MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - II - PAPER - 2 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Solve the following : (Any 5)

(i) $t_n = n^2 - 2n$

y $t_1 = 1 - 2(1) = 1 - 2 = -1$

y $t_2 = 2^2 - 2(2) = 4 - 4 = 0$

y $t_3 = 3^2 - 2(3) = 9 - 6 = 3$

y $t_4 = 4^2 - 2(4) = 16 - 8 = 8$

y $t_5 = 5^2 - 2(5) = 25 - 10 = 15$

y The first five terms of the sequence are – 1, 0, 3, 8 and 15. **1**

(ii) $\frac{5}{x} - 3 = x^2$

Multiplying throughout by x, we get;

$5 - 3x = x^3$

y $0 = x^3 + 3x - 5$

y $x^3 + 3x - 5 = 0$ ½

Here maximum index of the variable x is 3.

So it is not a quadratic equation. ½

(iii) $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix} = 31$

y $(m \times 7) - (2 \times -5) = 31$

y $7m + 10 = 31$ ½

y $7m = 31 - 10$

y $7m = 21$

y $m = \frac{21}{7}$

y $m = 3$ ½

(iv) $t_1 = 1,\ t_2 = 3,\ t_3 = 6,\ t_4 = 10$

$t_2 - t_1 = 3 - 1 = 2$

$t_3 - t_2 = 6 - 3 = 3$

$t_4 - t_3 = 10 - 6 = 4$ ½

∵ The difference between two consecutive terms is not constant.

y **The sequence is not an A.P.** ½

(v)	$x^2 + 2x + 1 = 0$, $x = -1$	
	Putting $x = -1$ in L.H.S. we get,	
	L.H.S. $= (-1)^2 + 2(-1) + 1$	½
	$= 1 - 2 + 1$	
	$= 2 - 2$	
	$= 0$	
	$=$ R.H.S.	
	y L.H.S. $=$ R.H.S.	
	Thus equation is satisfied.	
	So -1 is the root of the given quadratic equation.	½
(vi)	$D_y = -15$ and $D = -5$	
	By Cramer's rule,	
	$y = \frac{D_y}{D}$	
	y $y = \frac{-15}{-5}$	
	y $\boxed{y = 3}$	**1**
A.2.	**Solve the following : (Any 4)**	
(i)	For the given A.P. 12, 16, 20, 24, ...	
	Here, $a = t_1 = 12$	
	$d = t_2 - t_1 = 16 - 12 = 4$	½
	We know,	
	$t_n = a + (n - 1) d$	
	y $t_{25} = a + (25 - 1) d$	½
	y $t_{25} = 12 + 24 (4)$	
	y $t_{25} = 12 + 96$	
	y $t_{25} = 108$	
	y The twenty fifth term of A.P. is 108.	**1**
(ii)	$y^2 + 8y + 16 = 0$	
	y $y^2 + 4y + 4y + 16 = 0$	½
	y $y (y + 4) + 4 (y + 4) = 0$	
	y $(y + 4) (y + 4) = 0$	½
	y $(y + 4)^2 = 0$	
	Taking square root on both the sides, we get,	
	$y + 4 = 0$	
	y $y = -4$	
	$\therefore$ $\boxed{y = -4}$	**1**

