MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - II - PAPER - 5 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Solve the following : (Any 5)

(i) a = 5, d = 2

Here, $t_1 = a = 5$

$t_2 = t_1 + d = 5 + 2 = 7$

$t_3 = t_2 + d = 7 + 2 = 9$

$t_4 = t_3 + d = 9 + 2 = 11$

$t_5 = t_4 + d = 11 + 2 = 13$

y The first five terms of the A.P. are 5, 7, 9, 11 and 13. **1**

(ii) $2x^2 - x + 3 = 0$

Comparing with $ax^2 + bx + c = 0$

a = 2, b = – 1 c = 3 **1**

(iii) $\begin{vmatrix} -3 & 8 \\ 6 & 0 \end{vmatrix}$

= $(-3 \times 0) - (8 \times 6)$

= $0 - 48$

= -48 **1**

(iv) $t_n = n^3$

y $t_1 = 1^3 = 1$

y $t_2 = 2^3 = 8$

y $t_3 = 3^3 = 27$

y $t_4 = 4^3 = 64$

y $t_5 = 5^3 = 125$

y The first five terms of the sequence are 1, 8, 27, 64 and 125. **1**

(v) $x^2 - 4x + 4 = 0$, x = 0

Putting x = 0 in L.H.S., we get,

L.H.S. = $(0)^2 + 4(0) + 4$

= $0 - 0 + 4$

= 4 **½**

÷ R.H.S.

y L.H.S. ÷ R.H.S.
Thus equation is not satisfied.
So 0 is not the root of the given quadratic equation. ½

(vi) $12x + 13y = 29$ ......(i)
$13x + 12y = 21$ .....(ii)
Adding (i) and (ii),

$$\begin{array}{r} 12x + 13y = 29 \\ 13x + 12y = 21 \\ \hline 25x + 25y = 50 \\ \hline \end{array}$$ ½

Dividing throughout by 25 we get,

$$x + y = \frac{50}{25}$$

y $\boxed{x + y = 2}$ ½

A.2. Solve the following : (Any 4)

(i) $S_n = \frac{n (n + 1) (2n + 1)}{6}$

y $S_1 = \frac{1 (1 + 1) [2 (1) + 1]}{6} = \frac{1 \times 2 \times 3}{6} = \frac{6}{6} = 1$

y $S_2 = \frac{2 (2 + 1) [2 (2) + 1]}{6} = \frac{2 \times 3 \times 5}{6} = \frac{30}{6} = 5$

y $S_3 = \frac{3 (3 + 1) [2 (3) + 1]}{6} = \frac{3 \times 4 \times 7}{6} = 14$ 1

We know that,
$t_1 = S_1 = 1$
$t_2 = S_2 - S_1 = 5 - 1 = 4$
$t_3 = S_3 - S_2 = 14 - 5 = 9$

y $\boxed{\text{The first three terms of the sequence are 1, 4 and 9.}}$ 1

(ii) $21x = 196 - x^2$
y $x^2 + 21x - 196 = 0$
y $x^2 + 28x - 7x - 196 = 0$ 1
y $x (x + 28) - 7 (x + 28) = 0$
y $(x + 28) (x - 7) = 0$
y $x + 28 = 0$ or $x - 7 = 0$
∴ $\boxed{x = -28 \text{ or } x = 7}$ 1

(iii)	The equation of Y-axis is $x = 0$	**1**
	Let the point of intersection of the line $y = 3x + 2$ with Y-axis be (0, k)	
	$\because$ (0, k) lies on the line it satisfies the equation	
	y Substituting $x = 0$ and $y = k$ in the equation we get,	
	$k = 3(0) + 2$	½
	y $k = 2$	
	y **[boxed]** The point of intersection of the line $y = 3x + 2$ with Y-axis is (0, 2).	½
(iv)	$3x + y = 1$	
	$2x = 11y + 3$	
	y $2x - 11y = 3$	
	$D = \begin{vmatrix} 3 & 1 \\ 2 & -11 \end{vmatrix} = (3 \times -11) - (1 \times 2) = -33 - 2 = -35$	
	$D_x = \begin{vmatrix} 1 & 1 \\ 3 & -11 \end{vmatrix} = (1 \times -11) - (1 \times 3) = -11 - 3 = -14$	
	$D_y = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = (3 \times 3) - (1 \times 2) = 9 - 2 = 7$	**1**
	By Cramer's rule,	
	$x = \frac{D_x}{D} = \frac{-14}{-35} = \frac{2}{5}$	
	$y = \frac{D_y}{D} = \frac{7}{-35} = \frac{-1}{5}$	
	y **[boxed]** $x = \frac{2}{5}$ and $y = \frac{-1}{5}$ is the solution of given simultaneous equations.	**1**
(v)	$x^2 - 7x + k = 0$	
	$x = 4$ is the root of given quadratic equation.	
	So it satisfies the given equation.	
	Substituting $x = 4$ in the equation	**1**
	y $(4)^2 - 7(4) + k = 0$	
	y $16 - 28 + k = 0$	
	y $-12 + k = 0$	
	y **[boxed]** $k = 12$	**1**
(vi)	Sample space for the game of chance of spinning an arrow	
	$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$	
	$n(S) = 8$	**1**
	Let A be the event that the arrow points at 8	

