MT

2017 ___ ___ 1100

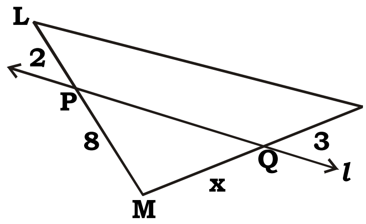
MT - GEOMETRY - SEMI PRELIM - II : PAPER - 3

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. **Attempt ANY FIVE of the following :**

(i) In aLMN,

line l || side LN [Given]



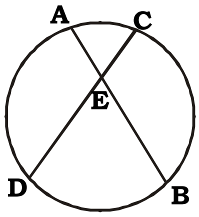
y $\frac{MP}{LP} = \frac{MQ}{NQ}$ [By B.P.T.] ½

y $\frac{8}{2} = \frac{x}{3}$

y $x = \frac{8 \times 3}{2}$

y $\boxed{x = 12}$ ½

(ii) Chords AB and CD intersect each other at point E inside the circle ½



y AE × BE = CE × DE

y AE × 3 = 4 × 6

y $AE = \frac{4 \times 6}{3}$

y $\boxed{AE = 8 \text{ units}}$ ½

(iii) F + V = E + 2 ½

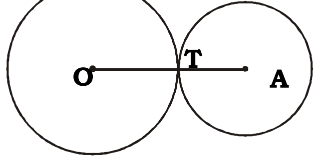
y F + 6 = 12 + 2

y F + 6 = 14

y F = 14 – 6 ½

y $\boxed{F = 8}$

(iv) **Given :** Two circles with centres O and A touch each other externally at point T.

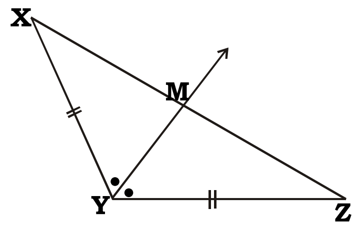


To Prove : OA = OT + AT

Proof : O - T - A [If two circles are touching circles then the common point lies on the line joining their centres] ½

y **OA = OT + AT** [∵ O - T - A] ½

(v)



In ΔXYZ,

ray YM bisects ∠XYZ [Given]

∴ $\frac{XY}{YZ} = \frac{XM}{MZ}$ [Property of angle bisector of a triangle] ½

∴ $1 = \frac{XM}{MZ}$ [∵ XY = YZ]

∴ **XM = MZ** ½

(vi) A cylinder and cone have equal height and equal radii

y Volume of cone = $\frac{1}{3}$ × volume of cylinder ½

= $\frac{1}{3}$ × 300

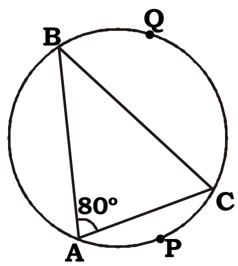
= 100 cm^3

y Volume of the cone is 100 cm^3. ½

A.2. Solve ANY FOUR of the following :

(i) (a) m ĎABC = $\frac{1}{2}$ m(arc APC) ½

[Inscribed angle theorem]



y m ĎABC = $\frac{1}{2}$ × 60

y m ĎABC = 30° ½

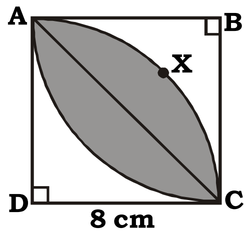
(b) m ĎBAC = $\frac{1}{2}$ m(arc BQC) [Inscribed angle theorem] ½

y 80 = $\frac{1}{2}$ m(arc BQC)

y m(arc BQC) = 80 × 2

y m(arc BQC) = 160° ½

(ii) Mark point X as shown in the figure



□ABCD is a square [Given]

side = 8 cm

Radius (r) = side of a square

y r = 8 cm

Measure of arc ( ) = 90° [Angle of a square] ½

Area of the segment AXC $= r^2 \left[\frac{\theta \times \pi}{360} - \frac{\sin\theta}{2}\right]$ (½)

$= 8^2 \left[\frac{3.14 \times 90}{360} - \frac{\sin 90}{2}\right]$

$= 64 \left[\frac{1.57}{2} - \frac{1}{2}\right]$

$= 64 \left[\frac{1.57 - 1}{2}\right]$ (½)

$= \frac{64 \times 0.57}{2}$

$= \frac{36.48}{2}$ cm²

Area of shaded region $= 2 \times$ Area of segment AXC (½)

$= 2 \times \frac{36.48}{2}$

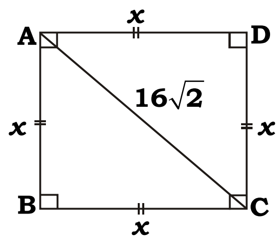
$= 36.48$ cm²

∴ **Area of shaded region is 36.48 cm².**

(iii) **Given :** □ABCD is a square.

