MT

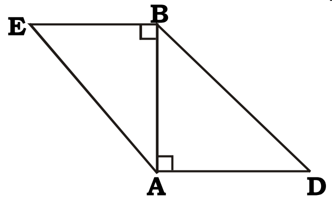
2017 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - II : PAPER - 1

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

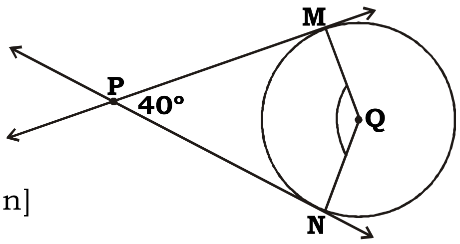
(i) $\frac{\text{A (aABE)}}{\text{A (aABD)}} = \frac{BE}{AD}$ [Triangles with common base] ½



∴ $\frac{\text{A (aABE)}}{\text{A (aABD)}} = \frac{6}{9}$

∴ $\boxed{\frac{\text{A (aABE)}}{\text{A (aABD)}} = \frac{2}{3}}$ ½

(ii)



In □MQNP,

m ĎMPN = 40° [Given]

m ĎPMQ = 90°
m ĎPNQ = 90° } [Radius is perpendicular to the tangent] ½

y $\boxed{\text{m ĎMQN = 140°}}$ [Remaining angle] ½

(iii) F + V = E + 2 ½

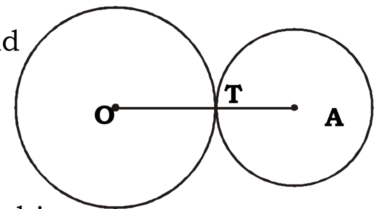
y F + 6 = 12 + 2

y F + 6 = 14

y F = 14 – 6 ½

y $\boxed{F = 8}$

(iv) **Given :** Two circles with centres O and A touch each other externally at point T.

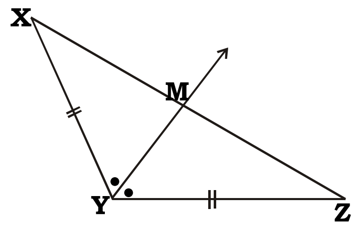


To Prove : OA = OT + AT ½

Proof : O - T - A [If two circles are touching circles then the common point lies on the line joining their centres]

y **OA = OT + AT** [∵ O - T - A] ½

(v)



In ΔXYZ,

ray YM bisects ∠XYZ [Given]

∴ $\frac{XY}{YZ} = \frac{XM}{MZ}$ [Property of angle bisector of a triangle] ½

∴ $1 = \frac{XM}{MZ}$ [∵ XY = YZ] ½

∴ **XM = MZ**

(vi) A cylinder and cone have equal height and equal radii

y Volume of cone $= \frac{1}{3} \times$ volume of cylinder ½

$= \frac{1}{3} \times 300$

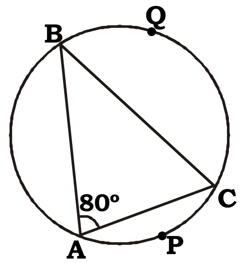
$= 100\ cm^3$

y Volume of the cone is $100\ cm^3$. ½

A.2. Solve ANY FOUR of the following :

(i) (a) m ĎABC $= \frac{1}{2}$ m(arc APC) ½

[Inscribed angle theorem]



y m ĎABC $= \frac{1}{2} \times 60$

y m ĎABC = 30° ½

(b) m ĎBAC $= \frac{1}{2}$ m(arc BQC) [Inscribed angle theorem] ½

y 80 $= \frac{1}{2}$ m(arc BQC)

y m(arc BQC) = 80 × 2

y m(arc BQC) = 160° ½

(ii) Mark point X as shown in the figure

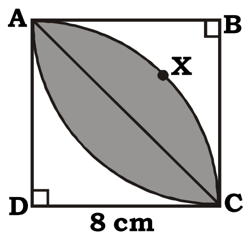
□ABCD is a square [Given]

