MT

2017 ____ ____ 1100 **Seat No.**

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - II - PAPER - 1 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Solve the following : (Any 5)

(i)

$t_1 = 3^1 = 3$
$t_2 = 3^2 = 9$
$t_3 = 3^3 = 27$
$t_4 = 3^4 = 81$
$t_5 = 3^5 = 243$
$t_6 = 3^6 = 729$
$t_7 = 3^7 = 2187$
$t_8 = 3^8 = 6561$

y The next four terms of the sequence are 243, 729, 2187 and 6561. **1**

(ii)

$(y - 2)(y + 2) = 0$
y $(y)^2 - (2)^2 = 0$ $[\because a^2 - b^2 = (a + b)(a - b)]$
y $y^2 - 4 = 0$ ½
y $y^2 + 0y - 4 = 0$
Here a = 1, b = 0, c = – 4 are real numbers
where a ÷ 0
So it is a quadratic equation in variable y. ½

(iii)

$$\begin{vmatrix} 1.2 & 0.03 \\ 0.57 & -0.23 \end{vmatrix}$$

$= (1.2 \times -0.23) - (0.03 \times 0.57)$ ½
$= -0.276 - 0.0171$
$= -0.2931$ ½

(iv)

$t_1 = 3, t_2 = 6, t_3 = 12, t_4 = 24$
$t_2 - t_1 = 6 - 3 = 3$
$t_3 - t_2 = 12 - 6 = 6$
$t_4 - t_3 = 24 - 12 = 12$
∵ The difference between two consecutive terms is not constant.
y **The sequence is not an A.P.** **1**

(v)	$^2 - 2x + 1 = 0,\ x = 1$	
	Putting x = 1 in L.H.S. we get,	
	L.H.S. $= (1)^2 - 2(1) + 1$	
	$= 1 - 2(1) + 1$	
	$= 2 - 2$	
	$= 0$	½
	= R.H.S.	
	y L.H.S. = R.H.S.	
	Thus equation is satisfied.	½
	So 1 is the root of the given quadratic equation.	
(vi)	$D_x = -18$ and $D = 3$	
	By Cramer's rule,	
	$x = \frac{D_x}{D}$	
	y $x = \frac{-18}{3}$	
	y $\boxed{x = -6}$	1
A.2.	**Solve the following : (Any 4)**	
(i)	$S_n = n^2(n + 1)$	
	y $S_1 = 1^2(1 + 1) = 1(2) = 2$	
	y $S_2 = 2^2(2 + 1) = 4(3) = 12$	
	y $S_3 = 3^2(3 + 1) = 9(4) = 36$	1
	We know that,	
	$t_1 = S_1 = 2$	
	$t_2 = S_2 - S_1 = 12 - 2 = 10$	
	$t_3 = S_3 - S_2 = 36 - 12 = 24$	
	y The first three terms of the sequence are 2, 10 and 24.	1
(ii)	$x^2 + 10x + 24 = 0$	
	y $x^2 + 6x + 4x + 24 = 0$	
	y $x(x + 6) + 4(x + 6) = 0$	½
	y $(x + 6)(x + 4) = 0$	
	y $x + 6 = 0$ or $x + 4 = 0$	½
	$\therefore$ $\boxed{x = -6}$ or $\boxed{x = -4}$	1
(iii)	Substituting x = 2 and y = 5 in the L.H.S. of the equation	
	$3x - y = 1$	
	L.H.S. $= 3x - y$	
	$= 3(2) - 5$	½
	$= 6 - 5$	

= 1

= R.H.S. ½

y x = 2 and y = 5 satisfies the equation 3x – y = 1

Hence (2, 5) lies on the graph of the equation 3x – y = 1. **1**

(iv) 3x – y = 7

x + 4y = 11

$D = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - (-1 \times 1) = 12 + 1 = 13$

$D_x = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = (7 \times 4) - (-1 \times 11) = 28 + 11 = 39$

$D_y = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = (3 \times 11) - (7 \times 1) = 33 - 7 = 26$ **1**

By Cramer's rule,

$x = \frac{D_x}{D} = \frac{39}{13} = 3$

$y = \frac{D_y}{D} = \frac{26}{13} = 2$

y x = 3 and y = 2 is the solution of given simultaneous equations. **1**

(v) $y^2 - (k - 4)y - 4k = 0.$

By putting y = k in L.H.S. we get

L.H.S. $= (k)^2 - (k - 4)(k) - 4k$ ½

$= k^2 - (k^2 - 4k) - 4k$

$= k^2 - k^2 + 4k - 4k$

$= 0$

= R.H.S. ½

y L.H.S. = R.H.S.

Thus equation is satisfied.