AC = $16\sqrt{2}$ cm

To find : Side of a square



Sol. □ABCD is a square [Given] (½)

Let the sides of the square be x cm

In ΔABC,

m∠ABC = 90° [Angle of a square]

∴ $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem] (½)

∴ $(16\sqrt{2})^2 = x^2 + x^2$

∴ $256 \times 2 = 2x^2$

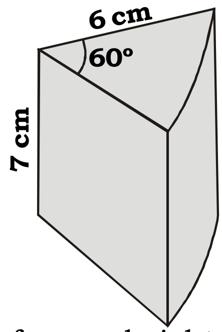
∴ $x^2 = \frac{256 \times 2}{2}$

∴ $x^2 = 256$

∴ $x = 16$ [Taking square roots] (1)

∴ **The side of a square is 16 cm.**

(iv)



For a sector,
Measure of arc (¯) = 60°
Radius (r) = 6 cm

Curved surface area of the cheese = Length of arc × height ½

$= \frac{\bar{}}{360} \times 2®r \times h$ ½

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6 \times 7$ ½

$= 44 \text{ cm}^2$

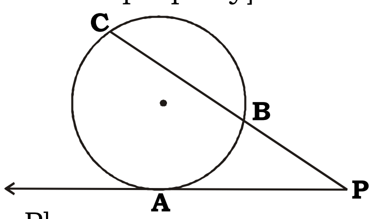
y **The curved surface area of the cheese is 44 cm².** ½

(v) Line PBC is a secant intersecting the circle at points B and C and line PA is a tangent to the circle at point A.

y $CP \times BP = AP^2$ [Tangent secant property] ½

y $CP \times 10 = (15)^2$

y $CP \times 10 = 225$



y $CP = \frac{225}{10}$

y $CP = 22.5$ units ½

$CP = BC + BP$ [∵ C - B - P]

y $22.5 = BC + 10$ ½

y $BC = 22.5 - 10$

y **BC = 12.5 units** ½

(vi) In aPQR,
seg PM is the median [Given]

y $PQ^2 + PR^2 = 2PM^2 + 2QM^2$ [By Appollonius theorem] ½

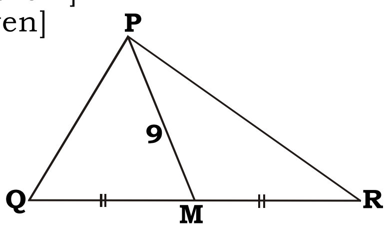
y $290 = 2(9)^2 + 2QM^2$ [Given]

y $290 = 2(81) + 2QM^2$ ½

y $290 = 162 + 2QM^2$

y $290 - 162 = 2QM^2$

y $128 = 2QM^2$



y $QM^2 = \frac{128}{2}$ ½

y $QM^2 = 64$

y $QM = 8$ units [Taking square roots]

y QM = $\frac{1}{2}$ QR [∵ M is midpoint of side QR]

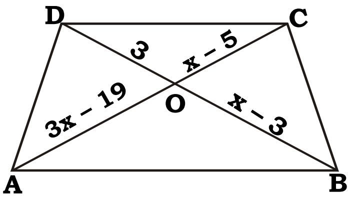
y 8 = $\frac{1}{2}$ QR

y 8 × 2 = QR

y **QR = 16 units** ½

A.3. Solve ANY THREE of the following :

(i)



seg AB || seg DC [Given]

y On transversal BD,

ĎCDB] ĎABD [Converse of alternate angles test] ½

y ĎCDO] ĎABO ....(i) [D - O - B]

In aDOC and aBOA,

ĎCDO] ĎABO [From (i)]