side = 8 cm

Radius (r) = side of a square

y r = 8 cm

Measure of arc (‾) = 90° [Angle of a square] ½



Area of the segment AXC = $r^2 \left[\frac{®¯}{360} - \frac{\sin ¯}{2}\right]$ ½

$= 8^2 \left[\frac{3.14 \times 90}{360} - \frac{\sin 90}{2}\right]$

$= 64 \left[\frac{1.57}{2} - \frac{1}{2}\right]$

$= 64 \left[\frac{1.57 - 1}{2}\right]$ ½

$= \frac{64 \times 0.57}{2}$

$= \frac{36.48}{2}$ cm^2

Area of shaded region = 2 × Area of segment AXC ½

$= 2 \times \frac{36.48}{2}$

$= 36.48$ cm^2

y **Area of shaded region is 36.48 cm^2.**

(iii) In aABC and aDAC,

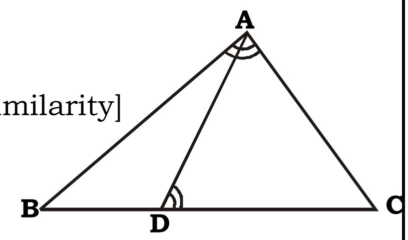
ĎBAC] ĎADC [Given]

ĎACB] ĎACD [Common angle] ½

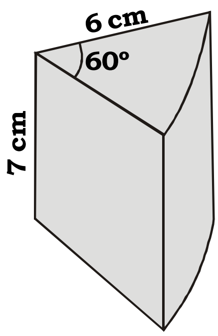
y aABC ~ aDAC [By AA test of similarity] ½

y $\frac{AC}{DC} = \frac{BC}{AC}$ [c.s.s.t.] ½

y **$AC^2 = BC \times DC$** ½



(iv)



For a sector,

Measure of arc (¯) = 60°

Radius (r) = 6 cm

(a) Curved surface area of the cheese = Length of arc × height ½

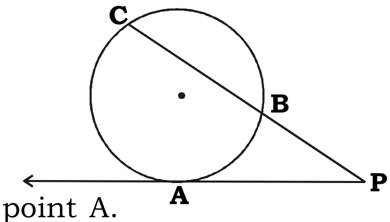
$= \frac{¯}{360} \times 2®r \times h$ ½

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6 \times 7$ ½

$= 44 \text{ cm}^2$

y **The curved surface area of the cheese is 44 cm².** ½

(v)



Line PBC is a secant intersecting the circle at points B and C and line PA is a tangent to the circle at point A.

y $CP \times BP = AP^2$ [Tangent secant property] ½

y $CP \times 10 = (15)^2$

y $CP \times 10 = 225$

y $CP = \frac{225}{10}$

y $CP = 22.5$ units ½

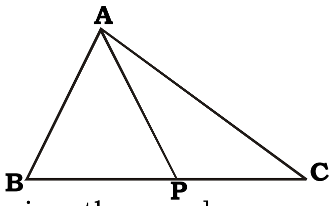
$CP = BC + BP$ [∵ C - B - P] ½

y $22.5 = BC + 10$

y $BC = 22.5 - 10$

y **BC = 12.5 units** ½

(vi)



In aABC,

seg AP is the median [Given]

y $AB^2 + AC^2 = 2AP^2 + 2BP^2$ [By Appollonius theorem] ½

y $260 = 2(7)^2 + 2BP^2$ [Given]

y $260 = 2(49) + 2BP^2$

y $260 = 98 + 2BP^2$

y $260 - 98 = 2BP^2$

y $2BP^2 = 162$

y $BP^2 = \frac{162}{2}$

y $BP^2 = 81$ ½

y $BP = 9$ units [Taking square roots]

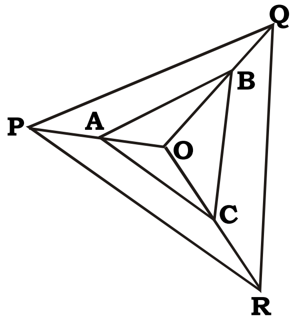
$BP = \frac{1}{2} BC$ [∵ P is the midpoint of seg BC] ½

y $9 = \frac{1}{2} BC$

y **BC = 18 units** ½

A.3. Solve ANY THREE of the following :

(i)



