MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - II - PAPER - 4 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1.	**Solve the following : (Any 5)**	
(i)	$t_1 = 3,\ t_2 = 5,\ t_3 = 7,\ t_4 = 9,\ t_5 = 11$	
	$t_2 - t_1 = 5 - 3 = 2$	
	$t_3 - t_2 = 7 - 5 = 2$	
	$t_4 - t_3 = 9 - 7 = 2$	
	$t_5 - t_4 = 11 - 9 = 2$	
	∵ The difference between two consecutive terms is 2 which is constant.	
	∴ **The sequence is an A.P.**	**1**
(ii)	$\frac{y^2}{23} - 3 = 0$	
	Multiplying throughout by 23, we get	
	$y^2 - 69 = 0$	
	∴ $y^2 + 0y - 69 = 0$	**1**
(iii)	$\begin{vmatrix} 3\sqrt{6} & -4\sqrt{2} \\ 5\sqrt{3} & 2 \end{vmatrix}$	
	$= \left(3\sqrt{6} \times 2\right) - \left(-4\sqrt{2} \times 5\sqrt{3}\right)$	**½**
	$= \left(6\sqrt{6}\right) - \left(-20\sqrt{6}\right)$	
	$= 6\sqrt{6} + 20\sqrt{6}$	
	$= \boxed{26\sqrt{6}}$	**½**
(iv)	$t_1 = 0 + 2^0 = 0 + 1 = 1$	
	$t_2 = 1 + 2^1 = 1 + 2 = 3$	
	$t_3 = 3 + 2^2 = 3 + 4 = 7$	
	$t_4 = 7 + 2^3 = 7 + 8 = 15$	
	$t_5 = 15 + 2^4 = 15 + 16 = 31$	
	$t_6 = 31 + 2^5 = 31 + 32 = 63$	
	$t_7 = 63 + 2^6 = 63 + 64 = 127$	
	$t_8 = 127 + 2^7 = 127 + 128 = 255$	
	$t_9 = 255 + 2^8 = 255 + 256 = 511$	
	∴ The next four terms of the sequence are 63, 127, 255 and 511.	**1**

(v) $x^2 + x - 1 = 0, x = 2$

Putting x = 2 in L.H.S., we get,

L.H.S. = $(2)^2 + 2 - 1$

= 4 + 1

= 5 — ½

÷ R.H.S.

y L.H.S. ÷ R.H.S.

Thus equation is not satisfied.

So 2 is not the root of the given quadratic equation. — ½

(vi) 5x = 10 – 2y

y 5x + 2y = 10

y = 3x – 11

y – 3x + y = – 11

$$D_x = \begin{vmatrix} 10 & 2 \\ -11 & 1 \end{vmatrix}$$ — 1

A.2. Solve the following : (Any 4)

(i) For the given A.P. 1, 7, 13, 19, ...

Here, $a = t_1 = 1$

$d = t_2 - t_1 = 7 - 1 = 6$

We know,

$t_n = a + (n - 1)\, d$ — 1

y $t_{18} = a + (18 - 1)\, d$

y $t_{18} = 1 + 17\,(6)$

y $t_{18} = 1 + 102$

y $t_{18} = 103$

y Eighteenth term of A.P. is 103. — 1

(ii) $x^2 - 17x + 60 = 0$

y $x^2 - 12x - 5x + 60 = 0$

y $x(x - 12) - 5(x - 12) = 0$ — 1

y $(x - 12)(x - 5) = 0$

y $x - 12 = 0$ or $x - 5 = 0$

∴ $x = 12$ or $x = 5$ — 1

(iii) The equation of X-axis is y = 0 — 1

Let the point of intersection of graph x + y = 3 with X-axis be (h, 0)

