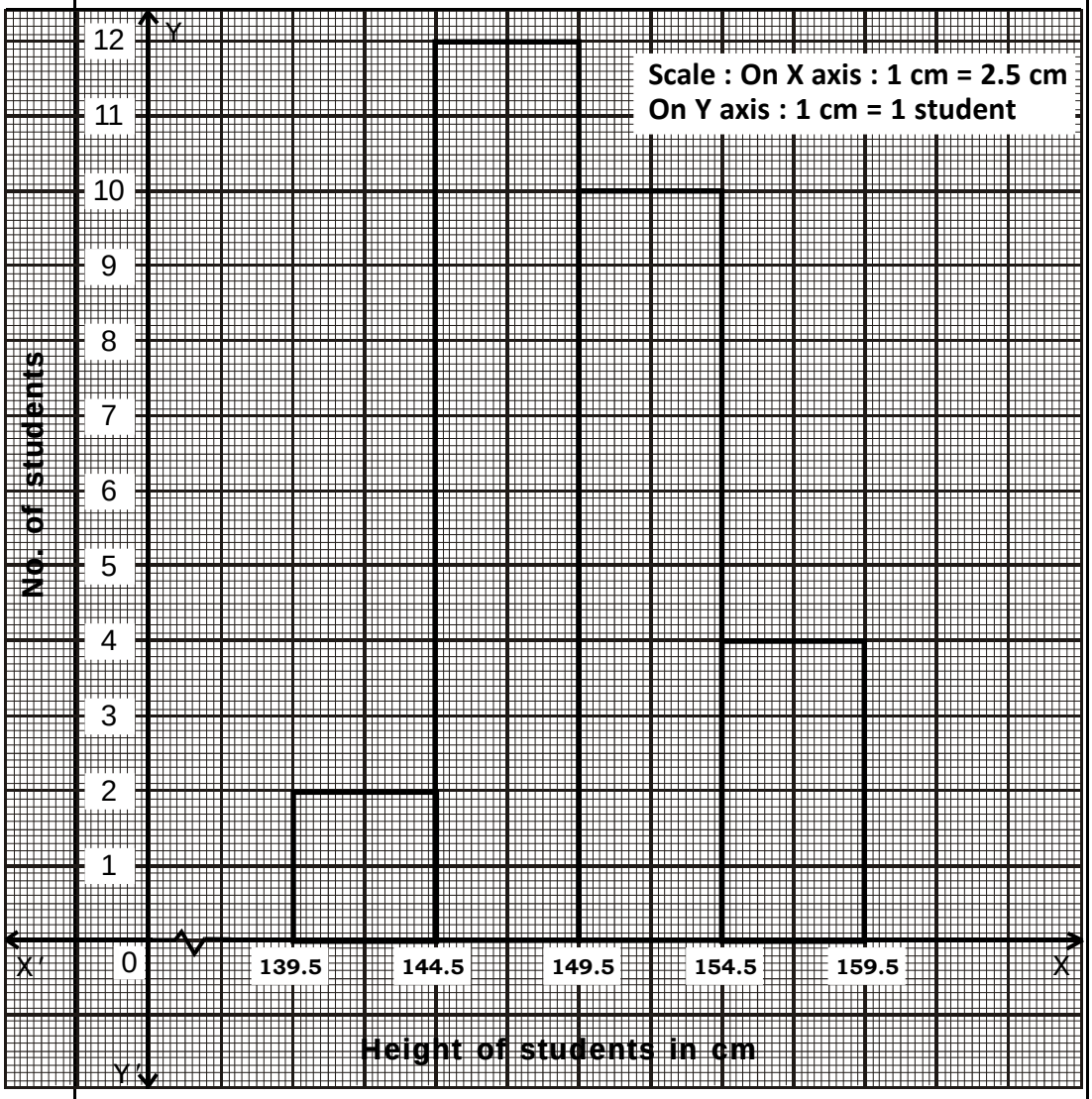
$= 40.6$

y Median of size of farm is 40.6 acres. **1**

(iii)

Height of students	Continuous classes	Frequency No. of students
140 - 144	139.5 - 144.5	2
145 - 149	144.5 - 149.5	12
150 - 154	149.5 - 154.5	10
155 - 159	154.5 - 159.5	4

1



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A.5. **Solve the following : (Any 2)**

(i) Let the time taken to fill a tank by a bigger tap alone be x hrs.

y The time taken by smaller tap alone is (x + 5) hrs.