	$A = \{8\}$	
	∴ $n(A) = 1$	½
	∴ $P(A) = \frac{n(A)}{n(S)}$	
	∴ $\boxed{P(A) = \frac{1}{8}}$	½
A.3.	**Solve the following : (Any 3)**	
(i)	The three digit natural numbers that are divisible by 4 are as follows 100, 104, 108, ... 996. These numbers form an A.P. with $a = t_1 = 100$, $d = t_2 - t_1 = 104 - 100 = 4$.	½
	Let, $t_n = 996$ We know that for an A.P. $t_n = a + (n-1)d$	
	∴ $996 = 100 + (n-1)4$	1
	∴ $996 = 100 + 4n - 4$ ∴ $996 = 96 + 4n$ ∴ $4n = 996 - 96$ ∴ $4n = 900$ ∴ $n = \frac{900}{4}$	
	∴ $n = 225$	1
	∴ There are 225 three digit natural numbers that are divisible by 4.	½
(ii)	$z^2 + 4z - 7 = 0$ ∴ $z^2 + 4z = 7$ ..... (i) Third term $= \left(\frac{1}{2} \times \text{coefficient of } z\right)^2$ $= \left(\frac{1}{2} \times 4\right)^2 = (2)^2$ $= 4$	1
	Adding 4 to both the sides of (i) we get, $z^2 + 4z + 4 = 7 + 4$ ∴ $(z+2)^2 = 11$ ∴ $(z+2)^2 = \left(\sqrt{11}\right)^2$	1
	Taking square root on both the sides, we get; $z + 2 = \pm\sqrt{11}$ ∴ $z = -2 \pm \sqrt{11}$ ∴ $z = -2 + \sqrt{11}$ or $z = -2 - \sqrt{11}$	
	∴ $-2 + \sqrt{11}$ and $-2 - \sqrt{11}$ are the roots of the given quadratic equation.	1

(iii) Let the measures of two acute angles of a right angled triangle be x° and y°. ½

As per the given condition,

$x = y + 20$

y $x - y = 20$ ....(i) ½

We know,

$x + y = 90$ ......(ii) ½

[Acute angles of a right angled triangle are complementary]

Adding (i) and (ii),

$$\begin{aligned} x - y &= 20 \\ x + y &= 90 \\ \hline 2x &= 110 \end{aligned}$$

$x = \frac{110}{2}$

y $x = 55$ ½

Substituting $x = 55$ in (ii),

$55 + y = 90$ ½

y $y = 90 - 55$

y $y = 35$

y **The measures of the two acute angles of a right angled triangle are 55° and 35°.** ½

