MT

2017 ___ ___ 1100

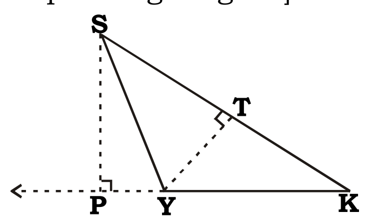
MT - GEOMETRY - SEMI PRELIM - II : PAPER - 2

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) $\frac{A\ (aSYK)}{A\ (aYTK)} = \frac{YK \times SP}{TK \times YT}$ [The ratio of the areas of two triangles is equal to the ratio of the products of their bases and corresponding heights] ½

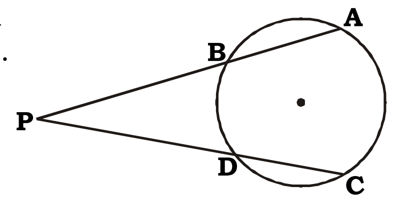
y $\frac{A\ (aSYK)}{A\ (aYTK)} = \frac{13 \times 6}{12 \times 5}$ [Given]



y $\frac{A\ (aSYK)}{A\ (aYTK)} = \frac{13}{10}$

y **A (aSYK) : A (aYTK) = 13 : 10** ½

(ii) Chords AB and CD intersect each other at point P outside the circle.



y PA × PB = PC × PD ½

y 6 × 3 = PC × 4

y PC = $\frac{6 \times 3}{4}$

y PC = $\frac{9}{2}$

y **PC = 4.5 units** ½

(iii) F + V = E + 2 ½

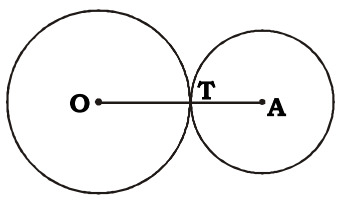
y F + 6 = 12 + 2

y F + 6 = 14

y F = 14 – 6

y **F = 8** ½

(iv) **Given :** Two circles with centres O and A touch each other externally at point T.

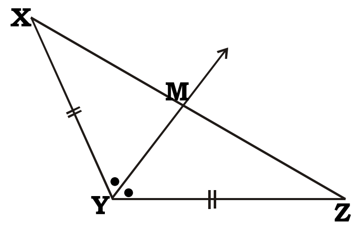


To Prove : OA = OT + AT

Proof : O - T - A [If two circles are touching circles then the common point lies on the line joining their centres] ½

y **OA = OT + AT** [∵ O - T - A] ½

(v)



In ΔXYZ,

ray YM bisects $\angle XYZ$ [Given]

$\therefore \frac{XY}{YZ} = \frac{XM}{MZ}$ [Property of angle bisector of a triangle] ½

$\therefore 1 = \frac{XM}{MZ}$ [$\because$ XY = YZ] ½

$\therefore$ **XM = MZ**

(vi) Volume of cuboid = Volume of cube [Given]

$\therefore 3 \times 9 \times x = (6)^3$ ½

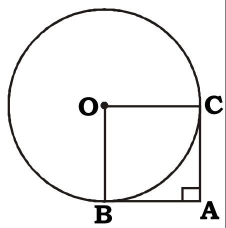
$\therefore 3 \times 9 \times x = 6 \times 6 \times 6$

$\therefore x = \frac{6 \times 6 \times 6}{3 \times 9}$

$\therefore \boxed{x = 8}$ ½

A.2. Solve ANY FOUR of the following :

(i)



In □BACO,

m$\angle$OBA = 90° } [Radius is perpendicular ½

m$\angle$OCA = 90° } to the tangent]

m$\angle$BAC = 90° [Given]

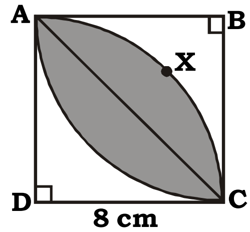
$\therefore$ m$\angle$BOC = 90° [Remaining angle]

$\therefore$ □BACO is a rectangle [By definition] ½

seg OB $\cong$ seg OC [Radii of the same circle]

$\therefore$ **□BACO is a square** [A rectangle in which adjacent sides are congruent is a square] 1

(ii)



Mark point X as shown in the figure

□ABCD is a square [Given]

side = 8 cm

Radius (r) = side of a square

$\therefore$ r = 8 cm

Measure of arc (θ) = 90° [Angle of a square] ½

Area of the segment AXC $= r^2 \left[\frac{\theta}{360} - \frac{\sin\theta}{2}\right]$ ½