∵ (h, 0) lies on the graph, it satisfies the equation

y Substituting x = h and y = 0 in the equation we get,

h + 0 = 3 — ½

y h = 3

y The line x + y = 3 intersects the X-axis at (3, 0). — ½

(iv)	$x + 18 = 2y$	
	y $x - 2y = -18$	
	$y = 2x - 9$	
	y $-2x + y = -9$	
	$D = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = (1 \times 1) - (-2 \times -2) = 1 - 4 = -3$	
	$D_x = \begin{vmatrix} -18 & -2 \\ -9 & 1 \end{vmatrix} = (-18 \times 1) - (-2 \times -9) = -18 - 18 = -36$	
	$D_y = \begin{vmatrix} 1 & -18 \\ -2 & -9 \end{vmatrix} = (1 \times -9) - (-18 \times -2) = -9 - 36 = -45$	1
	By Cramer's rule,	
	$x = \frac{D_x}{D} = \frac{-36}{-3} = 12$	
	$y = \frac{D_y}{D} = \frac{-45}{-3} = 15$	
	y **x = 12 and y = 15 is the solution of given simultaneous equations.**	1
(v)	$kx^2 - 7x + 12 = 0$	
	$x = 3$ is root of given quadratic equation.	
	So it satisfies the given equation	
	Substituting $x = 3$ in the given equation.	
	y $k(3)^2 - 7(3) + 12 = 0$	1
	y $k(9) - 21 + 12 = 0$	
	y $9k - 9 = 0$	
	y $9k = 9$	
	y $k = \frac{9}{9}$	
	y **$k = 1$**	1
(vi)	Sample space for the game of chance of spinning an arrow	
	$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$	
	$n(S) = 8$	1
	Let A be the event that the arrow points at 8	
	$A = \{8\}$	
	y $n(A) = 1$	½
	y $P(A) = \frac{n(A)}{n(S)}$	
	y **$P(A) = \frac{1}{8}$**	½

A.3.	**Solve the following : (Any 3)**	
(i)	$S_{55} = 3300$ [Given]	
	$S_n = \frac{n}{2}[2a + (n-1)d]$	
	∴ $S_{55} = \frac{55}{2}[2a + (55-1)d]$	½
	∴ $3300 = \frac{55}{2}[2a + 54d]$	
	∴ $3300 = \frac{55}{2} \times 2\ [a + 27d]$	
	∴ $\frac{3300}{55} = a + 27d$	
	∴ $\frac{300}{5} = a + 27d$	
	∴ $a + 27d = 60$ ......(i)	1
	$t_n = a + (n-1)d$	
	∴ $t_{28} = a + (28-1)d$	½
	∴ $t_{28} = a + 27d$	
	∴ $t_{28} = 60$ [From (i)]	
	∴ Twenty eighth term of A.P. is 60.	1
(ii)	$p^2 - 12p + 32 = 0$	
	∴ $p^2 - 12p = -32$ ......(i)	
	Third term $= \left(\frac{1}{2} \times \text{coefficient of } p\right)^2$	
	$= \left(\frac{1}{2} \times -12\right)^2$	
	$= (-6)^2$	
	$= 36$	1
	Adding 36 to both sides of (i), we get,	
	$p^2 - 12p + 36 = -32 + 36$	
	∴ $(p-6)^2 = 4$	1
	Taking square root on both the sides we get,	
	$p - 6 = \pm 2$	
	∴ $p = 6 \pm 2$	
	∴ $p = 6 + 2$ or $p = 6 - 2$	
	∴ $p = 8$ or $p = 4$	
	∴ 8 and 4 are the roots of the given quadratic equations.	1