Time taken by both the taps together to fill the same tank is 6 hrs.

y Portion of tank filled in 1 hr by bigger tap = $\frac{1}{x}$

Portion of tank filled in 1 hr by smaller tap = $\frac{1}{x+5}$

Portion of tank filled in 1 hr by both taps together = $\frac{1}{6}$ **1**

As per the given condition,

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

y $$\frac{x+5+x}{x(x+5)} = \frac{1}{6}$$ **1**

y $$\frac{2x+5}{x^2+5x} = \frac{1}{6}$$

y $6(2x+5) = 1(x^2+5x)$

y $12x + 30 = x^2 + 5x$

y $0 = x^2 + 5x - 12x - 30$

$x^2 - 7x - 30 = 0$

y $x^2 + 3x - 10x - 30 = 0$ **1**

y $x(x+3) - 10(x+3) = 0$

y $(x+3)(x-10) = 0$

y $x + 3 = 0$ or $x - 10 = 0$

y $x = -3$ or $x = 10$ **1**

∵ x is the time taken bybigger tap

y $x \neq -3$

Hence x = 10

y $x + 5 = 10 + 5 = 15$

y Time taken by bigger tap alone is 10 hrs and smaller tap alone is 15 hrs. **1**

(ii) Class width (h) = 5, Assumed mean (A) = 22.5

No. of trees	**Class Mark (x_i)**	**$d_i = x_i - A$**	**$u_i = \frac{d_i}{h}$**	**No. of societies (f_i)**	**$f_i u_i$**
10 - 15	12.5	– 10	– 2	2	– 4
15 - 20	17.5	– 5	– 1	7	– 7
20 - 25	22.5 ← A	0	0	9	0
25 - 30	27.5	5	1	8	8
30 - 35	32.5	10	2	6	12
35 - 40	37.5	15	3	4	12
Total				**36**	**21**

2

$\bar{u} = \frac{p f_i u_i}{p f_i}$

y $\bar{u} = \frac{21}{36}$ **1**

y $\bar{u} = 0.583$

Mean $(\bar{x}) = A + h\bar{u}$ **1**

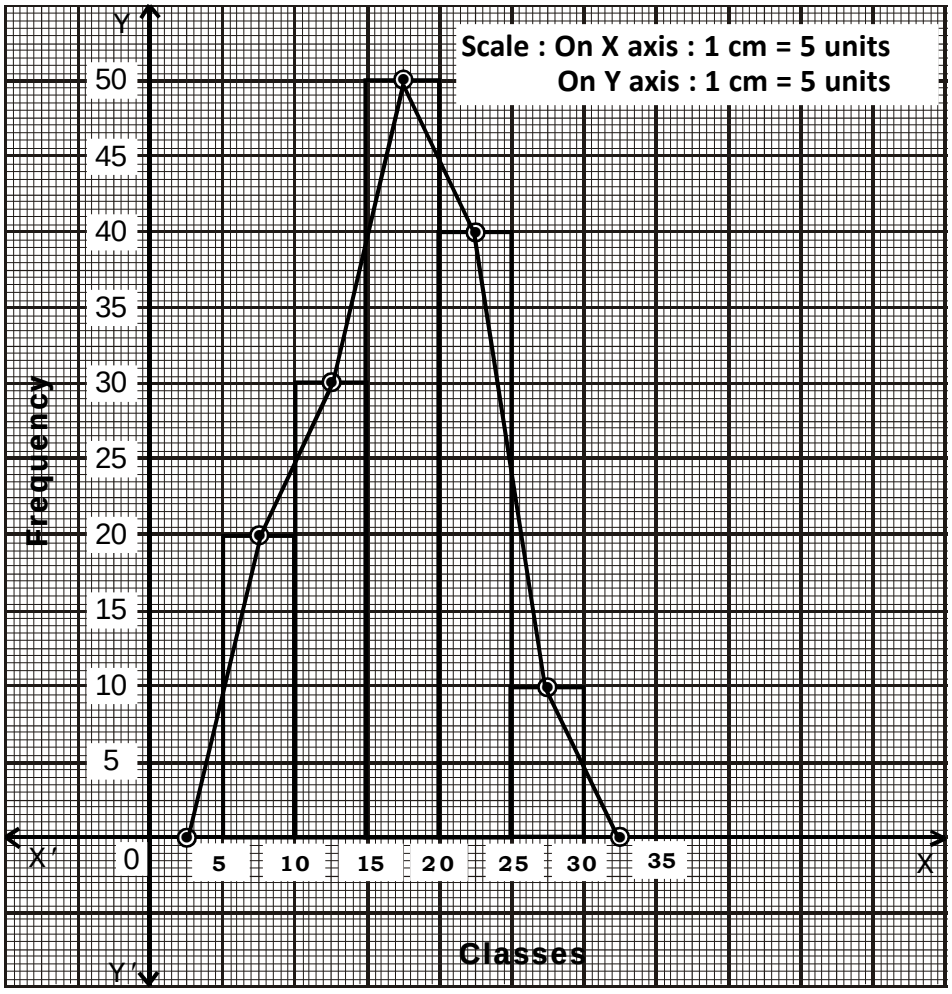
$= 22.5 + 5\ (0.583)$

$= 22.5 + 2.92$

$= 25.42$

y Mean of trees planted by societies 25.42 trees. **1**

(iii)



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