ĎDOC] ĎBOA [Vertically opposite angles]

y aDOC ~ aBOA [By AA test of similarity] ½

y $\frac{DO}{BO} = \frac{OC}{OA}$ [c.s.s.t.] ½

y $\frac{3}{x-3} = \frac{x-5}{3x-19}$

y $3(3x - 19) = (x - 5)(x - 3)$ ½

y $9x - 57 = x^2 - 3x - 5x + 15$

y $9x - 57 = x^2 - 8x + 15$

y $x^2 - 8x - 9x + 15 + 57 = 0$

y $x^2 - 17x + 72 = 0$ ½

y $x^2 - 9x - 8x + 72 = 0$

y $x(x - 9) - 8(x - 9) = 0$

y $(x - 9)(x - 8) = 0$

y $x - 9 = 0$ or $x - 8 = 0$

y **x = 9 or x = 8** ½

(ii) Diameter of the roller = 0.9 m

y its radius (r) = $\frac{0.9}{2}$ ½

= 0.45 m

its length (h) = 1.8 m

Curved surface area of the roller = 2®rh ½

= 2 × 3.14 × 0.45 × 1.8

= 6.28 × 0.81

= 5.0868 m^2 ½

Area of the ground pressed by the roller in 1 revolution = curved surface area of roller ½

y Area of the ground pressed in one revolution = $5.0868\ m^2$

Area of the ground pressed in 500 revolution = 500×5.0868 ½

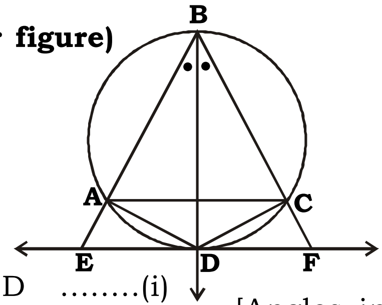
$= 500 \times \frac{50868}{10000}$

$= 2543.4\ m^2$ ½

y **Area of the ground pressed by the roller is $2543.4\ m^2$.**

(iii)

(1 mark for figure)



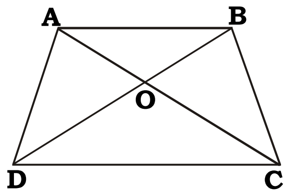
ĎEDA] ĎABD ....(i)

=ĎFDC] ĎCBD ....(ii) [Angles in alternate segment] 1

But, ĎABD] ĎCBD ....(iii) [∵ Ray BD bisects ĎABC]

y ĎEDA] ĎFDC [From (i), (ii) and (iii)] 1

(iv)



□ABCD is a trapezium

side AB || side DC [Given]

y On transversal AC,

ĎBAC] ĎDCA [Converse of alternate angles test] ½

y ĎBAO] ĎDCO ......(i) [∵ A - O - C] ½

In aAOB and aCOD,

ĎBAO] ĎDCO [From (i)]

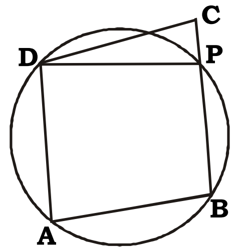
ĎAOB] ĎCOD [Vertically opposite angles]

y aAOB ~ aCOD [By AA test of similarity] 1

y $\frac{AO}{CO} = \frac{BO}{DO}$ [c.s.s.t.]

y $\mathbf{\frac{AO}{BO} = \frac{CO}{DO}}$ [By Alternendo] 1

(v)



❑ABPD is cyclic [By definition]

y ĎDPC] ĎDAB .....(i) [An exterior angle of cyclic quadrilateral is congruent to the angle opposite to adjacent interior angle] **1**

❑ABCD is parallelogram

y ĎDCB] ĎDAB ....(ii) [Opposite angles of a parallelogram are congruent] **1**

y ĎDPC] ĎDCB ......(iii) [From (i) and (ii)]

In aDPC,

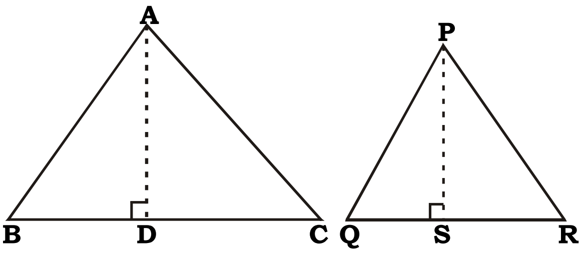
ĎDPC] ĎDCP [From (ii) and C - P - B]

y seg DP] seg DC [Converse of isosceles triangle theorem] **1**

y **DP = DC**

A.4. **Solve ANY TWO of the following :**

(i)



Given : aABC ~ aPQR.