(iv) (a) Two coins are tossed

$S = \{ HH, HT, TH, TT \}$

$n(S) = 4$ **1**

Let A be the event that head appears on both the coins

$A = \{ HH \}$

$n(A) = 1$

$P(A) = \frac{n(A)}{n(S)}$

y $\boxed{P(A) = \frac{1}{4}}$ **1**

Let B be the event that head does not appear

$B = \{ TT \}$

$n(B) = 1$

$P(B) = \frac{n(B)}{n(S)}$

y $\boxed{P(B) = \frac{1}{4}}$ **1**

(v) y $0 = 6x^2 - 7x - 1$

Comparing with $ax^2 + bx + c = 0$ we have $a = 6$, $b = -7$, $c = -1$ ½

	$b^2 - 4ac = (-7)^2 - 4(6)(-1)$	
	$= 49 + 24$	
	$= 73$	**1**
	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
	$= \dfrac{-(-7) \pm \sqrt{73}}{2(6)}$	½
	$= \dfrac{7 \pm \sqrt{73}}{12}$	
	y $x = \dfrac{7 + \sqrt{73}}{12}$ or $x = \dfrac{7 - \sqrt{73}}{12}$	½
	y $\dfrac{7 + \sqrt{73}}{12}$ and $\dfrac{7 - \sqrt{73}}{12}$ are the roots of the given quadratic equation.	½
A.4.	**Solve the following : (Any 2)**	
(i)	Let three consecutive terms in an A.P. be	
	$a - d, a, a + d$	½
	As per the first given condition,	
	$a - d + a + a + d = -3$	½
	y $3a = -3$	
	y $a = -1$	½
	As per the second given condition,	
	$(a - d)^3 a^3 (a + d)^3 = 512$	
	y $[(a - d) a (a + d)]^3 = 512$	½
	Taking cube roots on both sides	
	$(a - d) a (a + d) = \sqrt[3]{512}$	½
	y $(a - d) a (a + d) = 8$	
	y $a (a - d)(a + d) = 8$	
	y $a (a^2 - d^2) = 8$	
	y $-1 [(-1)^2 - d^2] = 8$	
	y $-1 (1 - d^2) = 8$	
	y $d^2 - 1 = 8$	
	y $d^2 = 8 + 1$	
	y $d^2 = 9$	
	y $d = \pm 3$	½
	If d = 3	
	y $a - d = -1 - 3 = -4$ $a - d = -1-(-3) = 2$	
	y $a + d = -1 + 3 = 2$ $a + d = -1 - 3 = 4$	
	y The three consecutive terms of A.P. are – 4, – 1, 2 or 2, –1, –4	**1**

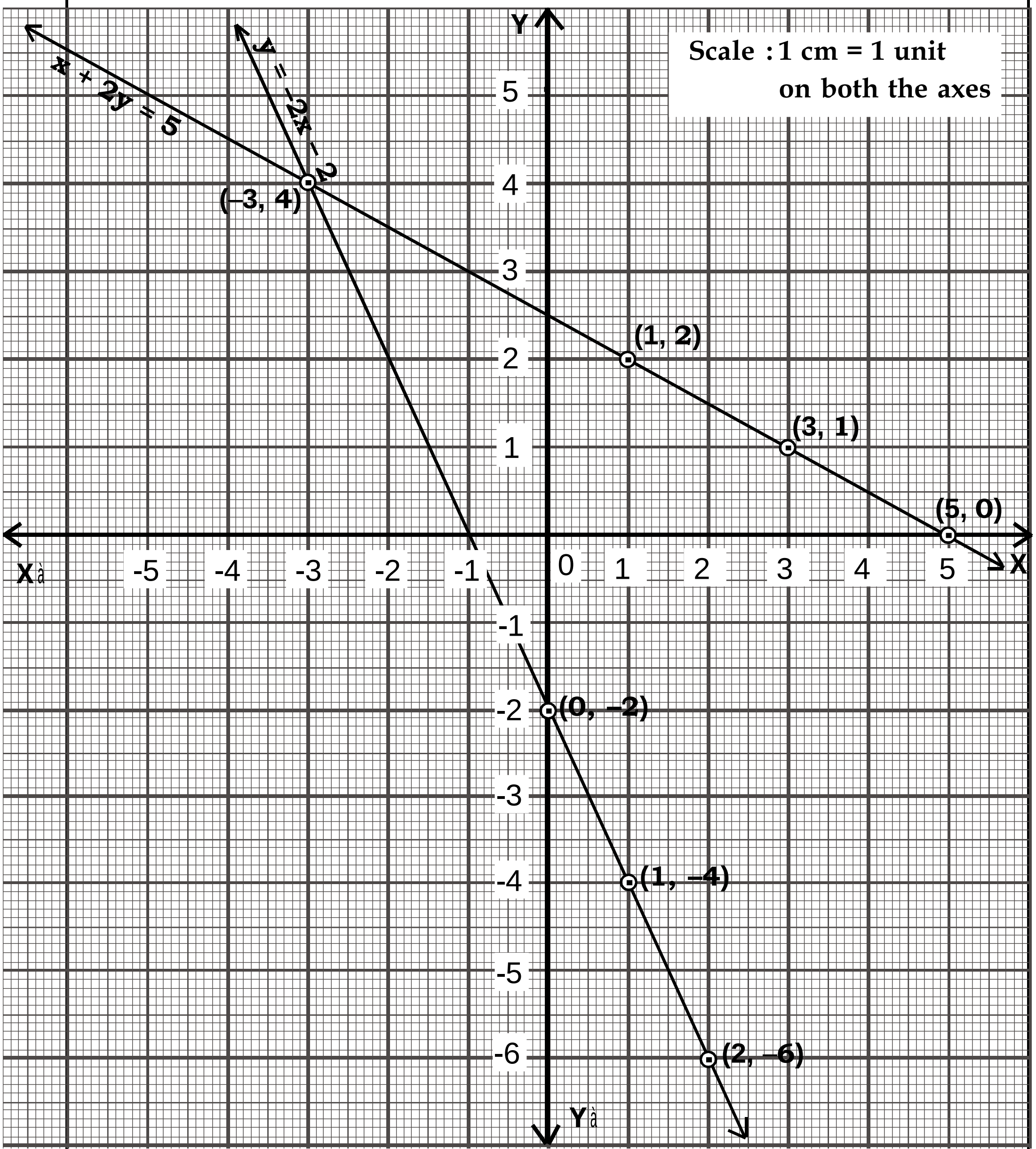
(ii) x + 2y = 5 y = –2x – 2

∴ x = 5 – 2y

x	5	3	1
y	0	1	2
(x, y)	(5, 0)	(3, 1)	(1, 2)

x	0	1	2
y	–2	–4	–6
(x, y)	(0, –2)	(1, –4)	(2, –6)

