MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - II - PAPER - 3 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Solve the following : (Any 5)

(i)

$t_1 = 0.1$

$t_2 = \frac{0.1}{10} = 0.01$

$t_3 = \frac{0.01}{10} = 0.001$

$t_4 = \frac{0.001}{10} = 0.0001$

$t_5 = \frac{0.0001}{10} = 0.00001$

$t_6 = \frac{0.00001}{10} = 0.000001$

$t_7 = \frac{0.000001}{10} = 0.0000001$

$t_8 = \frac{0.0000001}{10} = 0.00000001$

∴ The next four terms of the sequence are 0.00001, 0.000001, 0.0000001 and 0.00000001. **1**

(ii)

$p(p - 6) = 0$

∴ $p^2 - 6p = 0$

∴ $p^2 - 6p + 0 = 0$ **1**

(iii)

$$\begin{vmatrix} -\frac{4}{7} & -\frac{6}{35} \\ 5 & -\frac{2}{5} \end{vmatrix}$$

$= \left(\frac{-4}{7} \times \frac{-2}{5}\right) - \left(\frac{-6}{35} \times 5\right)$ **½**

$= \frac{8}{35} + \frac{30}{35}$

$= \boxed{\frac{38}{35}}$ **½**

(iv) $a = 3, d = 4$

Here, $t_1 = a = 3$

$t_2 = t_1 + d = 3 + 4 = 7$

$t_3 = t_2 + d = 7 + 4 = 11$

$t_4 = t_3 + d = 11 + 4 = 15$

$t_5 = t_4 + d = 15 + 4 = 19$

y The first five terms of the A.P. are 3, 7, 11, 15 and 19. **1**

(v) $x^2 - x = 0, x = 0$

Putting $x = 0$ in L.H.S., we get,

L.H.S. $= (0)^2 - 0$

$= 0 - 0$

$= 0$ **½**

$=$ R.H.S.

y L.H.S. = R.H.S.

Thus equation is satisfied.

So 0 is the root of the given quadratic equation. **½**

(vi) $3x + 4y = 8$

$x - 2y = 5$

$D_x = \begin{vmatrix} 8 & 4 \\ 5 & -2 \end{vmatrix}$ **1**

A.2. Solve the following : (Any 4)

(i) The natural numbers from 1 to 140 that are divisible by 4 are as follows : 4, 8, 12, 16, ..., 140

These numbers from an A.P. with $a = 4$, $d = t_2 - t_1 = 8 - 4 = 4$

Let, 140 be the n^{th} term of A.P.

$t_n = 140$

$t_n = a + (n - 1) d$

y $140 = 4 + (n - 1) 4$ **½**

y $140 = 4 + 4n - 4$

y $140 = 4n$

y $n = \frac{140}{4}$

y $n = 35$ **½**

y 140 is 35 term of A.P.

y We have to find sum of 35 terms i.e. S_{35},

$S_n = \frac{n}{2} [2a + (n - 1)d]$

y $S_{35} = \frac{35}{2} [2 (4) + (35 - 1) 4]$ **½**

	y $S_{35} = \frac{35}{2}[8 + 34(4)]$	
	y $S_{35} = \frac{35}{2}[8 + 136)$	
	y $S_{35} = \frac{35}{2}[144]$	
	y $S_{35} = 2520$	
	y **Sum of natural numbers from 1 to 140 that are divisible by 4 is 2520.**	½
(ii)	$y^2 - 16y + 63 = 0$	
	y $y^2 - 9y - 7y + 63 = 0$	
	y $y(y - 9) - 7(y - 9) = 0$	**1**
	y $(y - 9)(y - 7) = 0$	
	y $y - 9 = 0$ or $y - 7 = 0$	
	y $\boxed{y = 9 \text{ or } y = 7}$	**1**
(iii)	$\because$ (a, 3) is a point lying on the graph of the equation $5x + 2y = -4$, it satisfies the equation.	
	y Substituting $x = a$ and $y = 3$ in the equation we get,	
	$5(a) + 2(3) = -4$	**1**
	y $5a + 6 = -4$	
	y $5a = -4 - 6$	
	y $5a = -10$	
	y $a = \frac{-10}{5}$	
	y $\boxed{a = -2}$	**1**
(iv)	$3x + 2y + 11 = 0$	
	y $3x + 2y = -11$	
	$7x - 4y = 9$	
	$D = \begin{vmatrix} 3 & 2 \\ 7 & -4 \end{vmatrix} = (3 \times -4) - (2 \times 7) = -12 - 14 = -26$	
	$D_x = \begin{vmatrix} -11 & 2 \\ 9 & -4 \end{vmatrix} = (-11 \times -4) - (2 \times 9) = 44 - 18 = 26$	
	$D_y = \begin{vmatrix} 3 & -11 \\ 7 & 9 \end{vmatrix} = (3 \times 9) - (-11 \times 7) = 27 - (-77) = 27 + 77 = 104$	**1**