(iii)	Let the measures of two acute angles of a right angled triangle be x° and y°. As per the given condition, $x = y + 20$ ∴ $x - y = 20$ ....(i) We know, $x + y = 90$ ......(ii) [Acute angles of a right angled triangle are complementary] Adding (i) and (ii), $x - y = 20$ $x + y = 90$ $2x = 110$ $x = \frac{110}{2}$ ∴ $x = 55$ Substituting $x = 55$ in (ii), $55 + y = 90$ ∴ $y = 90 - 55$ ∴ $y = 35$ ∴ **The measures of the two acute angles of a right angled triangle are 55° and 35°.**	½ ½ 1 ½ ½
(iv)	The two digit numbers that can be formed using the digits 0, 1, 2, 3, 4 without repeating digits are ∴ S = { 10, 12, 13, 14, 20, 21, 23, 24, 30, 31, 32, 34, 40, 41, 42, 43 } ∴ n (S) = 16 (a) Let A be the event that the number formed is even number A = { 10, 12, 14, 20, 24, 30, 32, 34, 40, 42 } n (A) = 10 $P(A) = \frac{n(A)}{n(S)}$ ∴ $P(A) = \frac{10}{16}$ ∴ $P(A) = \frac{5}{8}$ (b) Let B be the event that number formed is greater than 40 B = { 41, 42, 43 } n (B) = 3 $P(B) = \frac{n(B)}{n(S)}$ ∴ $P(B) = \frac{3}{16}$	½ ½ ½ ½

(c) Let C be the event that number formed is prime number

$C = \{13, 23, 31, 41, 43\}$

$n(C) = 5$ ½

$P(C) = \frac{n(C)}{n(S)}$

y $\boxed{P(C) = \frac{5}{16}}$ ½

(v) $2x^2 + 5x - 2 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 2, b = 5, c = -2$

$b^2 - 4ac = (5)^2 - 4(2)(-2)$

$= 25 + 16$

$= 41$ ½

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-5 \pm \sqrt{41}}{2(2)}$ **1**

$= \frac{-5 \pm \sqrt{41}}{4}$

y $x = \frac{-5 + \sqrt{41}}{4}$ or $x = \frac{-5 - \sqrt{41}}{4}$ **1**

y $\frac{-5 + \sqrt{41}}{4}$ and $\frac{-5 - \sqrt{41}}{4}$ are the roots of the given quadratic equation. ½

A.4. Solve the following : (Any 2)