So k is the root of the given quadratic equation. **1**

(vi) Two coins are tossed

S = { HH, HT, TH, TT }

n (S) = 4 ½

Let A be the event that head appears on both the coins

A = { HH }

n (A) = 1 ½

$P(A) = \frac{n(A)}{n(S)}$

y $P(A) = \frac{1}{4}$ **1**

A.3. Solve the following : (Any 3)

(i) **Given :** For an A.P. $t_4 = 12$, $d = -10$

Find : General term $\{ t_n \}$

Sol.

$t_n = a + (n - 1)\,d$

y $t_4 = a + (4 - 1)\,d$

y $12 = a + 3\,(-10)$

y $a = 12 + 30$

y $a = 42$ **1**

$t_n = a + (n - 1)\,d$

y $t_n = 42 + (n - 1)(-10)$ **1**

y $t_n = 42 - 10n + 10$

y $t_n = 52 - 10n$

y The general term of A.P. is $52 - 10n$. **1**

(ii) $x^2 + 8x + 9 = 0$

y $x^2 + 8x = -9$ .....(i)

Third term $= \left(\frac{1}{2} \times \text{coefficient of } x\right)^2$

$= \left(\frac{1}{2} \times 8\right)^2$

$= (4)^2$

$= 16$ **1**

Adding 16 to both sides of (i) we get,

$x^2 + 8x + 16 = -9 + 16$

y $(x + 4)^2 = 7$

y $(x + 4)^2 = \left(\sqrt{7}\right)^2$

Taking square root on both the sides we get,

$x + 4 = \pm\sqrt{7}$ **1**

y $x = -4 \pm \sqrt{7}$

y $x = -4 + \sqrt{7}$ or $x = -4 - \sqrt{7}$

y $-4 + \sqrt{7}$ and $-4 - \sqrt{7}$ are the roots of the given quadratic equations. **1**

(iii) Let the two numbers be x and y

We know,

Dividend = Divisor × Quotient + Remainder

As per the first given condition,

$x + y = 15 \times 2 + 10$

y $x + y = 30 + 10$

y $x + y = 40$ ......(i) **1**

	As per the second given condition, $x - y = 3 \times 4 + 2$ y $x - y = 12 + 2$ y $x - y = 14$ ....(ii)	1
	Adding (i) and (ii), $x + y = 40$ $x - y = 14$ $2x = 54$ y $x = \frac{54}{2}$ y $x = 27$ Substituting x = 27 in (i), $27 + y = 40$ y $y = 40 - 27$ y $y = 13$ y The two numbers are 27 and 13.	1
(iv)	Two digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 without repeating digits are as follows : S = { 10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43, 45, 50, 51, 52, 53, 54 } n (S) = 25	1
	P is the event that the number so formed is even P = { 10, 12, 14, 20, 24, 30, 32, 34, 40, 42, 50, 52, 54 } y n (P) = 13	1
	Q is the event that the number so formed is divisible by 3 Q = { 12, 15, 21, 24, 30, 42, 45, 51, 54 } y n (Q) = 9	½
	R is the event that the number so formed is greater than 50 R = { 51, 52, 53, 54 } y n (R) = 4	½
(v)	$x^2 + 3x - 10 = 0$ Comparing with $ax^2 + bx + c = 0$ we have a = 1, b = 3, c = – 10 $b^2 - 4ac = (3)^2 - 4\ (1)\ (-10)$ $= 9 + 40$ $= 49$	½
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-3 \pm \sqrt{49}}{2\ (1)}$	1
	$= \frac{-3 \pm 7}{2}$	

	y $x = \frac{-3+7}{2}$ or $x = \frac{-3-7}{2}$	½
	y $x = \frac{4}{2}$ or $x = \frac{-10}{2}$	
	y $x = 2$ or $x = -5$	
	y 2 and – 5 are the roots of given quadratic equation.	1
A.4.	**Solve the following : (Any 2)**	
(i)	The odd natural numbers from 1 to 150 are as follows	
	1, 3, 5, 7, 9, ..., 149.	
	These numbers form an A.P. with a = 1, d = 2	½
	Let, 149 be n^{th} term of an A.P.	
	$t_n = 149$	
	$t_n = a + (n-1) d$	½
	$149 = 1 + (n-1) 2$	
	$149 = 1 + 2n - 2$	
	$149 = 2n - 1$	
	$149 + 1 = 2n$	
	y $2n = 150$	
	y $n = 75$	½
	y 149 is 75^{th} term of A.P.	
	y We have to find sum of 75 terms i.e. S_{75}	
	$S_n = \frac{n}{2} [2a + (n-1) d]$	½
	y $S_{75} = \frac{75}{2} [2 (1) + (75 - 1) 2]$	
	y $S_{75} = \frac{75}{2} [2 + 74 [2)]$	1
	$= \frac{75}{2} [2 + 148]$	
	$= \frac{75}{2} (150)$	½
	$= 75 (75)$	
	y $S_{75} = 5625$	
	y Sum of all odd natural from 1 to 150 is 5625.	½