	By Cramer's rule,	
	$x = \frac{D_x}{D} = \frac{26}{-26} = -1$	
	$y = \frac{D_y}{D} = \frac{104}{-26} = -4$	
	y **x = – 1 and y = – 4 is the solution of given simultaneous equations.**	1
(v)	$3x^2 + kx - 2 = 0$	
	x = 4 is the solution of given quadratic equation.	
	Substituting x = 4 in given quadratic equation, it will get satisfied.	
	y $3(4)^2 + k(4) - 2 = 0$	1
	y $3(16) + 4k - 2 = 0$	
	y $48 + 4k - 2 = 0$	
	y $4k + 46 = 0$	
	y $4k = -46$	
	y $k = \frac{-46}{4}$	
	y $\boxed{k = \frac{-23}{2}}$	1
(vi)	No. of male fishes in the tank = 5	
	No. of female fishes in the tank = 8	
	y Total no. of fishes in the tank n (S) = 5 + 8 = 13	½
	Let A be the event that the fish picked is a male fish	
	No. of male fishes n (A) = 5	½
	$P(A) = \frac{n(A)}{n(S)}$	
	y $\boxed{P(A) = \frac{5}{13}}$	1
A.3.	**Solve the following : (Any 3)**	
(i)	Since the taxi fare increases by Rs. 2 every kilometer after the first, the successive taxi fares form an A.P.	
	The taxi fare for first kilometer (a) = Rs. 14	½
	Increase in taxi fare in every kilometer after first kilometer (d) = 2	½
	No. of kilometers covered by taxi (n) = 10	
	Taxi fare for 10 kilometers = t_{10} = ?	

	$t_n = a + (n + 1)\ d$	
	y $t_{10} = a + (10 - 1)\ d$	**1**
	y $t_{10} = 14 + 9\ (2)$	
	y $t_{10} = 14 + 18$	
	y $t_{10} = 32$	
	y **[** Taxi fare for ten kilometers is Rs. 32. **]**	**1**
(ii)	$y^2 = 3 + 4y$	
	y $y^2 - 4y = 3$ ....(i)	
	Third term $= \left(\frac{1}{2} \times \text{coefficient of } y\right)^2$	
	$= \left(\frac{1}{2} \times -4\right)^2$	
	$= (-2)^2$	
	$= 4$	**1**
	Adding 4 to both sides of (i) we get,	
	$y^2 - 4y + 4 = 3 + 4$	
	y $(y - 2)^2 = 7$	**1**
	Taking square root on both the sides we get,	
	$y - 2 = \pm\sqrt{7}$	
	y $y = 2 \pm \sqrt{7}$	
	y $y = 2 + \sqrt{7}$ or $y = 2 - \sqrt{7}$	
	y **[** $2 + \sqrt{7}$ and $2 - \sqrt{7}$ are the roots of the given quadratic equations. **]**	**1**
(iii)	Let greater number be x and smaller number be y.	
	As per first given condition,	
	$x + y = 60$ ....(i)	½
	As per second given condition,	
	$x = 3y + 8$	
	y $x - 3y = 8$ ....(ii)	½
	Subtracting (ii) from (i),	
	$x + y = 60$	
	$x - 3y = 8$	
	$(-)\ (+)\ \ (-)$	
	$4y = 52$	
	$y = \frac{52}{4}$	
	y $y = 13$	**1**
	Substituting y = 13 in (i),	
	$x + 13 = 60$	
	y $x = 60 - 13$	
	y $x = 47$	½
	y **[** The greater number is 47 and smaller number is 13. **]**	½