(iii)	∵ (3, 2) lies on the graph of the equation $5x + y = 19$.	
	It satisfies the equation,	
	y Substituting $x = 3$ and $y = 2$ in the equation we get,	
	$5(3) + a(2) = 19$	1
	y $15 + 2a = 19$	
	y $2a = 19 - 15$	
	y $2a = 4$	
	y $a = \frac{4}{2}$	
	y $\boxed{a = 2}$	1
(iv)	$4x + 3y - 4 = 0$	
	y $4x + 3y = 4$	
	$6x = 8 - 5y$	
	y $6x + 5y = 8$	
	$D = \begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix} = (4 \times 5) - (6 \times 3) = 20 - 18 = 2$	
	$D_x = \begin{vmatrix} 4 & 3 \\ 8 & 5 \end{vmatrix} = (4 \times 5) - (3 \times 8) = 20 - 24 = -4$	
	$D_y = \begin{vmatrix} 4 & 4 \\ 6 & 8 \end{vmatrix} = (4 \times 8) - (4 \times 6) = 32 - 24 = 8$	1
	By Cramer's rule,	
	$x = \frac{D_x}{D} = \frac{-4}{2} = -2$	
	$y = \frac{D_y}{D} = \frac{8}{2} = 4$	
	y $\boxed{x = -2 \text{ and } y = 4 \text{ is the solution of given simultaneous equations.}}$	1
(v)	$x^2 - 7x + k = 0$	
	$x = 4$ is the root of given quadratic equation.	
	So it satisfies the given equation.	1
	Substituting $x = 4$ in the equation	
	y $(4)^2 - 7(4) + k = 0$	
	y $16 - 28 + k = 0$	
	y $-12 + k = 0$	
	y $\boxed{k = 12}$	1
(vi)	Two coins are tossed	
	$S = \{HH, HT, TH, TT\}$	½
	$n(S) = 4$	½

	Let B be the event that head does not appear	
	B = { TT }	½
	n (B) = 1	
	$P(B) = \frac{n(B)}{n(S)}$	
	y $\boxed{P(B) = \frac{1}{4}}$	½
A.3.	**Solve the following : (Any 3)**	
(i)	$S_n = 3n + n^2$	
	$S_1 = 3(1) + (1)^2$	
	y $S_1 = 3 + 1$	
	y $\boxed{S_1 = 4}$	**1**
	y $S_1 = t_1 = 4$	
	$S_2 = 3(2) + (2)^2$	
	y $S_2 = 6 + 4$	
	$\boxed{S_2 = 10}$	½
	$t_2 = S_2 - S_1$	
	y $t_2 = 10 - 4$	
	y $\boxed{t_2 = 6}$	½
	Now,	
	$t_1 + t_2 = 4 + 6$	
	y $t_1 + t_2 = 10$	
	$a = 4$	
	$d = 6 - 4 = 2$	
	$t_3 = t_2 + d$	
	y $t_3 = 6 + 2$	
	y $\boxed{t_3 = 8}$	½
	$t_n = a + (n - 1)d$	
	y $t_{15} = a + (15 - 1)d$	
	y $t_{15} = a + 14d$	
	y $t_{15} = 4 + 14(2)$	
	y $t_{15} = 4 + 28$	
	y $\boxed{t_{15} = 32}$	½
(ii)	$z^2 + 6z - 8 = 0$	
	y $z^2 + 6z = 8$ ....(i)	
	Third term $= \left(\frac{1}{2} \times \text{coefficient of } z\right)^2$	
	$= \left(\frac{1}{2} \times 6\right)^2$	
	$= (3)^2$	**1**
	$= 9$	

	Adding 9 to both sides of (i), we get,	
	$z^2 + 6z + 9 = 8 + 9$	
	y $(z + 3)^2 = 17$	1
	Taking square root on both the sides we get,	
	$z + 3 = \pm\sqrt{17}$	
	y $z = -3 \pm \sqrt{17}$	
	y $z = -3 + \sqrt{17}$ or $z = -3 - \sqrt{17}$	
	y **$-3 + \sqrt{17}$ and $-3 - \sqrt{17}$ are the roots of the given quadratic equations.**	1
(iii)	Let the larger number be x and smaller number be y.	
	As per the first given condition,	
	$x + y = 97$ ....(i)	½
	We know,	
	Dividend = Divisor × Quotient + Remainder	
	As per the second condition,	
	$x = y \times 7 + 1$	
	y $x = 7y + 1$	
	y $x - 7y = 1$ ....(ii)	½
	Subtracting (ii) from (i),	
	$x + y = 97$	
	$x - 7y = 1$	
	(–) (+) (–)	
	$8y = 96$	
	y $y = \frac{96}{8}$	
	y $y = 12$	1
	Substituting y = 12 in (i),	
	$x + 12 = 97$	
	y $x = 97 - 12$	½
	y $x = 85$	
	y **The two numbers are 85 and 12.**	½
(iv)	A die is thrown	
	y $S = \{ 1, 2, 3, 4, 5, 6 \}$	
	y $n(S) = 6$	½
	A is the event that a prime number comes up	
	y $A = \{ 2, 3, 5 \}$	
	y $n(A) = 3$	½
	B is the event that a number divisible by 3 comes up	
	y $B = \{ 3, 6 \}$	
	y $n(B) = 2$	½