To Prove : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Construction :

(i)Draw seg AD⊥{ side BC, B - D - C ½

(ii)Draw seg PS { side QR, Q - S - R

Proof : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC \times AD}{QR \times PS}$ [The ratio of the areas of two triangles is equal to ratio of the products of a base and its corresponding height] ½

$\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ ......(i)

aABC ~ aPQR [Given]

y $\frac{AB}{PQ} = \frac{BC}{QR}$ ......(ii) [c.s.s.t.] ½

Also, ĎB]∷ĎQ .....(iii) [c.a.s.t.] ½

In aADB and aPSQ,
ĎADB]∷ĎPSQ [Each is a right angle]
ĎB] ĎQ [From (ii)]
y aADB ~ aPSQ [By A-A test of similarity] ½

y $\frac{AD}{PS} = \frac{AB}{PQ}$ .....(iv) [c.s.s.t.] ½

y $\frac{A\,(aABC)}{A\,(aPQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (i), (ii) and (iv)] ½

y $\frac{A\,(aABC)}{A\,(aPQR)} = \frac{AB^2}{PQ^2}$ ....(vi)

Similarly we can prove

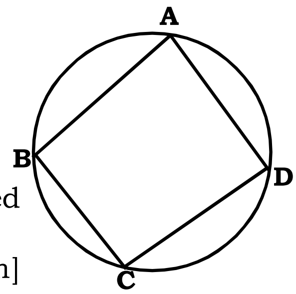
$\frac{A\,(aABC)}{A\,(aPQR)} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ ....(vii)

y $\mathbf{\frac{A(aABC)}{A(aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}}$ [From (vi) and (viii)] ½

(ii) **Given :** □ABCD is a cyclic

To Prove : m ĎABC + m ĎADC = 180° ½

m ĎBAD + m ĎBCD = 180°



Proof :

m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i)
m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii) } [Inscribed angle theorem] 1

Adding (i) and (ii), we get **(½ mark for figure)**

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC)

y m ĎABC + m ĎADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)] ½

y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y **m ĎABC + m ĎADC = 180°** ....(iii)

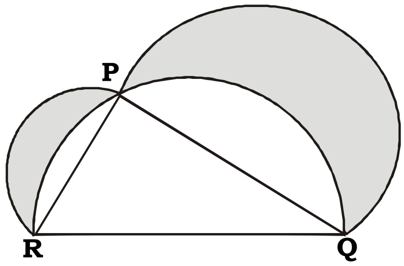
In □ABCD,

m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°
[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(iii)



Diameter PR = 6 units

y Its radius (r_1) = 3 units

Diameter PQ = 8 units

y Its radius (r_2) = 4 units

In aPQR,

m ĎRPQ = 90° ......(i) [Angle subtended by a semicircle] ½

$QR^2 = PR^2 + PQ^2$ [By Pythagoras theorem]

y $QR^2 = 6^2 + 8^2$

y $QR^2 = 36 + 64$

y QR = 100

y QR = 10 units [Taking square roots] ½

Diameter QR = 10 units

y Its radius (r_3) = 5 units

aPQR is a right angled triangle [From (i)]

A (aPQR) = $\frac{1}{2}$ × product of perpendicular sides

= $\frac{1}{2}$ × PR × PQ

= $\frac{1}{2}$ × 6 × 8

= 24 sq. units. **1**

Area of shaded portion

= Area of semicircle with diameter PR + Area of semicircle with diameter PQ + Area of aPQR – Area of semicircle with diameter QR ½

= $\frac{1}{2}$®r_1^2 + $\frac{1}{2}$®r_2^2 + 24 – $\frac{1}{2}$®r_3^2

= $\left(\frac{1}{2}\right.$®r_1^2 + $\frac{1}{2}$®r_2^2 – $\frac{1}{2}$®$\left.r_3^2\right)$ + 24 ½