$= 8^2 \left[\frac{3.14 \times 90}{360} - \frac{\sin 90}{2}\right]$

$= 64\left[\frac{1.57}{2} - \frac{1}{2}\right]$

$= 64\left[\frac{1.57 - 1}{2}\right]$

$= \frac{64 \times 0.57}{2}$

$= \frac{36.48}{2}$ cm²

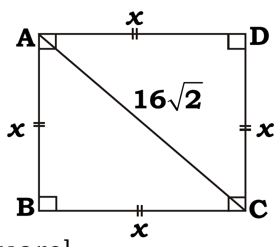
Area of shaded region $= 2 \times$ Area of segment AXC ½

$= 2 \times \frac{36.48}{2}$

$= 36.48$ cm²

y **Area of shaded region is 36.48 cm².** ½

(iii) **Given :** □ABCD is a square.



AC = $16\sqrt{2}$ cm

To find : Side of a square

Sol. □ABCD is a square [Given] ½

Let the sides of the square be x cm

In aABC,

m ĎABC = 90° [Angle of a square]

y $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem] ½

y $(16\sqrt{2})^2 = x^2 + x^2$

y $256 \times 2 = 2x^2$

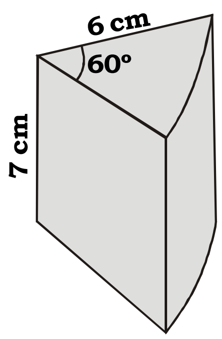
y $x^2 = \frac{256 \times 2}{2}$ ½

y $x^2 = 256$

y $x = 16$ [Taking square roots] ½

y **The side of a square is 16 cm.**

(iv)



For a sector,

Measure of arc (̄) = 60°

Radius (r) = 6 cm

(a) Curved surface area of the cheese = Length of arc × height ½

$= \frac{-}{360} \times 2®r \times h$ ½

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6 \times 7$ ½

$= 44$ cm²

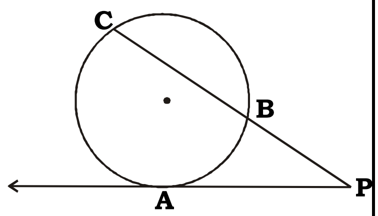
y The curved surface area of the cheese is 44 cm². ½

(v) Line PBC is a secant intersecting the circle at points B and C and line PA is a tangent to the circle at point A.

y $CP \times BP = AP^2$ [Tangent secant property] ½

y $CP \times 10 = (15)^2$

y $CP \times 10 = 225$



y $CP = \frac{225}{10}$ ½

y CP = 22.5 units

CP = BC + BP [∵ C - B - P] ½

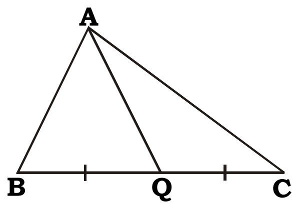
y 22.5 = BC + 10

y BC = 22.5 – 10

y BC = 12.5 units ½

(vi) In aABC,

seg AQ is the median [Given]



y $BQ = QC = \frac{1}{2} \times BC$

y $BQ = QC = \frac{1}{2} \times 10$ [Given] ½

y BQ = QC = 5 units ......(i)

$AB^2 + AC^2 = 2AQ^2 + 2BQ^2$ [By Appollonius theorem] ½

y $122 = 2AQ^2 + 2(5)^2$ [From (i) and given]

y $122 = 2AQ^2 + 2(25)$

y $122 = 2AQ^2 + 50$

y $2AQ^2 = 122 - 50$ ½

y $2AQ^2 = 72$

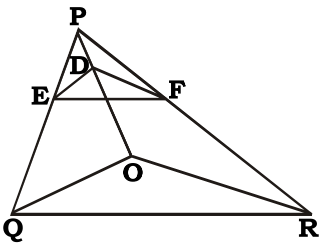
y $AQ^2 = 36$ [Taking square roots]

y AQ = 6 units ½

A.3. Solve ANY THREE of the following :

