MT

2017 ___ ___ 1100

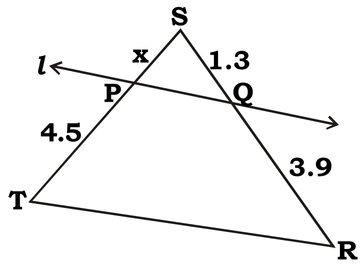
MT - GEOMETRY - SEMI PRELIM - II : PAPER - 4

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. **Attempt ANY FIVE of the following :**

(i) In aSTR,

line l || side TR [Given]



y $\frac{SP}{TP} = \frac{SQ}{RQ}$ [By B.P.T.] ½

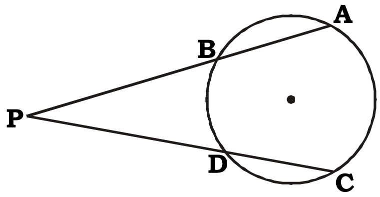
y $\frac{x}{4.5} = \frac{1.3}{3.9}$

y $x = \frac{4.5 \times 1.3}{3.9}$

y $x = \frac{4.5}{3}$

y $\boxed{x = 1.5}$ ½

(ii) Chords AB and CD intersect each other at point P outside the circle.



y PA × PB = PC × PD ½

y 6 × 3 = PC × 4

y $PC = \frac{6 \times 3}{4}$

y $PC = \frac{9}{2}$

y $\boxed{PC = 4.5 \text{ units}}$ ½

(iii) F + V = E + 2 ½

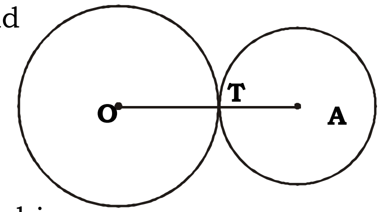
y F + 6 = 12 + 2

y F + 6 = 14

y F = 14 – 6

y $\boxed{F = 8}$ ½

(iv) **Given :** Two circles with centres O and A touch each other externally at point T.



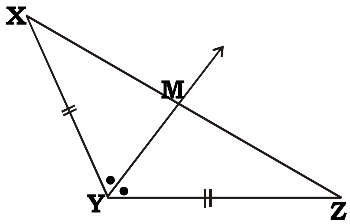
To Prove : OA = OT + AT

Proof : O - T - A ½

[If two circles are touching circles then the common point lies on the line joining their centres]

y **OA = OT + AT** [∵ O - T - A] ½

(v)



In ΔXYZ,

ray YM bisects ∠XYZ [Given]

$\therefore \quad \frac{XY}{YZ} = \frac{XM}{MZ}$ [Property of angle bisector of a triangle] ½

$\therefore \quad 1 = \frac{XM}{MZ}$ [∵ XY = YZ]

∴ **XM = MZ** ½

(vi) Volume of cuboid = Volume of cube [Given] ½

y $3 \times 9 \times x = (6)^3$

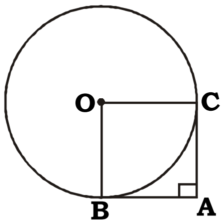
y $3 \times 9 \times x = 6 \times 6 \times 6$

y $x = \frac{6 \times 6 \times 6}{3 \times 9}$

y $\boxed{x = 8}$ ½

A.2. Solve ANY FOUR of the following :

(i) In □BACO,



m ĎOBA = 90° } [Radius is perpendicular ½

m ĎOCA = 90° } to the tangent]

m ĎBAC = 90° [Given]

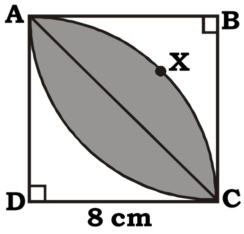
y m ĎBOC = 90° [Remaining angle] ½

y □BACO is a rectangle [By definition] ½

seg OB] seg OC [Radii of the same circle]

y **□BACO is a square** [A rectangle in which adjacent sides are congruent is a square] ½

