MT

2017 ____ ____ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - II - PAPER - 6 (E)

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1.	**Solve the following : (Any 5)**	
(i)	$t_1 = 4, t_2 = 3, t_3 = 2, t_4 = 1,$ $t_2 - t_1 = 3 - 4 = -1$ $t_3 - t_2 = 2 - 3 = -1$ $t_4 - t_3 = 1 - 2 = -1$	
	$\because$ The difference between two consecutive terms is –1 which is constant.	½
	y **The sequence is an A.P.**	½
(ii)	$x^2 - 7x + 4 = 0$ Comparing with $ax^2 + bx + c = 0$ $a = 1, b = -7, c = 4$	1
(iii)	$\begin{vmatrix} 5 & 2 \\ 7 & 4 \end{vmatrix}$ $= (5 \times 4) - (2 \times 7)$ $= 20 - 14$ $= \boxed{6}$	1
(iv)	$a = 6, d = 6$ Here, $t_1 = a = 6$ $t_2 = t_1 + d = 6 + 6 = 12$ $t_3 = t_2 + d = 12 + 6 = 18$ $t_4 = t_3 + d = 18 + 6 = 24$ $t_5 = t_4 + d = 24 + 6 = 30$	
	y The first five terms of A.P. are 6, 12, 18, 24 and 30.	1
(v)	$x^2 - 4x + 1 = 0, x = 1$ Putting $x = 1$ in L.H.S., we get, L.H.S. $= (1)^2 - 4(1) + 1$ $= 1 - 4 + 1$ $= 2 - 4$ $= -2$ ÷ R.H.S.	
	y L.H.S. ÷ R.H.S.	½
	Thus equation is not satisfied. So 1 is not the root of the given quadratic equation.	½

(vi)	$D_y = -15$ and $D = -5$	
	By Cramer's rule,	
	$y = \frac{D_y}{D}$	½
	y $y = \frac{-15}{-5}$	
	y $\boxed{y = 3}$	½
A.2.	**Solve the following : (Any 4)**	
(i)	The positive integers which are divisible by 6 are 6, 12, 18, 24, ...	
	The number form an A.P. with a = 6, d = 6.	½
	The sum of first 11 positive integers divisible by 6 is (S_{11})	
	$S_n = \frac{n}{2}[2a + (n - 1) d]$	½
	y $S_{11} = \frac{11}{2}[2a + (11 - 1) d]$	½
	$= \frac{11}{2}[2 (6) + 10 (6)]$	
	$= \frac{11}{2}[12 + 60]$	
	$= \frac{11}{2} \times 72$	
	$= 11 \times 36$	
	$S_n = 396$	
	y Sum of first 11 positive integers which are divisible by 6 is 396.	½
(ii)	$x^2 - x - 132 = 0$	
	y $x^2 - 12x + 11x - 132 = 0$	½
	y $x (x - 12) + 11 (x - 12) = 0$	½
	y $(x - 12) (x + 11) = 0$	
	y $x - 12 = 0$ or $x + 11 = 0$	½
	∴ $\boxed{x = 12 \text{ or } x = -11}$	½
(iii)	Substituting x = 2 and y = 5 in the L.H.S. of the equation 3x – y =1	½
	L.H.S. = 3x – y	½
	= 3 (2) – 5	
	= 6 – 5	
	= 1	
	= R.H.S.	
	y x = 2 and y = 5 satisfies the equation 3x – y = 1	½
	Hence (2, 5) lies on the graph of the equation 3x – y = 1.	½

(iv) $y = \dfrac{5x - 10}{2}$

y $2y = 5x - 10$

y $-5x + 2y = -10$

$4x + 5 = -y$

y $4x + y = -5$ ½

$D = \begin{vmatrix} -5 & 2 \\ 4 & 1 \end{vmatrix} = (-5 \times 1) - (2 \times 4) = -5 - 8 = -13$

$D_x = \begin{vmatrix} -10 & 2 \\ -5 & 1 \end{vmatrix} = (-10 \times 1) - (2 \times -5) = -10 + 10 = 0$ ½

$D_y = \begin{vmatrix} -5 & -10 \\ 4 & -5 \end{vmatrix} = (-5 \times -5) - (-10 \times 4) = 25 + 40 = 65$

By Cramer's rule,

$x = \dfrac{D_x}{D} = \dfrac{0}{-13} = 0$ ½

$y = \dfrac{D_y}{D} = \dfrac{65}{-13} = -5$

y **x = 0 and y = – 5 is the solution of given simultaneous equations.** ½

(v) $3x^2 + kx - 2 = 0$

x = 4 is the solution of given quadratic equation.