(iv) Two coins are tossed

y S = { HH, HT, TH, TT }

y n (S) = 4 ½

A is the event of getting at the most one head

y A = { HT, TH, TT }

y n (A) = 3 ½

B is the event of getting both heads.

y B = { HH }

y n (B) = 1 ½

C is the event of getting same face on both the coins

y C = { HH, TT }

y n (C) = 2 ½

A ą B = ¤ and A Ć B = S

y A and B are complementary and mutually exclusive events. ½

y A Ć C = S

y A and C are exhaustive events. ½

(v) $x^2 + 3x - 2 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 1$, $b = 3$, $c = -2$

$b^2 - 4ac = (3)^2 - 4(1)(-2)$

$= 9 + 8$

$= 17$ ½

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{17}}{2(1)}$ **1**

$= \frac{-3 \pm \sqrt{17}}{2}$

y $x = \frac{-3 + \sqrt{17}}{2}$ or $x = \frac{-3 - \sqrt{17}}{2}$ **1**

y $\frac{-3 + \sqrt{17}}{2}$ and $\frac{-3 - \sqrt{17}}{2}$ are the roots of the given quadratic equation. ½

A.4. Solve the following : (Any 2)

(i) $t_2 = 12$, $t_4 = 20$

$t_n = a + (n - 1)d$

$t_2 = a + (2 - 1)d$ ½

$12 = a + d$

y $a + d = 12$ ....(i) ½

$t_4 = a + (4 - 1)d$ ½

$20 = a + 3d$

y $a + 3d = 20$ ....(ii) ½

Subtracting (ii) from (i),

$$\begin{array}{rcl} a + d &=& 12 \\ a + 3d &=& 20 \\ (-)\ \ (-) && (-) \\ \hline -2d &=& -8 \end{array}$$

y $d = 4$ ½

Substituting d = 4 in (i),

$a + 4 = 12$

y $a = 12 - 4$

y $a = 8$ ½

$S_n = \frac{n}{2}[2a + (n - 1)d]$

y $S_{25} = \frac{25}{2}[2a + (25 - 1)d]$ ½

$= \frac{25}{2}[2(8) + 24\ (4)]$

$= \frac{25}{2}[16 + 96]$

$= \frac{25}{2}[112]$

$S_{25} = 1400$ ½

y **Sum of 25 terms of the A.P is 1400.**

(ii) $x + y = 8$

y $y = 8 - x$

x	0	1	2
y	8	7	6
(x, y)	(0, 8)	(1, 7)	(2, 6)