(ii) Mark point X as shown in the figure



□ABCD is a square [Given]

side = 8 cm

Radius (r) = side of a square

y r = 8 cm

Measure of arc (¯) = 90° [Angle of a square] ½

Area of the segment AXC $= r^2 \left[\frac{®¯}{360} - \frac{\sin ¯}{2}\right]$ ½

$= 8^2 \left[\frac{3.14 \times 90}{360} - \frac{\sin 90}{2}\right]$

$= 64 \left[\frac{1.57}{2} - \frac{1}{2}\right]$

$= 64 \left[\frac{1.57 - 1}{2}\right]$

$= \frac{64 \times 0.57}{2}$

$= \frac{36.48}{2}$ cm²

Area of shaded region = 2 × Area of segment AXC ½

$= 2 \times \frac{36.48}{2}$

= 36.48 cm²

y **Area of shaded region is 36.48 cm².** ½

(iii) $(61)^2 = 3721$ ......(i) ½

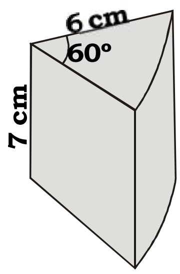
$(11)^2 + (60)^2 = 121 + 3600$

$= 3721$ ......(ii) ½

y $(61)^2 = (11)^2 + (60)^2$ [From (i) and (ii)]

y The given sides form a right angled triangle. [By Converse of Pythagoras theorem] 1

(iv)



For a sector,

Measure of arc (¯) = 60°

Radius (r) = 6 cm

(a) Curved surface area of the cheese = Length of arc × height ½

= $\frac{-}{360} \times 2$®$r \times h$ ½

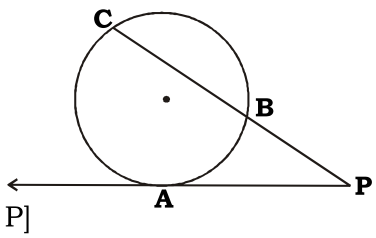
= $\frac{60}{360} \times 2 \times \frac{22}{7} \times 6 \times 7$ ½

= 44 cm^2

y The curved surface area of the cheese is 44 cm^2. ½

(v) Line PBC is a secant intersecting the circle at points B and C and line PA is a tangent to the circle at point A.

y $CP \times BP = AP^2$ [Tangent secant property] ½



y $CP \times 10 = (15)^2$

y $CP \times 10 = 225$

y $CP = \frac{225}{10}$

y $CP = 22.5$ units ½

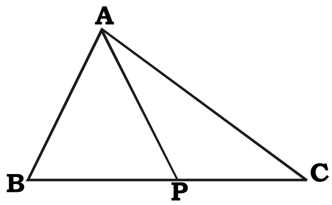
$CP = BC + BP$ [∵ C - B - P]

y $22.5 = BC + 10$

y $BC = 22.5 - 10$

y $BC = 12.5$ units 1

(vi)



In aABC,

seg AP is the median [Given]

y $AB^2 + AC^2 = 2AP^2 + 2BP^2$ [By Appollonius theorem] ½

y $260 = 2(7)^2 + 2BP^2$ [Given]

y $260 = 2(49) + 2BP^2$

y $260 = 98 + 2BP^2$ ½

y $260 - 98 = 2BP^2$

y $2BP^2 = 162$

y $BP^2 = \frac{162}{2}$

y $BP^2 = 81$

y $BP = 9$ units [Taking square roots] ½

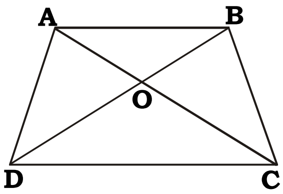
$BP = \frac{1}{2}BC$ [∵ P is the midpoint of seg BC]

y $9 = \frac{1}{2}BC$

y $BC = 18$ units ½

A.3. **Solve ANY THREE of the following :**

(i)

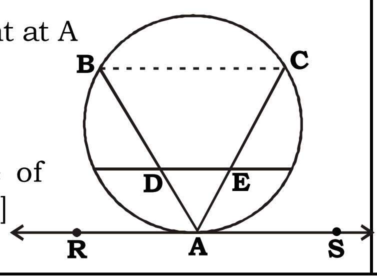


□ABCD is a trapezium
side AB || side DC [Given]
y On transversal AC,
ĎBAC] ĎDCA [Converse of alternate angles test] ½
y ĎBAO] ĎDCO ......(i) [∵ A - O - C]
In aAOB and aCOD,
ĎBAO] ĎDCO [From (i)]
ĎAOB] ĎCOD [Vertically opposite angles] **1**
y aAOB ~ aCOD [By AA test of similarity]
y $\frac{AO}{CO} = \frac{BO}{DO}$ [c.s.s.t.] ½
y $\mathbf{\frac{AO}{BO} = \frac{CO}{DO}}$ [By Alternendo] **1**