C is the event that a perfect square number comes up

y C = { 1, 4 } ½

y n (C) = 2

B ą C = ¤ ½

B and C are mutually exclusive events.

A ą C = ¤ ½

y A and C are mutually exclusive events.

(v) $m^2 - 3m - 10 = 0$

Comparing with $am^2 + bm + c = 0$ we have $a = 1$, $b = -3$, $c = -10$

$b^2 - 4ac = (-3)^2 - 4(1)(-10)$

$= 9 + 40$

$= 49$ ½

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ½

$= \frac{-(-3) \pm \sqrt{49}}{2(1)}$ ½

$= \frac{3 \pm 7}{2}$ ½

y $m = \frac{3+7}{2}$ or $m = \frac{3-7}{2}$

y $m = \frac{10}{2}$ or $m = \frac{-4}{2}$

y $m = 5$ or $m = -2$

∴ 5 and –2 are the roots of the given quadratic equation. 1

A.4. Solve the following : (Any 2)

(i) The first n even natural numbers are as follows 2, 4, 6, 8, ...

These numbers form an A.P. with $a = 2$, $d = t_2 - t_1 = 4 - 2 = 2$ ½

$S_n = \frac{n}{2}[2a + (n-1)d]$

y $S_n = \frac{n}{2}[2(2) + (n-1)2]$ ½

y $S_n = \frac{n}{2}[4 + 2n - 2]$

y $S_n = \frac{n}{2}[2n + 2]$ ½

y $S_n = \frac{n}{2} \times 2(n+1)$

y $S_n = n(n+1)$ 1

y $S_{20} = 20(20+1)$ ½

y $S_{20} = 420$

y Sum of first twenty even natural numbers is 420. 1

(ii) $3x + 4y + 5 = 0$

y $3x = -5 - 4y$

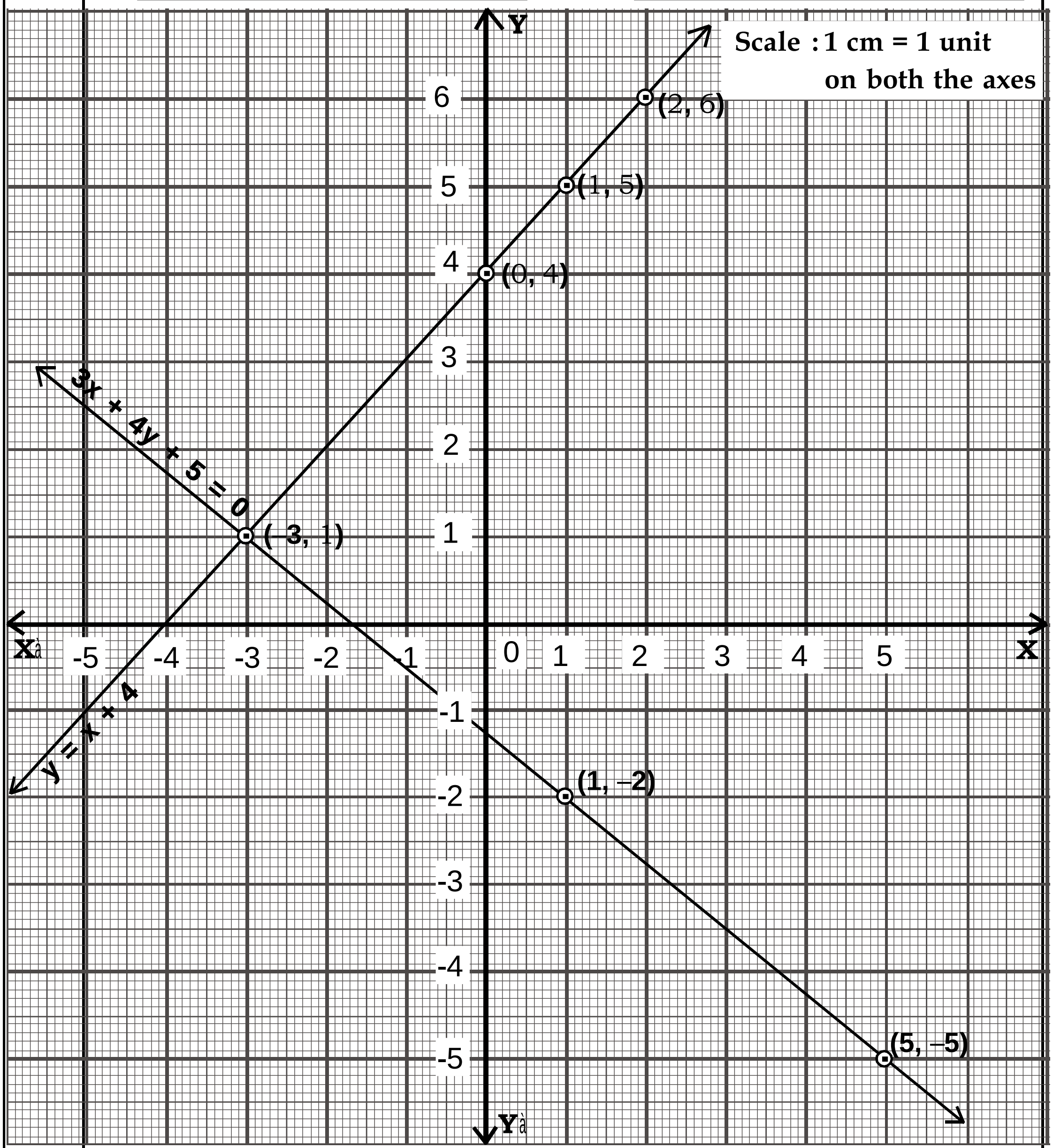
y $x = \dfrac{-5 - 4y}{3}$

x	–3	1	5
y	1	–2	–5
(x, y)	(–3, 1)	(1, –2)	(5, –5)

$y = x + 4$

x	0	1	2
y	4	5	6
(x, y)	(0, 4)	(1, 5)	(2, 6)

1