Substituting x = 4 in given quadratic equation, ½

it will get satisfied.

y $3(4)^2 + k(4) - 2 = 0$

y $3(16) + 4k - 2 = 0$ ½

y $48 + 4k - 2 = 0$

y $4k + 46 = 0$

y $4k = -46$ ½

y $k = \dfrac{-46}{4}$

y $\boxed{k = \dfrac{-23}{2}}$ ½

(vi) Sample space for the game of chance of spinning an arrow

S = { 1, 2, 3, 4, 5, 6, 7, 8 }

n (S) = 8 ½

Let C be the event that the arrow points at a number greater than 2

C = { 3, 4, 5, 6, 7, 8 }

y n (C) = 6 ½

	$P(C) = \frac{n(C)}{n(S)}$	½
	∴ $P(C) = \frac{6}{8}$	
	∴ $\boxed{P(C) = \frac{3}{4}}$	½
A.3.	**Solve the following : (Any 3)**	
(i)	– 5, 2, 9, 16, 23, 30, ...	
	$t_1 = -5, t_2 = 2, t_3 = 9, t_4 = 16, t_5 = 23, t_6 = 30$	
	$t_2 - t_1 = 2 - (-5) = 2 + 5 = 7$	
	$t_3 - t_2 = 9 - 2 = 7$	
	$t_4 - t_3 = 16 - 9 = 7$	1
	$t_5 - t_4 = 23 - 16 = 7$	
	$t_6 - t_5 = 30 - 23 = 7$	
	∵ The difference between two consecutive terms is 7 which is a constant.	
	∴ The sequence is an A.P. with $a = t_1 = -5$.	1
	Common difference (d) = 7	
	$t_n = a + (n - 1) d$	
	$t_n = -5 + (n - 1) 7$	
	$t_n = -5 + 7n - 7$	
	$t_n = 7n - 12$	
	∴ $\boxed{\text{The general term of A.P. is } 7n - 12.}$	1
(ii)	$m^2 = 4 + 5m$	
	∴ $m^2 - 5m = 4$ ..... (i)	
	Third term $= \left(\frac{1}{2} \times \text{coefficient of } m\right)^2$	
	$= \left(\frac{1}{2} \times -5\right)^2$	
	$= \left(\frac{-5}{2}\right)^2$	
	$= \frac{25}{4}$	½
	Adding $\frac{25}{4}$ to both the sides of (i) we get,	
	$m^2 - 5m + \frac{25}{4} = 4 + \frac{25}{4}$	½
	∴ $\left(m - \frac{5}{2}\right)^2 = \frac{16 + 25}{4}$	
	∴ $\left(m - \frac{5}{2}\right)^2 = \frac{41}{4}$	

Taking square root on both the sides we get,

$m - \frac{5}{2} = \pm\frac{\sqrt{41}}{2}$ **1**

y $m = \frac{5}{2} \pm \frac{\sqrt{41}}{2}$

y $m = \frac{5 \pm \sqrt{41}}{2}$

y $m = \frac{5+\sqrt{41}}{2}$ or $m = \frac{5-\sqrt{41}}{2}$

y $\boxed{\frac{5+\sqrt{41}}{2} \text{ and } \frac{5-\sqrt{41}}{2} \text{ are the roots of the given quadratic equation.}}$ **1**

(iii) Let the measures of ĎA be x°
and measure of ĎB be y°.
Seg AB is the diameter of a circle
and C is a point on the circumference.

y m ĎC = 90° [∵ Diameter subtends a right angle at any point on the circle]

In aABC,

m ĎA + m ĎB + m ∠C = 180° [∵ Sum of measures of the angles of a triangle is 180°]

y $x + y + 90 = 180$

y $x + y = 180 - 90$

y $x + y = 90$ ....(i) **½**

As the given condition,

$y = x - 10$

y $-x + y = -10$ ....(ii) **½**

Adding (i) and (ii),

$x + y = 90$

$-x + y = -10$

$2y = 80$

y $y = \frac{80}{2}$

y $y = 40$ **½**

Substituting y = 40 in (i),

$x + 40 = 90$

y $x = 90 - 40$

y $x = 50$ **½**

y **The measures of angle of aABC are 50°, 40° and 90°.** **1**

(iv) Let 3 red balls, 3 white balls and 3 green balls be denoted as $R_1, R_2, R_3, W_1, W_2, W_3$ and G_1, G_2, G_3 respectively.