In aPOQ,

seg AB || side PQ [Given]

y $\frac{OA}{AP} = \frac{OB}{BQ}$ ......(i) [By B.P.T.] **1**

In aPOR,

seg AC || side PR [Given]

y $\frac{OA}{AP} = \frac{OC}{CR}$ .....(ii) [By B.P.T.] **1**

In aOQR,

$\frac{OB}{BQ} = \frac{OC}{CR}$ [From (i) and (ii)]

y **seg BC || side QR** [By converse of B.P.T.] **1**

(ii)

Curved surface area of the frustum of a cone = 180 cm²

Perimeters of circular bases are 18 cm and 6 cm ½

y 2®r_1 = 18 ....(i) ½

2®r_2 = 6 ....(ii)

Adding (i) and (ii), we get

2®r_1 + 2®r_2 = 18 + 6

y 2® (r_1 + r_2) = 24

y ® (r_1 + r_2) = $\frac{24}{2}$

y ® (r_1 + r_2) = 12 ....(iii) **1**

Curved surface area of the frustum of a cone = ®(r_1 + r_2) l ½

y 180 = ®(r_1 + r_2) l

y 180 = 12 × l

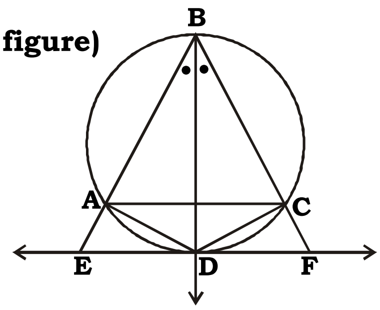
[From (iii)] ½

y l = 15 cm

y Slant height of the frustum of a cone is 15 cm.

(iii)

(1 mark for figure)



ĎEDA] ĎABD ....(i)

ĎFDC] ĎCBD ....(ii)

[Angles in alternate segment] ½

But, ĎABD] ĎCBD ....(iii) [∵ Ray BD bisects ĎABC] **1**

y **ĎEDA**] **ĎFDC** [From (i), (ii) and (iii)] ½

(iv) In aABC,

AB = 5 units

BC = 6 units [Given]

AC = 7 units

Perimeter of aPQR = 360 units [Given]

y PQ + QR + PR = 360 ......(i) ½

aABC ~ aPQR [Given]

y $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ [c.s.s.t.] ½

y $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR}$

y $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{5 + 6 + 7}{PQ + QR + PR}$ [By theorem on equal ratios] ½

y $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{18}{360}$ [From (i)]

y $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{1}{20}$ .....(ii)

$\frac{5}{PQ} = \frac{1}{20}$ [From (ii)]

y **PQ = 100 units** ½

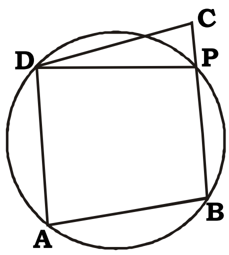
$\frac{6}{QR} = \frac{1}{20}$ [From (ii)]

y **QR = 120 units** ½

$\frac{7}{PR} = \frac{1}{20}$ [From (ii)]

y **PR = 140 units** ½

(v)



☐ABPD is cyclic [By definition]

y ĎDPC] ĎDAB .....(i) [An exterior angle of cyclic quadrilateral is congruent to the angle opposite to adjacent interior angle] 1

□ABCD is parallelogram

y ĎDCB] ĎDAB ....(ii) [Opposite angles of a parallelogram are congruent] 1

y ĎDPC] ĎDCB ......(iii) [From (i) and (ii)]

In aDPC,

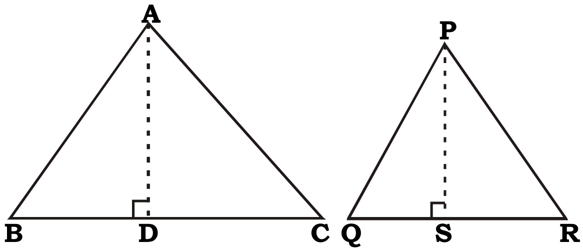
ĎDPC] ĎDCP [From (ii) and C - P - B] ½

y seg DP] seg DC [Converse of isosceles triangle theorem]

y **DP = DC** ½

A.4. **Solve ANY TWO of the following :**

(i)



Given : aABC ~ aPQR.