(i) In aPQO,

seg ED || side QO [Given]



y $\frac{PE}{EQ} = \frac{PD}{DO}$ ......(i) [By B.P.T.] **1**

In aPRO,

seg DF || seg OR [Given]

y $\frac{PF}{FR} = \frac{PD}{DO}$ .....(ii) [By B.P.T.] **1**

In aPQR,

$\frac{PE}{EQ} = \frac{PF}{FR}$ [From (i) and (ii)]

y **seg EF || side QR** [By converse of B.P.T.] **1**

(ii) Diameters of circular ends of frustum are 18 cm and 8 cm

y $r_1 = \frac{18}{2} = 9$ cm and $r_2 = \frac{8}{2} = 4$ cm ½

Slant height (l) = 13 cm

Curved surface area of frustum of cone = ® $(r_1 + r_2)\ l$

= ® (9 + 4) × 13 ½

= ® × 13 × 13

= 169® cm^2 ½

Radius of a cylinder (r_2) = 4 cm

Its height (h) = 10 cm

Curved surface area of a cylinder = 2®rh

= 2 × ® × 4 × 10 ½

= 80® cm^2

Surface area of tin required to make the funnel

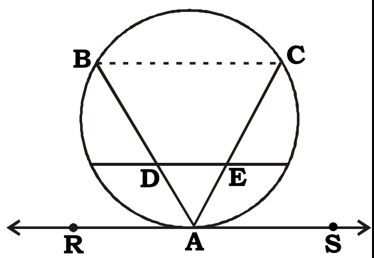
= Curved surface area of frustum + curved surface area of cylinder ½

= 169® + 80®

= 249 ®cm^2

y The surface area of the tin required to make the funnel is 249 cm^2. ½

(iii)



Construction : Draw seg BC. ½

Proof : Take points R and S on the tangent at A as shown in the figure

line DE || line RS [Given]

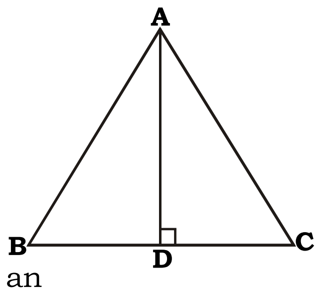
y On transversal AD,

∠EDA] ĎDAR [Converse of alternate angles test] ½

y ĎEDA] ĎBAR ....(i) [∵ B - D - A]
ĎBAR] ĎBCA ....(ii) [Angles in alternate segment] ½

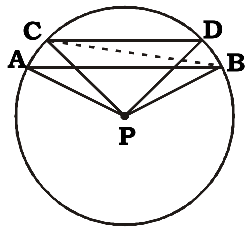
y ĎEDA] ĎBCA ......(iii) [From (i) and (ii)]
Similarly, we can prove that
ĎDEA] ĎCBA ......(iv) ½
In aABC,
seg AB] seg AC [Given]
y ĎBCA] ĎCBA ....(v) [Isosceles triangle theorem] ½
In aDEA,
ĎEDA] ĎDEA [From (iii), (iv) and (v)]
y seg AD] seg AE [Converse of isosceles triangle theorem] ½
y **AD = AE**

(iv)



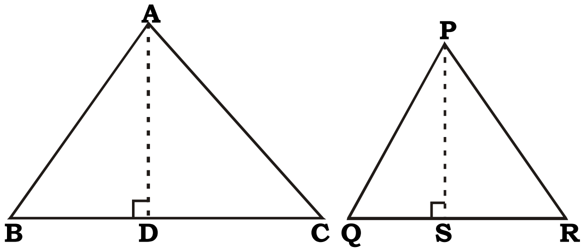
To prove : $3AB^2 = 4AD^2$
Proof : aABC is an equilateral triangle [Given]
In aADB,
m ĎADB = 90° [Given]
m ĎABD = 60° [Angle of an equilateral triangle]
y m ĎBAD = 30° [Remaining angle] 1
y aADB is a 30° - 60° - 90° triangle
y By 30° - 60° - 90° triangle theorem,
$AD = \frac{\sqrt{3}}{2} AB$ [Side opposite to 60°] 1
y $2AD = \sqrt{3}\, AB$
y $4AD^2 = 3AB^2$ [Squaring both sides] 1
y $\mathbf{3AB^2 = 4AD^2}$

(v)