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y $x = -3$ and $y = 1$ is the solution of given simultaneous equations. 1

(iii)	Let three boys be denoted as B_1, B_2, B_3 and two girls be denoted as G_1, G_2. A committee of two can be formed in the following ways ∴ S = { $B_1 B_2$, $B_1 B_3$, $B_1 G_1$, $B_1 G_2$, $B_2 B_3$, $B_2 G_1$, $B_2 G_2$, $B_3 G_1$, $B_3 G_2$, $G_1 G_2$ } ∴ n (S) = 10	1
	(a) Let A be event that committee contains atleast one girl A = { $B_1 G_1$, $B_1 G_2$, $B_2 G_1$, $B_2 G_2$, $B_3 G_1$, $B_3 G_2$, $G_1 G_2$ } n (A) = 7 $P(A) = \frac{n(A)}{n(S)}$ ∴ $\boxed{P(A) = \frac{7}{10}}$	1
	(b) Let B be the event that committee contains one boy and one girl B = { $B_1 G_1$, $B_1 G_2$, $B_2 G_1$, $B_2 G_2$, $B_3 G_1$, $B_3 G_2$ } n (B) = 6 $P(B) = \frac{n(B)}{n(S)}$ ∴ $P(B) = \frac{6}{10}$ ∴ $\boxed{P(B) = \frac{3}{5}}$	1
	(c) Let C be the event that committee contain only boys C = { $B_1 B_2$, $B_1 B_3$, $B_2 B_3$ } n (C) = 3 $P(C) = \frac{n(C)}{n(S)}$ ∴ $\boxed{P(C) = \frac{3}{10}}$	1
A.5.	**Solve the following : (Any 2)**	
(i)	$\frac{2}{x} + \frac{6}{y} = 13$ ....(i) $\frac{3}{x} + \frac{4}{y} = 12$ ......(ii) Substituting $\frac{1}{x}$ = a and $\frac{1}{y}$ = b in(i) and (ii) we get, 2a + 6b = 13 ......(iii) 3a + 4b = 12 ......(iv) Multiplying (iii) by 2, 4a + 12b = 26 ....(v) Multiplying (iv) by 3, 9a + 12b = 36 ......(vi)	½ ½ ½ ½