(ii) Diameters of circular ends of frustum are 18 cm and 8 cm
y $r_1 = \frac{18}{2}$ = 9 cm and $r_2 = \frac{8}{2}$ = 4 cm **1**
Slant height (l) = 13 cm
Curved surface area of frustum of frustum = ® ($r_1 + r_2$) l ½
= ® (9 + 4) × 13
= ® × 13 × 13
= 169® cm² ½
Radius of a cylinder (r_2) = 4 cm
Its height (h) = 10 cm
Curved surface area of a cylinder = 2®rh
= 2 × ® × 4 × 10
= 80® cm² ½
Surface area of tin required to make the funnel
= Curved surface area of frustum + curved surface area of cylinder ½
= 169® + 80®
= 249 ®cm²

y The surface area of the tin required to make the funnel is 249 cm².

(iii) **Construction :** Draw seg BC.



Proof : Take points R and S on the tangent at A as shown in the figure
line DE || line RS [Given]
y On transversal AD,
∠EDA] ĎDAR [Converse of alternate angles test] ½

∴ ∠EDA ≅ ∠BAR ....(i) [∵ B - D - A]
∠BAR ≅ ∠BCA ....(ii) [Angles in alternate segment] ½
∴ ∠EDA ≅ ∠BCA ......(iii) [From (i) and (ii)]
Similarly, we can prove that
∠DEA ≅ ∠CBA ......(iv) ½
In △ABC,
seg AB ≅ seg AC [Given]
∴ ∠BCA ≅ ∠CBA ....(v) [Isosceles triangle theorem] ½
In △DEA,
∠EDA ≅ ∠DEA [From (iii), (iv) and (v)]
∴ seg AD ≅ seg AE [Converse of isosceles triangle theorem] **1**
∴ **AD = AE**

(iv) In △ABC,
AB = 5 units
BC = 6 units [Given]
AC = 7 units
Perimeter of △PQR = 360 units [Given]
∴ PQ + QR + PR = 360 ......(i)
△ABC ~ △PQR [Given] ½

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$ [c.s.s.t.] ½

$$\therefore \frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR}$$

$$\therefore \frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{5+6+7}{PQ+QR+PR}$$ [By theorem on equal ratios] ½

$$\therefore \frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{18}{360}$$ [From (i)]

$$\therefore \frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{1}{20}$$ .....(ii)

$$\frac{5}{PQ} = \frac{1}{20}$$ [From (ii)] ½

∴ $\boxed{PQ = 100 \text{ units}}$

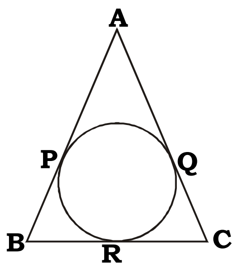
$$\frac{6}{QR} = \frac{1}{20}$$ [From (ii)] ½

∴ $\boxed{QR = 120 \text{ units}}$

$$\frac{7}{PR} = \frac{1}{20}$$ [From (ii)] ½

∴ $\boxed{PR = 140 \text{ units}}$

(v)



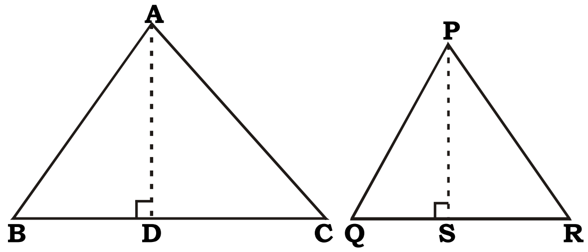
Let, AP = AQ = x ......(i)
BP = BR = y ......(ii)
CR = CQ = z ......(iii)
[The lengths of the two tangent segments to a circle drawn from an external point are equal] ½

AB = AP + PB [∵ A - P - B]
y AB = x + y .....(iv) [From (i) and (ii)] ½
Similarly, BC = y + z
y BC = 12 .....(v) ½
AC = x + z .....(vi) ½
Perimeter of aABC = 44 cm [Given]
y AB + BC + AC = 44
y x + y + y + z + x + z = 44 [From (iv), (v), and (vi)] ½
y 2x + 2y + 2z = 44
y 2(x + y + z) = 44
y x + y + z = 22
y x + 12 = 22 [From (v)]
y x = 22 – 12
y x = 10
y AP = AQ = 10 cm [From (i)]

y **Length of a tangent segment from A to the circle is 10 cm.** ½

A.4. **Solve ANY TWO of the following :**

(i)



Given : aABC ~ aPQR.