(i) Let the four consecutive terms in an A.P. be

$a - 3d, a - d, a + d, a + 3d$ ½

As per the first condition,

$a - 3d + a - d + a + d + a + 3d = 12$ ½

y $4a = 12$

y $a = 3$ ½

As per the second condition,

$a + d + a + 3d = 14$ ½

y $2a + 4d = 14$

y $2(3) + 4d = 14$

y $6 + 4d = 14$

y $4d = 14 - 6$

y $4d = 8$

y $d = 2$ ½

y $a - 3d = 3 - 3(2) = 3 - 6 = -3$

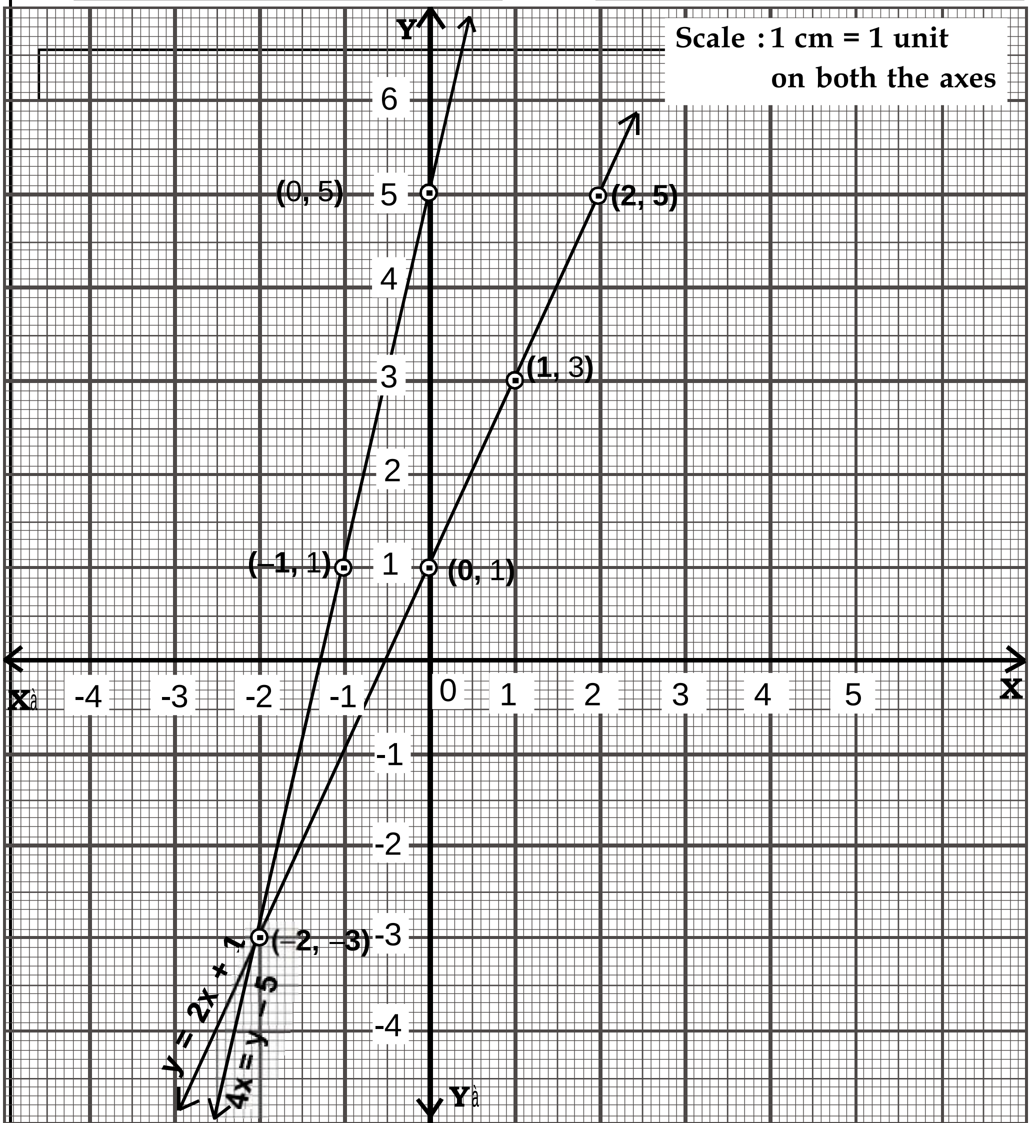
$a - d = 3 - 2 = 1$

	a + d = 3 + 2 = 5 a + 3d = 3 + 3 (2) = 9	1
	y The four consecutive terms of A.P. are – 3, 1, 5 and 9.	½
(ii)	4x = y – 5 y 4x +5 = y y y = 4x + 5 y = 2x + 1	

x	0	–1	–2
y	5	1	–3
(x, y)	(0, 5)	(–1, 1)	(–2, –3)

x	0	1	2
y	1	3	5
(x, y)	(0, 1)	(1, 3)	(2, 5)

1



2

y x = – 2 and y = – 3 is the solution of given simultaneous equations. 1

(iii)	When two dice are thrown	
	S = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }	
	y n (S) = 36	½
	A is the event that sum of the numbers on their upper faces is atleast nine	
	A = { (3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6) }	
	n (A) = 10	**1**
	B is the event that sum of numbers on their upper faces is divisible by 8.	
	B = { (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) }	
	n (B) = 5	**1**
	C is the event that same number appears an upper faces of both dice	
	C = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }	
	n (C) = 6	**1**
	A ą B = ¤	
	y A and B are mutually exclusive events.	½
A.5.	**Solve the following : (Any 2)**	
(i)	$\frac{2}{x} + \frac{6}{y} = 13$ ....(i)	
	$\frac{3}{x} + \frac{4}{y} = 12$ ......(ii)	
	Substituting $\frac{1}{x}$ = a and $\frac{1}{y}$ = b in(i) and (ii) we get,	
	2a + 6b = 13 ......(iii)	½
	3a + 4b = 12 ......(iv)	½
	Multiplying (iii) by 2,	
	4a + 12b = 26 ....(v)	
	Multiplying (iv) by 3,	
	9a + 12b = 36 ......(vi)	½
	Substracting (vi) from (v),	
	4a + 12b = 26	
	9a + 12b = 36	
	(–) (–) (–)	
	–5a = –10	
	y a = $\frac{-10}{-5}$	
	y a = 2	**1**