Construction : Draw seg BC.
Proof :
m ĎCPA = m (arc CA) ....(i)
m ĎDPB = m (arc DB) ......(ii)
[Definition of measure of minor arc] ½

m ĎABC = $\frac{1}{2}$ m (arc CA) ......(iii)
m ĎBCD = $\frac{1}{2}$ m (arc DB) ......(iv)
[Inscribed angle theorem] ½

	chord CD || chord AB	[Given]	
	y On transversal BC,		
	ĎABC] ĎBCD ……(v)	[Converse of alternate angles test]	½
	y $\frac{1}{2}$ m (arc CA) = $\frac{1}{2}$ m(arc DB)	[From (iii), (iv) and (v)]	½
	y m (arc CA) = m (arc DB) ……(vi)		
	y **m ĎCPA = m ĎDPB**	[From (i), (ii) and (vi)]	1
A.4.	**Solve ANY TWO of the following :**		
(i)			
	Given : aABC ~ aPQR.		
	To Prove : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$		
	Construction :		
	(i)Draw seg AD꞊{ side BC, B - D - C		½
	(ii)Draw seg PS { side QR, Q - S - R		
	Proof : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC \times AD}{QR \times PS}$	[The ratio of the areas of two triangles is equal to ratio of the products of a base and its corresponding height]	½
	$\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ ……(i)		
	aABC ~ aPQR	[Given]	
	y $\frac{AB}{PQ} = \frac{BC}{QR}$ ……(ii)	[c.s.s.t.]	½
	Also, ĎB]꞊ĎQ …..(iii)	[c.a.s.t.]	½
	In aADB and aPSQ,		
	ĎADB]꞊ĎPSQ	[Each is a right angle]	
	ĎB] ĎQ	[From (ii)]	
	y aADB ~ aPSQ	[By A-A test of similarity]	½
	y $\frac{AD}{PS} = \frac{AB}{PQ}$ …..(iv)	[c.s.s.t.]	½
	y $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$	[From (i), (ii) and (iv)]	½

y $\frac{A\,(aABC)}{A\,(aPQR)} = \frac{AB^2}{PQ^2}$ ....(vi)

Similarly we can prove

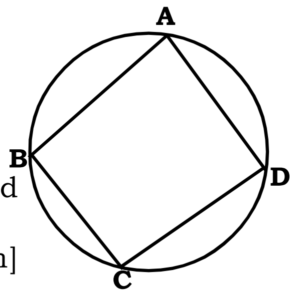
$\frac{A\,(aABC)}{A\,(aPQR)} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ ....(vii)

y $\mathbf{\frac{A(aABC)}{A(aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}}$ [From (vi) and (viii)] ½

(ii) **Given :** □ABCD is a cyclic

To Prove : m ĎABC + m ĎADC = 180° ½

m ĎBAD + m ĎBCD = 180°



Proof :

m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i) } [Inscribed angle theorem]

m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii) } 1

Adding (i) and (ii), we get **(½ mark for figure)**

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC)

y m ĎABC + m ĎADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)] ½

y m ĎABC + m ĎADC = $\frac{1}{2} \times 360°$ [∵ Measure of a circle is 360°] ½

y **m ĎABC + m ĎADC = 180°** **.......(iii)**

In □ABCD,

m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°

[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(iii) Height of the cylindrical container (h) = 14cm

Its radius (r) = 6 cm

Volume of cylindrical container = ® r^2h

= ® × 6 × 6 × 14

= 504® cm^3 1

But, volume of ink filled in the cylindrical container = 91% of 504 ®

= $\frac{91}{100} \times 504$® cm^3 ½

Length of ball pen refill (h_1) = 12m

its inner diameter = 2 mm

y Its radius (r_1) = 1 mm

= $\frac{1}{10}$ cm

Volume of the refill = $®r_1^2 h_1$

= $® \times \frac{1}{10} \times \frac{1}{10} \times 12$

= $\frac{12 ®}{100}$ cm^3 **1**

y But, volume of ink filled = 84% of $\frac{12 ®}{100}$

= $\frac{84}{100} \times \frac{12 ®}{100}$ cm^3 **½**

Number of refills that can be filled with ink

= $\frac{\text{Volume of ink filled in the cylindrical container}}{\text{Volume of ink filled in each refill}}$

= $\dfrac{\frac{91 \times 504 ®}{100}}{\frac{84 \times 12 ®}{100 \times 100}}$

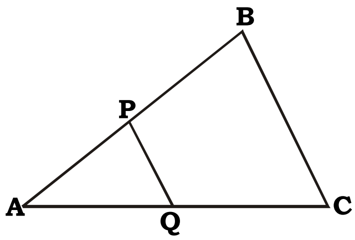
= $\frac{91 \times 504 ®}{100} \times \frac{100 \times 100}{84 \times 12 ®}$ **1**

= 4550

y Number of refills that can be filled with this ink is 4550.