To Prove : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Construction :
(i)Draw seg AD{ side BC, B - D - C ½
(ii)Draw seg PS { side QR, Q - S - R

Proof : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC \times AD}{QR \times PS}$ [The ratio of the areas of two triangles is equal to ratio of the products of a base and its corresponding height] ½

$\frac{A\,(aABC)}{A\,(aPQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ ......(i)

aABC ~ aPQR [Given]

y $\frac{AB}{PQ} = \frac{BC}{QR}$ ......(ii) [c.s.s.t.] ½

Also, ĎB]∷ĎQ .....(iii) [c.a.s.t.] ½

In aADB and aPSQ,

ĎADB]∷ĎPSQ [Each is a right angle]

ĎB] ĎQ [From (ii)]

y aADB ~ aPSQ [By A-A test of similarity] ½

y $\frac{AD}{PS} = \frac{AB}{PQ}$ .....(iv) [c.s.s.t.] ½

y $\frac{A\,(aABC)}{A\,(aPQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (i), (ii) and (iv)] ½

y $\frac{A\,(aABC)}{A\,(aPQR)} = \frac{AB^2}{PQ^2}$ ....(vi)

Similarly we can prove

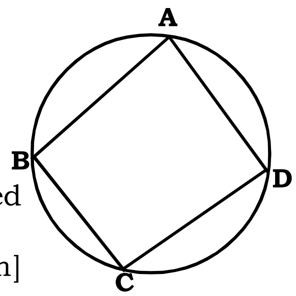
$\frac{A\,(aABC)}{A\,(aPQR)} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ ....(vii)

y $\mathbf{\frac{A\,(aABC)}{A\,(aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}}$ [From (vi) and (viii)] ½

(ii) **Given :** ☐ABCD is a cyclic

To Prove : m ĎABC + m ĎADC = 180° ½

m ĎBAD + m ĎBCD = 180°



Proof :

m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii)

} [Inscribed angle theorem] 1

Adding (i) and (ii), we get **(½ mark for figure)**

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC)

y m ĎABC + m ĎADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)] ½

y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y **m ĎABC + m ĎADC = 180°** ....(iii)

In ☐ABCD,

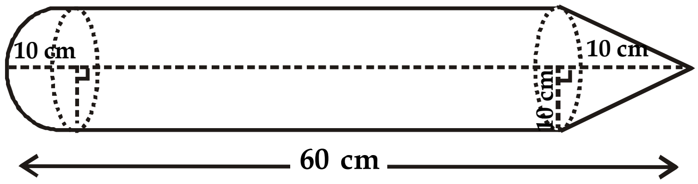
m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°

[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(iii)



A toy is a combination of cylinder, hemisphere and cone, each with radius 10 cm ½

y r = 10 cm

y Height of the conical part (h) = 10 cm

Height of the hemispherical part = its radius = 10cm

Total height of the toy = 60cm

y Height of the cylindrical part (h_1) = 60 – 10 – 10

= 60 – 20

= 40 cm 1

$l^2 = r^2 + h^2$

y $l^2 = 10^2 + 10^2$

y $l^2 = 100 + 100$

$l^2 = 200$

y $l = \sqrt{200}$ [Taking square roots]

$l = 10\sqrt{2}$ cm ½

Slant height of the conical part (l) = $10\sqrt{2}$

= 10 × 1.41

= 14.1 cm ½

Total surface area of the toy

= Curved surface area of the conical part + Curved surface area of the cylindrical part + Curved surface area of the hemispherical part ½

= ®rl + 2®rh_1 + 2®r^2 ½

= ®r (l + 2h_1 + 2r)

= 3.14 × 10 (14.1 + 2 × 40 + 2 × 10)

= 31.4 (14.1 + 80 + 20)

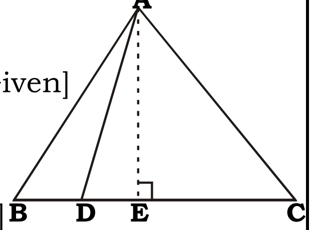
= 31.4 × 114.1

= 3582.74 cm^2

y **Total surface area of the toy is 3582.74 cm^2.** ½

A.5. Solve ANY TWO of the following :

(i) **Construction :** Draw seg AE { side BC, such that B - D - E - C.