(ii) 2x + y = 6

y y = 6 – 2x

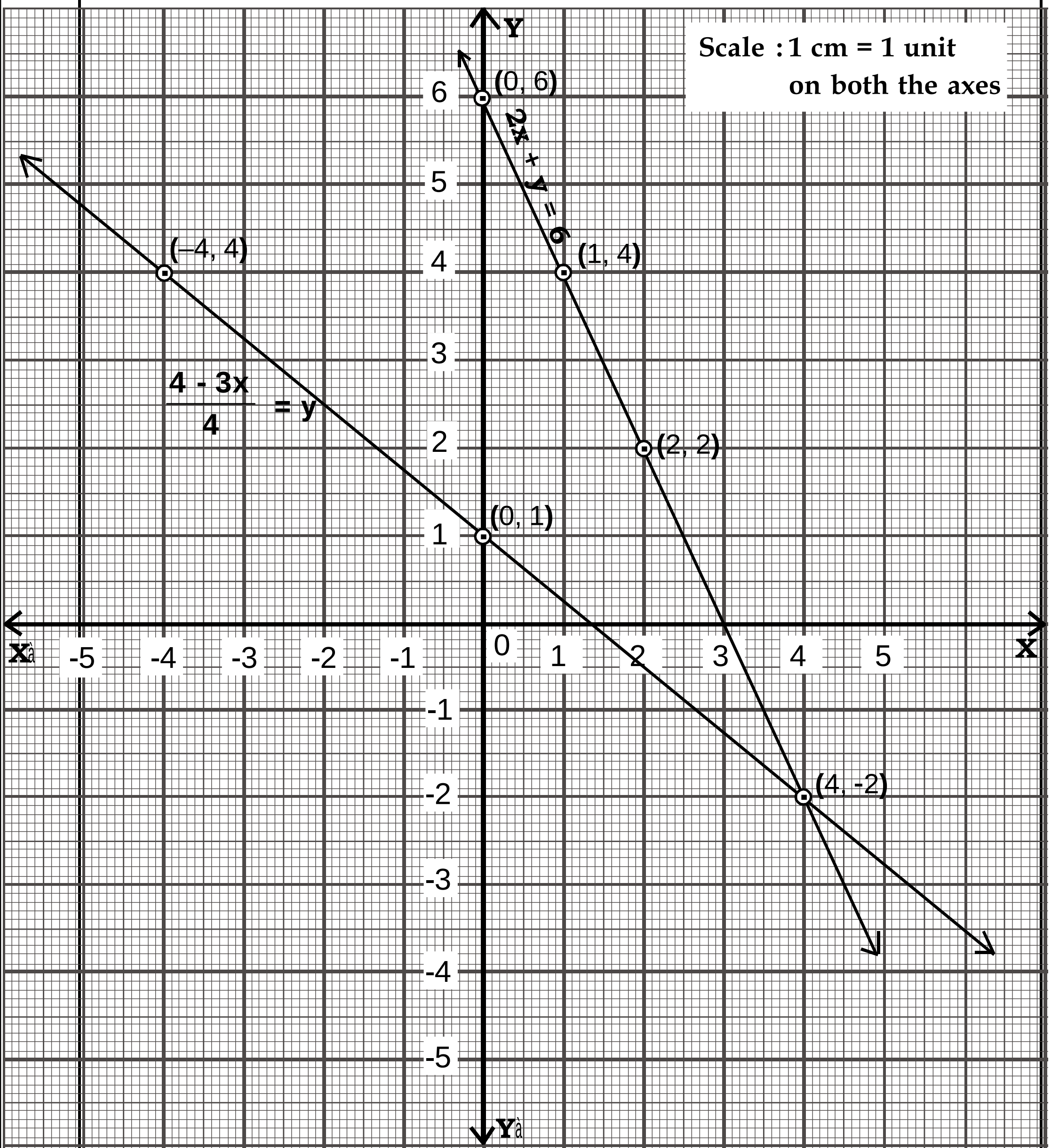
x	0	1	2
y	6	4	2
(x, y)	(0, 6)	(1, 4)	(2, 2)

$\frac{4 - 3x}{4} = y$

y $y = \frac{4 - 3x}{4}$

x	0	4	– 4
y	1	–2	4
(x, y)	(0, 1)	(4, –2)	(– 4, 4)