	Substracting (vi) from (v), $4a + 12b = 26$ $9a + 12b = 36$ $(-)\ (-)\qquad (-)$ $-5a = -10$ y $a = \frac{-10}{-5}$ y $a = 2$	1
	Substituting a = 2 in (iii), $2(2) + 6b = 13$ y $4 + 6b = 13$ y $6b = 13 - 4$ y $b = \frac{9}{6}$ y $b = \frac{3}{2}$	½
	Resubstituting the values of a and b $a = \frac{1}{x}$ $2 = \frac{1}{x}$ y $2x = 1$ y $x = \frac{1}{2}$	½
	$b = \frac{1}{y}$ $\frac{3}{2} = \frac{1}{y}$ y $3y = 2$ y $y = \frac{2}{3}$	½
	y $x = \frac{1}{2}$ and $y = \frac{2}{3}$ is the solution of given simultaneous equations.	½
(ii)	Total money repaid by Babubhai in 10 instalments $= (S_{10})$ $= 4000 + 500$ $=$ Rs. 4500	½

		No. of instalments (n) = 10	
		Difference between two consecutive instalments (d) = – 10	½
		First instalment = (a) = ?	
		Last instalment (t_{10}) = ?	
		$S_n = \frac{n}{2} [2a + (n - 1) d]$	
	y	$S_{10} = \frac{10}{2} [2a + (10 - 1) d]$	1
	y	$4500 = 5 [2a + 9 (-10)]$	
	y	$\frac{4500}{5} = 2a - 90$	
	y	$900 = 2a - 90$	
	y	$900 + 90 = 2a$	
	y	$990 = 2a$	
	y	$\frac{990}{2} = a$	1
	y	$a = 495$	
		$t_n = a + (n - 1) d$	
		$t_{10} = a + (10 - 1) d$	1
	y	$t_{10} = 495 + 9 (-10)$	
	y	$t_{10} = 495 - 90$	
	y	$t_{10} = 405$	
	y	First instalment is Rs. 495 and last instalment is Rs.405.	1
(iii)		Let the speed of bus be x km/hr. and time taken be y hrs.	
		Distance = Speed × Time	
	y	Distance = xy km	
		According to the first condition,	
		$(x + 15) (y - 2) = xy$	½
	y	$x (y - 2) + 15 (y - 2) = xy$	
	y	$xy - 2x + 15y - 30 = xy$	
	y	$-2x + 15y = 30$ ......(i)	½
		According to the second condition,	
		$(x - 5) (y + 1) = xy$	
	y	$x (y + 1) - 5 (y + 1) = xy$	½
	y	$xy + x - 5y - 5 = xy$	
	y	$x - 5y = 5$ ......(ii)	½
		Multiplying (ii) by 3 we get,	
		$3x - 15y = 15$ ......(iii)	
		Adding (i) and (iii) we get,	
		$-2x + 15y = 30$	
		$3x - 15y = 15$	
		$x = 45$	1

Substituting x = 45 in (ii),

y $45 - 5y = 5$

y $-5y = 5 - 45$

y $-5y = -40$

y $y = \frac{-40}{-5}$

y $y = 8$ **1**

y Distance = xy

= 45×8

= 360

y Distance covered by bus is 360 km. **1**

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