Proof : aABC is an equilateral triangle. [Given]

y AB = BC = AC ....(i) [Sides of an equilateral triangle] ½

In aAED,

m ĎAED = 90° [Construction]

y $AD^2 = AE^2 + DE^2$ ....(ii) [By Pythagoras Theorem] ½

In aAEB,
m ĎAEB = 90° [Construction]
m ĎABE = 60° [Angle of an equilateral triangle]
y m ĎBAE = 30° [Remaining angle] ½
y aAEB is a 30° - 60° - 90° triangle.
y By 30° - 60° - 90° triangle theorem,

$AE = \frac{\sqrt{3}}{2}AB$ ......(iii) [Side opposite to 60°] ½

$BE = \frac{1}{2}AB$ ......(iv) [Side opposite to 30°]

$DE = BE - BD$ [∵ B - D - E]

y $DE = \frac{1}{2}AB - \frac{1}{3}BC$ [From (iv) and Given]

y $DE = \frac{1}{2}AB - \frac{1}{3}AB$ [From (i)]

y $DE = \frac{3AB - 2AB}{6}$

y $DE = \frac{1}{6}AB$ ....(v) **1**

y $AD^2 = \left(\frac{\sqrt{3}}{2}AB\right)^2 + \left(\frac{1}{6}AB\right)^2$ [From (ii), (iii) and (v)] ½

y $AD^2 = \frac{3}{4}AB^2 + \frac{1}{36}AB^2$ ½

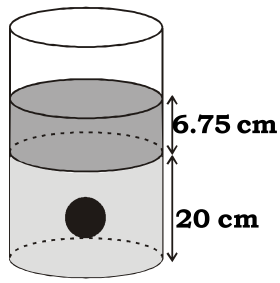
y $AD^2 = \frac{27AB^2 + AB^2}{36}$

y $AD^2 = \frac{28AB^2}{36}$ ½

y $AD^2 = \frac{7}{9}AB^2$

y $\mathbf{9AD^2 = 7AB^2}$ ½

(ii)



Radius of the cylinder (r) = 12 cm
A spherical iron ball is dropped into the cylinder and the water level rises by 6.75 cm ½ ½
y Volume of water displaced = volume of the iron ball
Height of the raised water level (h) = 6.75 m ½
Volume of water displaced $= \pi r^2 h$ ½
$= \pi \times 12 \times 12 \times 6.75\ cm^3$

y Volume of iron ball = ® × 12 × 12 × 6.75 cm³ ½

But, Volume of iron ball = $\frac{4}{3}$ ®r^3 ½

y ® × 12 × 12 × 6.75 = $\frac{4}{3}$ × ® × r^3

y $\frac{12 \times 12 \times 6.75 \times 3}{4}$ = r^3 ½

y r^3 = 3 × 12 × 6.75 × 3

y r^3 = 3 × 3 × 3 × 4 × 6.75

y r^3 = 3 × 3 × 3 × 27 ½

y r = $\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$ ½

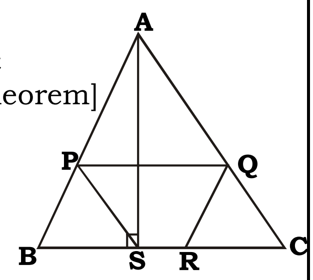
[Taking cube roots]

y r = 3 × 3

y r = 9

y **Radius of the iron ball is 9 cm.** ½

(iii) In aABC,



P and Q are midpoint of seg AB and seg AC

y seg PQ || seg BC [By midpoint theorem] ½

y seg PQ || seg BR [B - R - C]

Similarly, seg QR || seg PB

y □PBRQ is a parallelogram [By definition] ½

In aASB,

m ĎASB = 90° [Given]

seg SP is median to hypotenuse AB [Given]

y SP = $\frac{1}{2}$AB .....(i) [In a right angled triangle the median drawn to the hypotenuse is half of it] ½

But, PB = $\frac{1}{2}$AB .....(ii) [∵ P is the midpoint of side AB] ½

In aPBS, SP = PB [From (i) and (ii)]

y m ĎPBS = m ĎPSB [Isosceles triangle theorem]

□PBRQ is a parallelogram

y m ĎPBR = m ĎPQR [Opposite angles of a parallelogram are congruent] ½

y m ĎPBS = m ĎPQR .....(iv) [B - S - R]

y m ĎPSB = m ĎPQR ......(v) [From (iii) and (iv)] ½

But,

m ĎPSB + m ĎPSR = 180° [Linear pair axiom] 1

m ĎPQR + m ĎPSR = 180° [From (v)]

y **□PQRS is cyclic** [If opposite angles of a quadrilateral are supplementary then it is a cyclic quadrilateral] 1

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