1



2

y x = 4 and y = – 2 is the solution of given simultaneous equations. 1

(iii)	Two dice are thrown ∴ S = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) } ∴ n (S) = 36	1
	(a) Let A be the event that sum of numbers on their upper faces is divisible by 9 A = { (3, 6), (4, 5), (5, 4), (6, 3) } n (A) = 4 $P(A) = \frac{n(A)}{n(S)}$ ∴ $P(A) = \frac{4}{36}$ ∴ $\boxed{P(A) = \frac{1}{9}}$	½ ½
	(b) Let B be the event that sum of number on their upper faces is at the most 3. B = { (1, 1), (1, 2), (2, 1) } n (B) = 3 $P(B) = \frac{n(B)}{n(S)}$ ∴ $P(B) = \frac{3}{36}$ ∴ $\boxed{P(B) = \frac{1}{12}}$	½ ½
	(c) Let C be the event that number on the upper face of the first die is less than the number on the upper face of second die. C = { (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6) } n (C) = 15 $P(C) = \frac{n(C)}{n(S)}$ ∴ $P(C) = \frac{15}{36}$ ∴ $\boxed{P(C) = \frac{5}{12}}$	½ ½

A.5. Solve the following : (Any 2)

(i)

$\frac{16}{x+y} + \frac{2}{x-y} = 1$ ....(i)

$\frac{8}{x+y} - \frac{12}{x-y} = 7$ ....(ii)

Substituting $\frac{1}{x+y} = a$ and $\frac{1}{x-y} = b$ in (i) and (ii),

$16a + 2b = 1$ ....(iii)

$8a - 12b = 7$ ....(iv) **1**

Multiplying (iv) by 2,

$16a - 24b = 14$ ....(v)

Subtracting (v) from (iii),

$16a + 2b = 1$

$16a - 24b = 14$

(–) (+) (–)

$26b = -13$

y $b = \frac{-13}{26}$

y $b = \frac{-1}{2}$ **1**

Substituting $b = \frac{-1}{2}$ in (iii),

$16a + 2\left(\frac{-1}{2}\right) = 1$

y $16a - 1 = 1$

y $16a = 1 + 1$

y $16a = 2$

y $a = \frac{2}{16}$

y $a = \frac{1}{8}$ **½**

Resubstituting the values of a and b,

$a = \frac{1}{x+y}$

$\frac{1}{8} = \frac{1}{x+y}$

y $x + y = 8$ ......(vi) **½**

$b = \frac{1}{x-y}$

$\frac{-1}{2} = \frac{1}{x-y}$

y $x - y = -2$ .....(vii) **½**

	Adding (vi) and (vii),	
	$x + y = 8$	
	$x - y = -2$	
	$2x = 6$	
	$x = \frac{6}{2}$	
	y $x = 3$	½
	Substituting x = 3 in (vi) we get,	
	$3 + y = 8$	
	y $y = 8 - 3$	
	y $y = 5$	½
	y **x = 3 and y = 5 is the solution of given simultaneous equations.**	½
(ii)	Since the no. of literate people increases by 400 every year, the population of literate people every year forms an A.P.	
	No. of people in the year 2010 (a) = 4000	
	Increase in population every year (d) = 400	**1**
	No. of years from 2010-2020 (n) = 10	
	$t_n = a + (n - 1) d$	**1**
	y $t_{10} = 4000 + (10 - 1) 400$	
	y $t_{10} = 4000 + 9 (400)$	
	y $t_{10} = 4000 + 3600$	
	y $t_{10} = 7600$	**1**
	y There will be 7600 literate people till the year 2020	
	$t_n = a + (n - 1) d$	
	y $t_n = 4000 + (n - 1) 400$	**1**
	y $t_n = 4000 + 400n - 400$	
	y $t_n = 3600 + 400n$	
	There will be (3600 + 400n) literate people after n years.	**1**
(iii)	Let the no. of students standing in each row be x and let no. of rows be y.	
	y Total no. of students participating in the drill = xy	
	As per the first given condition,	
	$(x - 3)(y + 10) = xy$	½
	y $x (y + 10) - 3 (y + 10) = xy$	
	y $xy + 10x - 3y - 30 = xy$	
	y $10x - 3y = 30$ ....(i)	½
	As per the second given condition,	
	$(x + 5)(y - 10) = xy$	½
	y $x (y - 10) + 5 (y - 10) = xy$	
	y $xy - 10x + 5y - 50 = xy$	
	y $-10x + 5y = 50$ ......(ii)	½

Adding (i) and (ii),

$$\begin{aligned} 10x - 3y &= 30 \\ -10x + 5y &= 50 \\ \hline 2y &= 80 \end{aligned}$$

y $y = \frac{80}{2}$

y $y = 40$ **1**

Substituting y = 40 in (i),

$10x - 3(40) = 30$

y $10x - 120 = 30$

y $10x = 30 + 120$

y $10x = 150$

y $x = \frac{150}{10}$

y $x = 15$ **1**

y $xy = 15 \times 40 = 600$

y 600 students were participating in the drill. **1**

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