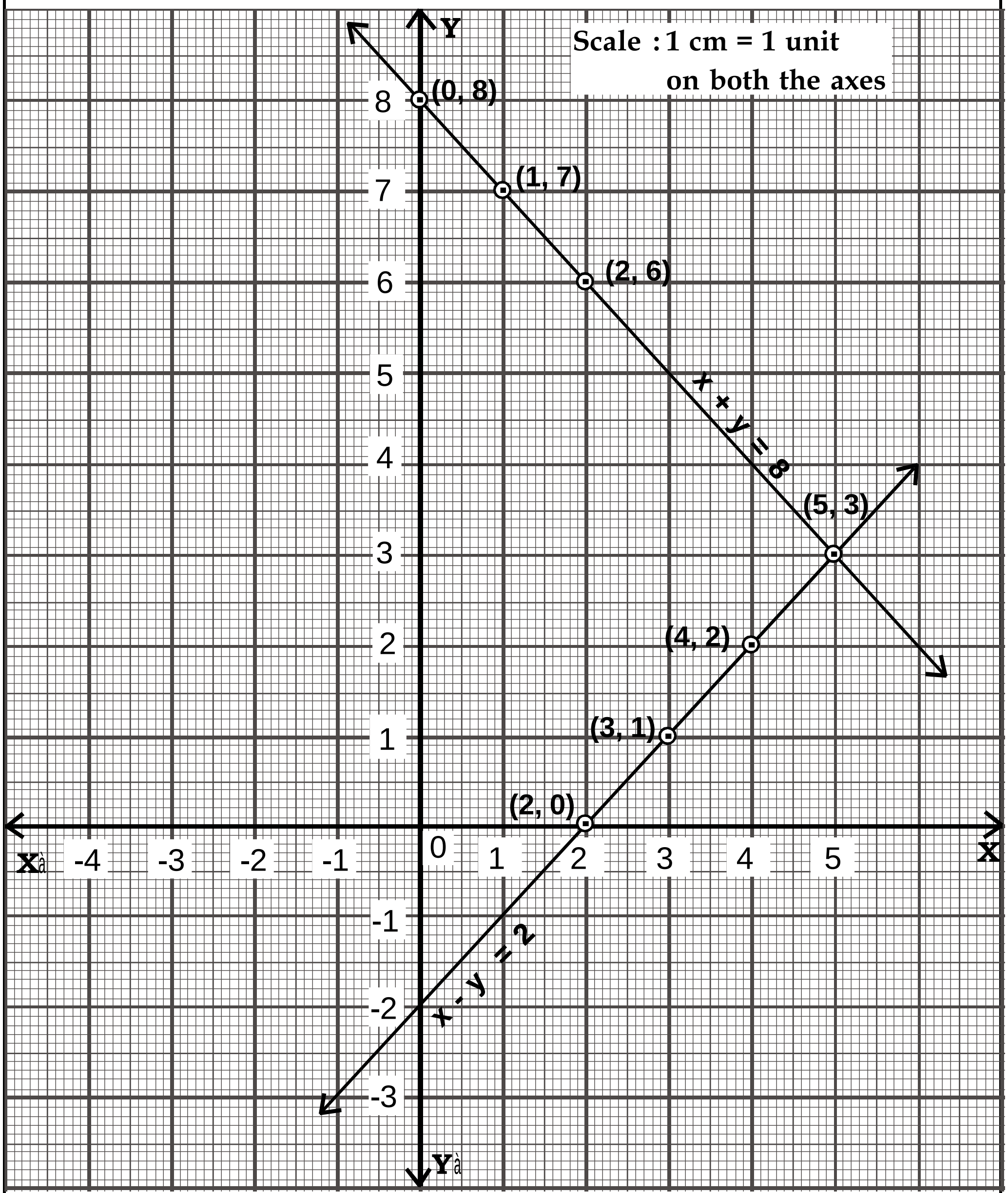
½

$x - y = 2$

y $x = 2 + y$

x	2	3	4
y	0	1	2
(x, y)	(2, 0)	(3, 1)	(4, 2)

½



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y x = 5 and y = 3 is the solution of given simultaneous equations. **1**

(iii) When a coin tossed three times

S = { HHH, HTH, THH, TTH, HHT, HTT, THT, TTT }

n (S) = 8 **1**

(a) Let A be the event of getting head on middle coin

A = { HHH, THH, HHT, THT }

n (A) = 4 **½**

$P(A) = \frac{n(A)}{n(S)}$

y $P(A) = \frac{4}{8}$

y $\boxed{P(A) = \frac{1}{2}}$ ½

(b) Let B be the event of getting exactly one tail

B = { HTH, THH, HHT }

n (B) = 3 ½

$P(B) = \frac{n(B)}{n(S)}$

y $\boxed{P(B) = \frac{3}{8}}$ ½

(c) Let C be the event of getting no tail

C = { HHH }

n (C) = 1 ½

$P(C) = \frac{n(C)}{n(S)}$

y $\boxed{P(C) = \frac{1}{8}}$ ½

A.5. Solve the following : (Any 2)