= $\frac{1}{2}$® $(r_1^2 + r_2^2 - r_3^2)$ + 24

= $\frac{1}{2}$ × 3.14 $(3^2 + 4^2 - 5^2)$ + 24

= $\frac{1}{2}$ × 3.14 × (9 + 16 – 25) + 24 ½

= $\frac{1}{2}$ × 3.14 (0) + 24

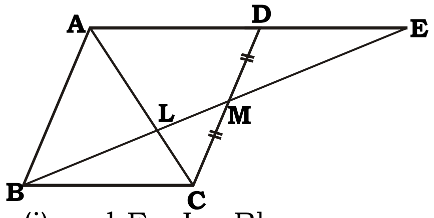
= 0 + 24

= 24 sq. units

y **Area of shaded portion is 24 sq.units** ½

A.5. **Solve ANY TWO of the following :**

(i)



In aELA and aBLC,

ĎAEL]⁼ĎCBL [From (i) and E - L - B]

ĎALE]⁼ĎCLB [Vertically opposite angles] ½

y aELA ~ aBLC [By AA test of similarity]

y $\frac{EL}{BL} = \frac{EA}{BC}$ .....(i) [c.s.s.t.] ½

☐ABCD is a parallelogram [Given]

y seg AD || seg BC [By definition]

y seg AE || seg BC [∵ A - D - E]

y On transversal BE,

ĎAEB]⁼ĎCBE .....(ii) [Converse of alternate angles test] ½

In aDME and aCMB,

side DM] side CM [Given]

ĎDME]⁼ĎCMB [Vertically opposite angles]

ĎDEM]⁼ĎCBM [From (ii) and A - D - E, B - M - E] ½

y aDME]⁼aCMB [By SAA test of congruence]

y DE = BC .....(iii) [c.s.c.t.]

But, AD = BC .....(iv) [Opposite sides of a parallelogram] ½

y DE = BC = AD .....(v) [From (iii) and (iv)]

y $\frac{EL}{BL} = \frac{ED + DA}{BC}$ [From (i) and E - D - A] **1**

y $\frac{EL}{BL} = \frac{BC + BC}{BC}$ [From (v)]

y $\frac{EL}{BL} = \frac{2BC}{BC}$ **1**

y $\frac{EL}{BL} = \frac{2}{1}$

y **EL = 2BL** ½

(ii)

Diameter of marble = 1.4 cm

y its radius (r) = $\frac{1.4}{2}$

= 0.7 cm ½

Volume of a marble = $\frac{4}{3}$®r^3

= $\frac{4}{3} \times$ ® $\times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$ cm^3 **1**

∵ Marbles are submerged fully in the water, water level rises by 5.6 cm

y Height of water displaced (h) = 5.6 cm

Diameter of beaker = 7 cm

y Its radius (r_1) = $\frac{7}{2}$ cm ½

Volume of water displaced = $®r_1^2h$

= $® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$ cm^3 1

Number of marbles = $\frac{\text{Volume of water displaced}}{\text{Volume of marble}}$ ½

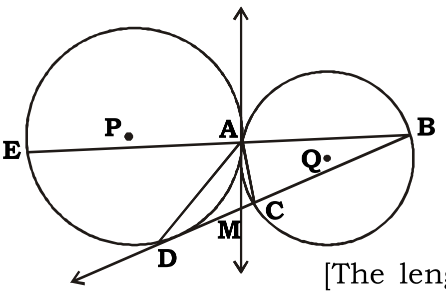
= $® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} \div \left(\frac{4}{3} \times ® \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}\right)$ ½

= $® \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} \times \frac{3}{4} \times \frac{1}{®} \times \frac{10}{7} \times \frac{10}{7} \times \frac{10}{7}$

= 150

y **Number of marbles is 150.** 1

(iii)



In aMAD,

MA = MD [The lengths of two tangent segments from an external point to a circle are equal] ½

y m ĎMAD = m ĎMDA [Isosceles triangle theorem]

Let,

m ĎMAD = m ĎMDA = x° .....(i)

ĎCAM]≅ĎABC [Angles in alternate segments] ½

Let,

m ĎCAM = m ĎABC = y° .....(ii) ½

m ĎCAD = m ĎCAM + m ĎMAD [Angles Addition property]

m ĎCAD = (x + y)° ....(iii) [From (i) and (ii)] ½

ĎDAE is an exterior angle of aADB

y m ĎDAE = m ĎADB + mĎABD [Remote Interior angles theorem] 1

y m ĎDAE = m ĎADM + m ĎABC [D - M - C - B]

y mĎDAE = (x + y)° ....(iv) [From (i) and (ii)] 1

y m ∠CAD = m≅ĎDAE [From (iii) and (iv)]

y **ray AD is an angle bisector of ĎCAE.** 1

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