To Prove : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Construction :

(i)Draw seg AD⊧{ side BC, B - D - C ½

(ii)Draw seg PS { side QR, Q - S - R

Proof : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC \times AD}{QR \times PS}$ [The ratio of the areas of two triangles is equal to ratio of the products of a base and its corresponding height] ½

$\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ ......(i)

aABC ~ aPQR [Given]

y $\frac{AB}{PQ} = \frac{BC}{QR}$ ......(ii) [c.s.s.t.] ½

Also, ĎB]∷ĎQ .....(iii) [c.a.s.t.] ½

In aADB and aPSQ,

ĎADB]∷ĎPSQ [Each is a right angle]

ĎB] ĎQ [From (ii)]

y aADB ~ aPSQ [By A-A test of similarity] ½

y $\frac{AD}{PS} = \frac{AB}{PQ}$ .....(iv) [c.s.s.t.] ½

y $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (i), (ii) and (iv)] ½ y $\frac{A\,(aABC)}{A\,(aPQR)} = \frac{AB^2}{PQ^2}$ ....(vi)

Similarly we can prove

$\frac{A\,(aABC)}{A\,(aPQR)} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ ....(vii)

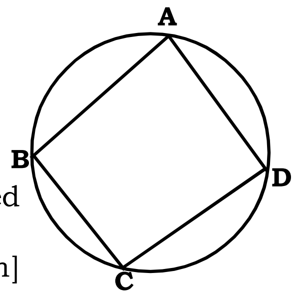
y $\mathbf{\frac{A(aABC)}{A(aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}}$ [From (vi) and (viii)] ½

(ii) **Given :** □ABCD is a cyclic

To Prove : m ĎABC + m ĎADC = 180° ½

m ĎBAD + m ĎBCD = 180°

Proof :



m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii)

} [Inscribed angle theorem] 1

(½ mark for figure)

Adding (i) and (ii), we get

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC)

y m ĎABC + m ĎADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)] ½

y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y **m ĎABC + m ĎADC = 180° ....(iii)**

In □ABCD,

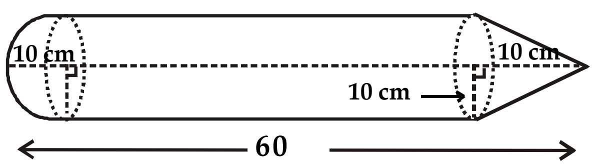
m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°

[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(iii)



A toy is a combination of cylinder, hemisphere and cone, each with radius 10 cm

y r = 10 cm

y Height of the conical part (h) = 10 cm ½

Height of the hemispherical part = its radius = 10cm

Total height of the toy = 60cm

y Height of the cylindrical part (h_1) = 60 – 10 – 10
= 60 – 20
= 40 cm **1**

l^2 = $r^2 + h^2$
y l^2 = $10^2 + 10^2$
y l^2 = 100 + 100
l^2 = 200
y l = $\sqrt{200}$
[Taking square roots]
l = $10\sqrt{2}$ cm

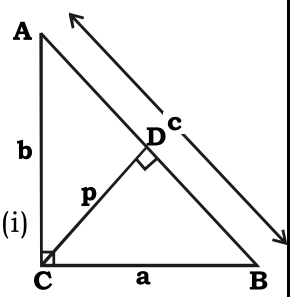
Slant height of the conical part (l) = $10\sqrt{2}$
= 10 × 1.41
= 14.1 cm ½

Total surface area of the toy
= Curved surface area of the conical part + Curved surface area of the cylindrical part + Curved surface area of the hemispherical part ½
= ®rl + 2®rh_1 + 2®r^2 ½
= ®r (l + 2h_1 + 2r)
= 3.14 × 10 (14.1 + 2 × 40 + 2 × 10)
= 31.4 (14.1 + 80 + 20) ½
= 31.4 × 114.1
= 3582.74 cm^2

y **Total surface area of the toy is 3582.74 cm^2.** ½

A.5. Solve ANY TWO of the following :