(i) $\frac{1}{3x} + \frac{1}{5y} = \frac{1}{15}$

Multiplying throught by 15 we get,

$15\left(\frac{1}{3x}\right) + 15\left(\frac{1}{5y}\right) = 15\left(\frac{1}{15}\right)$

y $\frac{5}{x} + \frac{3}{y} = 1$ ....(i) ½

$\frac{1}{2x} + \frac{1}{3y} = \frac{1}{12}$

Multipying throught by 12,

$12\left(\frac{1}{2x}\right) + 12\left(\frac{1}{3y}\right) = 12\left(\frac{1}{12}\right)$

y $\frac{6}{x} + \frac{4}{y} = 1$ ......(ii) ½

Substituting $\frac{1}{x}$ = a and $\frac{1}{y}$ = b in (i) and (ii),

5a + 3b = 1 ......(iii) ½

6a + 4b = 1 ....(iv) ½

	Multiplying by (iii) bt 4,	
	$20a + 12b = 4$ ....(v)	
	Multiplying (iv) by 3	
	$18a + 12b = 3$ ....(vi)	½
	Subtracting (vi) from (v)	
	$20a + 12b = 4$	
	$18a + 12b = 3$	
	(–) (–) (–)	
	$2a = 1$	
	y $a = \frac{1}{2}$	½
	Substituting $a = \frac{1}{2}$ in (iv),	
	$6\left(\frac{1}{2}\right) + 4b = 1$	
	y $3 + 4b = 1$	
	y $4b = 1 - 3$	
	y $4b = -2$	
	y $b = \frac{-2}{4}$	
	y $b = \frac{-1}{2}$	½
	Resubstituting the values of a and b,	
	$a = \frac{1}{x}$	
	$\frac{1}{2} = \frac{1}{x}$	½
	y $x = 2$	
	$b = \frac{1}{y}$	
	$\frac{-1}{2} = \frac{1}{y}$	
	y $y = -2$	½
	y **x = 2 and y = –2 is the solution of given simultaneous equations.**	½
(ii)	Let the four consecutive terms is an A.P. be $a - 3d$, $a - d$, $a + d$ and $a + 3d$	½
	As per first condition,	
	$a - 3d + a - d + a + d + a + 3d = -54$	½
	y $4a = -54$	
	y $a = \frac{-54}{4}$	
	y $a = -\frac{27}{2}$	1

As per the second condition,

$a - 3d + a + d = -30$ ½

y $2a - 2d = -30$

y $2\left(-\frac{27}{2}\right) - 2d = -30$

y $-27 - 2d = -30$

y $-2d = -30 + 27$

y $-2d = -3$

y $d = \frac{3}{2}$ **1**

y $a - 3d = -\frac{27}{2} - 3\left(\frac{3}{2}\right) = \frac{-27-9}{2} = \frac{-36}{2} = -18$

y $a - d = -\frac{27}{2} - \frac{3}{2} = \frac{-30}{2} = -15$ ½

y $a + d = -\frac{27}{2} + \frac{3}{2} = -\frac{27}{2} + \frac{3}{2} = \frac{-24}{2} = -12$

y $a + 3d = -\frac{27}{2} + 3\left(\frac{3}{2}\right) = \frac{-27+9}{2} = \frac{-27+9}{2} = \frac{-18}{2} = -9$ ½

y **The four consecutive terms of an A.P. are – 18, – 15, – 12 and – 9.** ½

(iii) Let the time for which the car travels at 60 km/hr be x hours and at 75 (i.e. 60 + 15) km/hr be y hours .

We know that, Distance = Speed × Time

Distance covered by car with the speed of 60 km/hr = 60 × x = 60x km ½

Distance covered by car with the speed of 75 km/hr = 75 × y = 75y km ½

As per first condition,

$60x + 75y = 555$

$15(4x + 5y) = 555$

$4x + 5y = \frac{555}{15}$

$4x + 5y = 37$ ....(i) **1**

As per second condition,

$x + y = 8$ ....(ii) **1**

Multiplying (ii) by 5, we get,

$5x + 5y = 40$ ....(iii) **1**

Subtracting (i) from (iii), we get

$$\begin{array}{l} 5x + 5y = 40 \\ 4x + 5y = 37 \\ -\ \ -\ \ \ \ \ - \\ \hline x = 3 \end{array}$$

y **Time taken by the car at the speed of 60 km/hr = 3 hours and Distance covered with the speed of 60km/hr = 60 × 3 = 180 km.** **1**

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