1



2

∴ x = – 3 and y = 4 is the solution of given simultaneous equations. 1

(iii)	S = { HHH, HTH, THH, TTH, HHT, HTT, THT, TTT }	
	y n (S) = 8	**1**
	P is the event of getting atleast two heads	
	P = { HHH, HTH, THH, HHT }	
	y n (P) = 4	**1**
	Q is the event of getting no head	
	Q = { TTT }	
	y n (Q) = 1	**1**
	R is the event of getting head on second coin	
	R = { HHH, THH, HHT, THT }	
	y n (R) = 4	
	Here P ą Q = ¤	
	y P and Q are mutually exclusive events.	
	Here Q ą R = ¤	**1**
	y Q and R are mutually exclusive events.	
A.5.	**Solve the following : (Any 2)**	
(i)	$\frac{1}{x} + \frac{1}{y} = 8$ ....(i)	
	$\frac{4}{x} - \frac{2}{y} = 2$ ....(ii)	
	Substituting $\frac{1}{x}$ = a and $\frac{1}{y}$ = b in (i) and (ii) we get	½
	a + b = 8 ......(iii)	½
	4a – 2b = 2 ....(iv)	½
	Multiplying (iii) by 2,	
	2a + 2b = 16 ....(v)	
	Adding (iv) and (v),	
	4a – 2b = 2	
	2a + 2b = 16	
	6a = 18	½
	y a = $\frac{18}{6}$	
	y a = 3	½
	Substituting a = 3 in (iii),	
	3 + b = 8	½
	y b = 8 – 3	
	y b = 5	½
	Resubstituting the values of a and b,	

$a = \frac{1}{x}$

$3 = \frac{1}{x}$ ½

y $3x = 1$

y $x = \frac{1}{3}$

$b = \frac{1}{y}$

$5 = \frac{1}{y}$

y $5y = 1$

y $y = \frac{1}{5}$ ½

y $x = \frac{1}{3}$ and $y = \frac{1}{5}$ is the solution of given simultaneous equations. ½

(ii) For the A.P $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots\ldots$

$a = \frac{1}{6}$

y $d = \frac{1}{4} - \frac{1}{6}$ ½

y $d = \frac{3}{12} - \frac{2}{12}$

y $d = \frac{1}{12}$ ½

$t_n = a + (n - 1)d$ ½

$t_{20} = \frac{1}{6} + (20 - 1)\left(\frac{1}{12}\right)$ ½

$= \frac{1}{6} + \frac{19}{12}$

$= \frac{2}{12} + \frac{19}{12}$

$= \frac{21}{12}$

y $t_{20} = \frac{7}{4}$ ½

Now,

	$S_n = \frac{n}{2}[2a + (n-1)d]$	½
	∴ $S_{10} = \frac{10}{2}[2a + (n-1)d]$	½
	$= 5\left[2\left(\frac{1}{6}\right) + 9\left(\frac{1}{12}\right)\right]$	½
	$= 5\left[\frac{1}{3} + \frac{3}{4}\right]$	
	$= 5\left[\frac{4+9}{12}\right]$	
	$= 5\left[\frac{13}{12}\right]$	
	∴ $\boxed{S_{10} = \frac{65}{12}}$	1
(iii)	Let the no. of Rs. 10 notes given to Durga be x and the no. of Rs.5 notes given to her be y. As per the first condition, $10x + 5y = 190$ …….(i)	1
	As per the second condition, $5x + 10y = 185$ ……(ii)	1
	Adding (i) and (ii), $15x + 15y = 375$ Dividing throughout by 15 we get, $x + y = \frac{375}{15}$ ∴ $x + y = 25$ ……(iii)	½
	Subtracting (ii) from (i), $5x - 5y = 5$	½
	Dividing throughout by 5 we get, $x - y = 1$ …….(iv)	½
	Adding (iii) and (iv), $x + y = 25$ $x - y = 1$ ∴ $2x = 26$ ∴ $x = 13$	½
	Substituting x = 13 in (iii), $13 + y = 25$ ∴ $y = 25 - 13$ ∴ $y = 12$	½
	∴ Durga had 13 notes of Rs. 10 rupee and 12 notes of Rs. 5.	½

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