$S = \{R_1, R_2, R_3, W_1, W_2, W_3, G_1, G_2, G_3\}$

n (S) = 9

		P is the event that the ball is red	
		P = { R_1, R_2, R_3 }	
	y	n (P) = 3	1
		Q is the event that the ball is not green	
		Q = { R_1, R_2, R_3, W_1, W_2, W_3 }	
	y	n (Q) = 6	1
		R is the event that the ball is red or white	
		R = { R_1, R_2, R_3, W_1, W_2, W_3 }	
	y	n (R) = 6	1
(v)	y	$2x^2 - x - 4 = 0$	
		Comparing with $ax^2 + bx + c = 0$ we have a = 2, b = – 1, c = – 4	½
		$b^2 - 4ac = (-1)^2 - 4(2)(-4)$	½
		$= 1 + 32$	
		$= 33$	
		$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
		$= \dfrac{-(-1) \pm \sqrt{33}}{2(2)}$	
		$= \dfrac{1 \pm \sqrt{33}}{4}$	1
	y	$x = \dfrac{1 + \sqrt{33}}{4}$ or $x = \dfrac{1 - \sqrt{33}}{4}$	
	y	$\dfrac{1 + \sqrt{33}}{4}$ and $\dfrac{1 - \sqrt{33}}{4}$ are the roots of the given quadratic equation.	1
A.4.		**Solve the following : (Any 2)**	
(i)		The savings done by Neela on each day is as follows 2, 4, 6, ...	
		These every day savings form an A.P. with	
		First day saving (a) = 2	½
		Difference in savings made in two successive days (d) = 2	
		Total no. of days in the month of February 2010 (n) = 28	½
	y	Total savings for the month of February (S_{28}) = ?	½
		$S_n = \dfrac{n}{2}[2a + (n-1)d]$	½
	y	$S_{28} = \dfrac{28}{2}[2(2) + (28-1)(2)]$	
		$= 14[4 + 27(2)]$	
		$= 14[4 + 54]$	1
	y	$S_{28} = 14[58]$	
	y	$S_{28} = 812$	
	y	Neela saved Rs. 812 in the month of February.	1