	Substituting a = 2 in (iii), $2(2) + 6b = 13$ y $4 + 6b = 13$ y $6b = 13 - 4$ y $b = \frac{9}{6}$ y $b = \frac{3}{2}$	1
	Resubstituting the values of a and b $a = \frac{1}{x}$ $2 = \frac{1}{x}$ y $2x = 1$ y $x = \frac{1}{2}$	½
	$b = \frac{1}{y}$ $\frac{3}{2} = \frac{1}{y}$ y $3y = 2$ y $y = \frac{2}{3}$	½
	y $x = \frac{1}{2}$ and $y = \frac{2}{3}$ is the solution of given simultaneous equations.	½
(ii)	for the A.P. 7, 14, 21 a = 7, d = 7 $S_n = 5740$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{n}{2}[2(7) + (n-1)7]$	1
	$5740 = \frac{n}{2}[14 + 7n - 7]$ $5740 = \frac{n}{2}[7n + 7]$ $11480 = 7n^2 + 7n$ y $7n^2 + 7n - 11480 = 0$	1

	Dividing through at by 7 we get,	
	$n^2 + n - 1640 = 0$	1
	y $n^2 + 41n - 40n - 1640 = 0$	
	y $n(n + 41) - 40(n + 41) = 0$	
	y $(n + 41)(n - 40) = 0$	
	y $n + 41 = 0$ **or** $n - 40 = 0$	
	y $n = -41$ **or** $n = 40$	1
	$n = -41$ is not acceptable because no of terms cannot be negative	
	y $n = 40$	½
	y For the given sequence 40 terms have to be considered for getting sum of 5740.	½
(iii)	Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.	
	y Speed of the boat upstream = $(x - y)$ km/hr	
	and speed of the boat downstream = $(x + y)$ km/hr	½
	We know that, Time = $\frac{\text{Distance}}{\text{Speed}}$	
	As per the first condition,	
	$\frac{8}{x - y} + \frac{32}{x + y} = 6$ ....(i)	½
	As per the second condition,	
	$\frac{20}{x - y} + \frac{16}{x + y} = 7$ ......(ii)	½
	Substituting $\frac{1}{x - y} = m$ and $\frac{1}{x + y} = n$ in (i) and (ii) we get,	
	$8m + 32n = 6$ .....(iii)	
	$20m + 16n = 7$ ......(iv)	½
	Multiplying (iv) by 2 we get,	
	$40m + 32n = 14$ ......(v)	
	Subtracting (v) from (iii),	
	$8m + 32n = 6$	
	$40m + 32n = 14$	
	(–) (–) (–)	
	$-32m = -8$	
	y $m = \frac{-8}{-32}$	
	y $m = \frac{1}{4}$	½

Substituting $m = \frac{1}{4}$ in (iii),

$8\left(\frac{1}{4}\right) + 32n = 6$

y $2 + 32n = 6$

y $32n = 6 - 2$

y $32n = 4$

y $n = \frac{4}{32}$

y $n = \frac{1}{8}$ ½

Resubstituting the values of m and n we get,

$m = \frac{1}{x - y}$

y $\frac{1}{4} = \frac{1}{x - y}$

y $x - y = 4$ ……(vi) ½

$n = \frac{1}{x + y}$

y $\frac{1}{8} = \frac{1}{x + y}$

y $x + y = 8$ ……(vii) ½

Adding (vi) and (vii),

$$\begin{aligned} x - y &= 4 \\ x + y &= 8 \\ \hline 2x &= 12 \end{aligned}$$

y $x = \frac{12}{2}$

y $x = 6$

Substituting x = 6 in (vii)

$6 + y = 8$

y $y = 8 - 6$

y $y = 2$ ½

y **The speed of boat in still water is 6 km/hr and speed of stream is 2 km/ hr.** ½

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