A.5. Solve ANY TWO of the following :

(i) seg PQ divides aABC into two parts of equal areas [Given]



y A (aAPQ) = $\frac{1}{2}$ A (aABC)

y $\frac{\text{A (aAPQ)}}{\text{A (aABC)}} = \frac{1}{2}$ ......(i) **½**

seg PQ || side BC [Given]

y On transversal AC,

ĎAQP] ĎACB . .....(ii) [Converse of corresponding angles test]

In aAPQ and aABC,

ĎAQP] ĎACB [From (ii)]

ĎPAQ] ĎBAC [Common angle]

y a**APQ ~** a**ABC** [By AA test of similarity] **½**

y $\frac{\text{A (aAPQ)}}{\text{A (aABC)}} = \frac{AP^2}{AB^2}$ [Areas of similar triangles]

y $\frac{1}{2} = \frac{AP^2}{AB^2}$ [From (i)] **1**

	y $\frac{1}{\sqrt{2}} = \frac{AP}{AB}$ [Taking square roots]	
	y $\frac{AP}{AB} = \frac{1}{\sqrt{2}}$	
	y $\frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$ [$\because$ A - P - B]	**1**
	y $\frac{AB}{AB} - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$	
	y $1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$	
	y $1 - \frac{1}{\sqrt{2}} = \frac{BP}{AB}$	**1**
	y $\boxed{\frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}}$	**1**
(ii)	Diameter of semicircle sheet = 28 cm	
	y Its radius (r) = $\frac{28}{2}$	
	= 14 cm	½
	A semicircular sheet is bent to form a open cone	
	y Slant height of a cone (l) = radius of a semicircular sheet	
	y l = 14 cm	½
	Circumference of a base of a cone = length of semicircle	
	= ®r	
	= $\frac{22}{7} \times 14$	
	= 44 cm	½
	Let the radius of a cone be r_1	
	Circumference of a base of a cone = 2®r_1	
	y 44 = 2®r_1	
	y $44 = 2 \times \frac{22}{7} \times r_1$	½
	y $\frac{44 \times 7}{2 \times 22} = r_1$	
	y r_1 = 7 cm	
	$l^2 = r_1^2 + h^2$	
	y $14^2 = 7^2 + h^2$	
	y $h^2 = 14^2 - 7^2$	
	y $h^2 = 196 - 49$	**1**
	y $h^2 = 147$	
	y $h = \sqrt{147}$	

y h = $\sqrt{49 \times 3}$

y h = $7\sqrt{3}$

y h = 7 × 1.73

y h = 12.11 cm **1**

Volume of conical cup = $\frac{1}{3}$®r_1^2h

= $\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12.11$

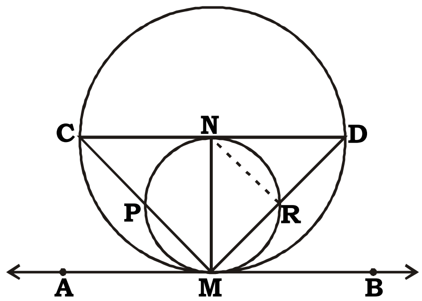
= $\frac{1864.94}{3}$

= 621.646

= 621.65 cm3

y Depth of a conical cup is 12.11 cm and volume of conical cup is 621.65 cm³. **1**

(iii)



Construction : Draw seg NR.

Proof :

ĎCMA] ∠CDM [Angles in alternate segments] **½**

Let,

m ĎCMA= mĎCDM = x° ....(i)

ĎNMA] ĎNRM [Angles in alternate segments] **½**

Let,

mĎNMA] ĎNRM = y° ....(ii)

m ĎNMC = m ĎNMA – m ĎCMA [Angle Addition property]

y m ĎNMC = (y – x)° ......(iii) [From (i) and (ii)] **1**

ĎNMR] ĎDNR ....(iv) [Angles in alternate segment]

ĎNRM is an exterior angle of a NDR

y m ĎNRM = m ĎNDR + mĎDNR [Remote interior angles] **1**

y m ĎNRM = ĎCDM + m ĎDNR [∵ C - N - D and D - R - M]

y y = x + m:ĎDNR [From (i) and (ii)]

y m ĎDNR = (y – x)° ......(v) **1**

y m ĎNMR = (y – x)° [From (iv) and (v)]

y m ĎNMD = (y – x)° ......(vi) [D - R - M]

y Ď**CMN**] Ď**DMN** [From (iii) and (vi)] **1**

❖❖❖❖