(ii) $4x = y - 5$

y $4x + 5 = y$

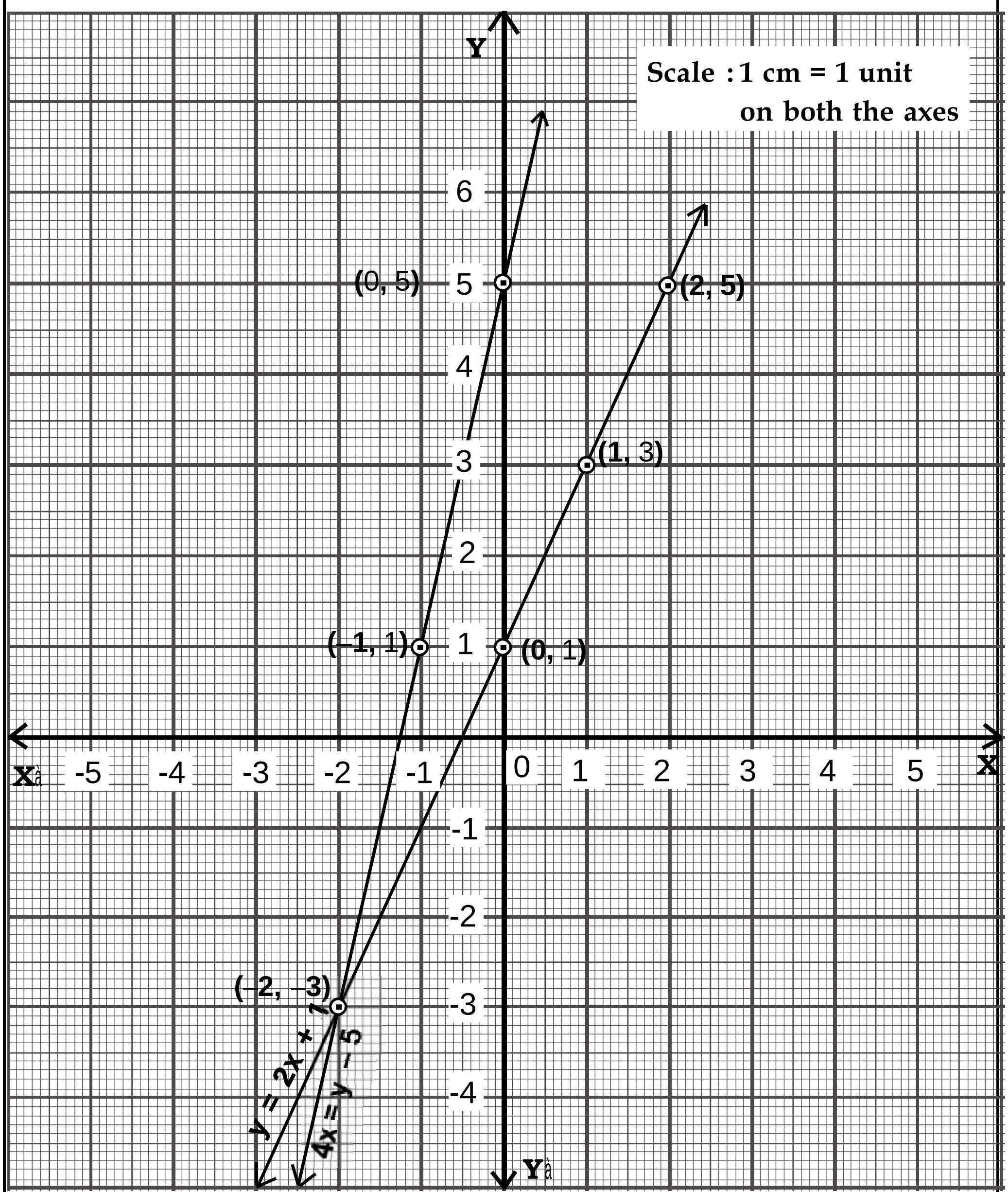
y $y = 4x + 5$

x	0	–1	–2
y	5	1	–3
(x, y)	(0, 5)	(–1, 1)	(–2, –3)

$y = 2x + 1$

x	0	1	2
y	1	3	5
(x, y)	(0, 1)	(1, 3)	(2, 5)

1



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y $x = -2$ and $y = -3$ is the solution of given simultaneous equations. 1

(iii)

(a) There are 52 cards in a pack of cards

y n (S) = 52

(1) Let A be the event that card drawn bears a number between 4 and 7 both inclusive

∵ There are 4 numbers from 4 to 7 inclusive of both and there are 4 types of cards

y Total no. of cards bearing numbers from 4 to 7 both inclusive in the pack of cards are 4 × 4 = 16

n (A) = 16 ½

$$P(A) = \frac{n(A)}{n(S)}$$

y $P(A) = \frac{16}{52}$

y $\boxed{P(A) = \frac{4}{13}}$ ½

(2) Let B be the event that card drawn bears a number between 3 and 8 both inclusive.

∵ There are 6 numbers from 3 to 8 both inclusive and there are 4 types of cards

y Total no. of cards bearing numbers from 3 to 8 both inclusive in the pack of cards is 6 × 4 = 24