(i) (a) Area of a triangle = $\frac{1}{2}$ × base × height



A (aABC) = $\frac{1}{2}$ × AB × CD

y A (aABC) = $\frac{1}{2}$ × c × p ......(i) ½

Also, A (aABC) = $\frac{1}{2}$ × BC × AC

y A (aABC) = $\frac{1}{2}$ × a × b ......(ii) ½

From (i) and (ii) we get,

$\frac{1}{2}$ × c × p = $\frac{1}{2}$ × a × b

y cp = ab ½

(b) $cp = ab$

y $\frac{1}{cp} = \frac{1}{ab}$ [By Invertendo] ½

y $\frac{1}{c^2p^2} = \frac{1}{a^2b^2}$ [Squaring both sides] ½

y $\frac{1}{p^2} = \frac{c^2}{a^2b^2}$ .....(iii) ½

In aACB,

m ĎACB = 90° [Given]

y $AB^2 = AC^2 + BC^2$ [By Pythagoras theorem] ½

y $c^2 = b^2 + a^2$ .....(iv)

y $\frac{1}{p^2} = \frac{b^2 + a^2}{a^2b^2}$ [From (iii) and (iv)] ½

y $\frac{1}{p^2} = \frac{b^2}{a^2 b^2} + \frac{a^2}{a^2 b^2}$ ½

y $\mathbf{\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}}$ ½

(ii) Length of a stair-step (l) = 50 cm

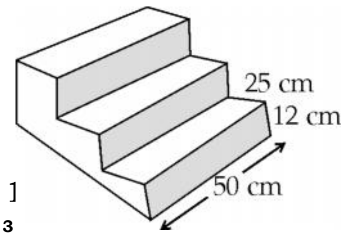
its breadth (b) = 25 cm ½

its height (h) = 12 cm

Volume of a stair-step = $l \times b \times h$ ½

= 50 × 25 × 1

= 15000 cm^3 ½



y Volume of 3 stair-step = 6 × 15000

= 90000 cm^3 ½

Length of a brick (l_1) = 12.5 cm

its breadth (b_1) = 6.25 cm

its height (h_1) = 4 cm

Volume of a brick = $l_1 \times b_1 \times h_1$ ½

= 12.5 × 6.25 × 4

= 312.5 cm^3 ½

Number of bricks required = $\frac{\text{Volume of 3 stair-steps}}{\text{Volume of each brick}}$ ½

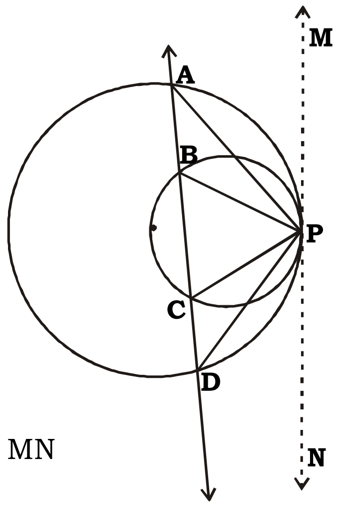
$= \frac{90000}{312.5}$ ½

$= \frac{6 \times 50 \times 25 \times 12}{12.5 \times 6.25 \times 4}$ ½

= 288

y **Number of bricks required is 288.** ½

(iii)



Construction : Draw a common tangent MN at point P. ½

Proof : ĎAPM]꞊ĎADP [Angles in alternate segment]

Let,

m ĎAPM = m ĎADP = x ......(i) ½

ĎBPM]꞊ĎBCP [Angles in alternate segment]

Let,

m ĎBPM = m ĎBCP = y .....(ii) ½

m ĎAPB = m ĎBPM – m ĎAPM [Angle addition proerty]

y m ĎAPB = (y – x) .....(iii) [From (i) and (ii)] **1**

ĎBCP is an exterior angle of aCPD,

y m ĎBCP = m ĎCPD + m ĎCDP [Remote interior angles theorem] **1**

y y = m ĎCPD + x [From (i) and A - C - D]

y m ĎCPD = (y – x) ......(iv) **1**

y m ĎAPB = m ĎCPD [From (iii) and (iv)] ½

y Ď**APB**] Ď**CPD**

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