n (B) = 24 ½

y $P(B) = \frac{n(B)}{n(S)}$

y $P(B) = \frac{24}{52}$

y $\boxed{P(B) = \frac{6}{13}}$ ½

(b) There are 52 cards in a pack

y n (S) = 52

(1) Let A be event that the card drawn is a spade card

y There are 13 spade cards

y n (A) = 13 ½

$$P(A) = \frac{n(A)}{n(S)}$$

y $P(A) = \frac{13}{52}$

y $\boxed{P(A) = \frac{1}{4}}$ ½

(b) Let B be event that the card drawn is not a diamond

There are 13 diamond cards ½

	y There are 39 cards which are not of diamond	
	y n (B) = 39	
	$P(B) = \frac{n(B)}{n(S)}$	
	y $P(B) = \frac{39}{52}$	
	y $\boxed{P(B) = \frac{3}{4}}$	½
A.5.	**Solve the following : (Any 2)**	
(i)	$\frac{27}{x-2} + \frac{31}{y+3} = 85$ ....(i)	
	$\frac{31}{x-2} + \frac{27}{y+3} = 89$ ....(ii)	
	Substituting $\frac{1}{x-2}$ = a and $\frac{1}{y+3}$ = b in (i) and (ii),	½
	27a + 31b = 85 ....(iii)	
	31a + 27b = 89 ....(iv)	**1**
	Adding (iii) and (iv),	
	58a + 58b = 174	
	Dividing throughout by 58 we get,	
	$a + b = \frac{174}{58}$	
	y a + b = 3 ....(v)	½
	Subtracting (iv) from (iii)	
	– 4a + 4b = – 4	
	Dividing throughout by – 4 we get,	
	a – b = 1 ....(vi)	½
	Adding (v) and (vi),	
	a + b = 3	
	a – b = 1	
	2a = 4	
	y a = 2	½
	Substituting a = 2 in (v),	
	2 + b = 3	
	y b = 3 – 2	
	y b = 1	½
	Resubstituting the values of a and b	

$a = \frac{1}{x-2}$

$2 = \frac{1}{x-2}$

y $2(x-2) = 1$

y $2x - 4 = 1$

y $2x = 1 + 4$

y $2x = 5$

y $x = \frac{5}{2}$ ½

$b = \frac{1}{y+3}$

$1 = \frac{1}{y+3}$

$y + 3 = 1$

y $y = 1 - 3$ ½

y $y = -2$

y $x = \frac{5}{2}$ and $y = -2$ is the solution of given simultaneous equations. ½

(ii) (a) $t_n = a + (n-1)d$

$t_{11} = a + (11-1)d$

$16 = a + 10d$ ....(i) ½

y $a + 10d = 16$

$t_{21} = a + (21-1)d$

$29 = a + 20d$

y $a + 20d = 29$ ......(ii) ½

Subtracting (ii) from (i),

$a + 10d = 16$

$a + 20d = 29$

(–) (–) (–)

$-10d = -13$

y $10d = 13$

y $d = \frac{13}{10}$

y $d = 1.3$ **1**

Substituting $d = 1.3$ in (i),

$a + 10(1.3) = 16$

y $a + 13 = 16$

y $a = 16 - 13$

y $a = 3$

y The first term is 3 and the common difference is 1.3 **1**

		Marks
	(b) $t_n = a + (n - 1) d$	½
	$t_{34} = a + (34 - 1) d$	
	$= 3 + 33 (1.3)$	
	$= 3 + 42.9$	
	$t_{34} = 45.9$	
	y Thirty fourth term of A.P. is 45.9.	½
	(c) $t_n = a + (n - 1) d$	½
	y $55 = 3 + (n - 1) d$	
	y $55 = 3 + (n - 1) 1.3$	
	y $55 = 3 + 1.3n - 1.3$	
	y $55 = 1.7 + 1.3n$	
	y $53.3 = 1.3n$	
	y $\frac{53.3}{1.3} = n$	
	y $n = \frac{533}{13}$	
	y $n = 41$	½
(iii)	Let the fixed charge for stay in hostel be Rs. x and cost of food per day be Rs. y	½
	Ram's total expenditure = Rs. (x + 20y)	½
	Rahim's total expenditure = Rs. (x + 24y)	½
	As per the first given condition,	
	$x + 20y = 1700$ ......(i)	½
	As per the second given condition,	
	$x + 24y = 1900$ .....(ii)	½
	Substracting (ii) from (i),	
	$x + 20y = 1700$	
	$x + 24y = 1900$	
	(–) (–) (–)	
	$-4y = -200$	
	y $y = \frac{-200}{-4}$	
	y $y = 50$	1
	Substituting y = 50 in (i),	
	$x + 20 (50) = 1700$	
	y $x + 1000 = 1700$	
	y $x = 1700 - 1000$	
	y $x = 700$	1
	y Fixed charge for stay in the hostel is Rs. 700 and cost of food per day is Rs. 50.	½

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