MT

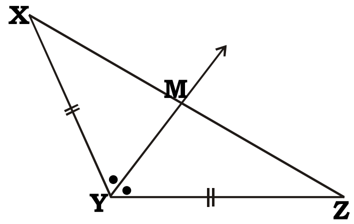
2017 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - II : PAPER - 5

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. **Attempt ANY FIVE of the following :**

(i)



In ΔXYZ,

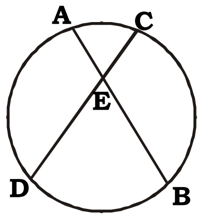
ray YM bisects $\angle XYZ$ [Given]

$\therefore \frac{XY}{YZ} = \frac{XM}{MZ}$ [Property of angle bisector of a triangle] ½

$\therefore 1 = \frac{XM}{MZ}$ [$\because$ XY = YZ]

$\therefore$ **XM = MZ** ½

(ii) Chords AB and CD intersect each other at point E inside the circle ½



y AE × BE = CE × DE

y AE × 3 = 4 × 6

y AE = $\frac{4 \times 6}{3}$

y AE = 8 units ½

(iii) F + V = E + 2 ½

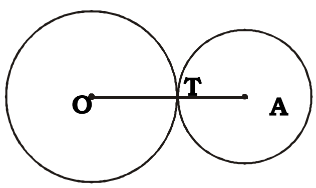
y F + 6 = 12 + 2

y F + 6 = 14

y F = 14 – 6

y $\boxed{F = 8}$ ½

(iv) **Given :** Two circles with centres O and A touch each other externally at point T.

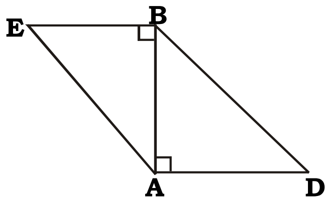


To Prove : OA = OT + AT

Proof : O - T - A [If two circles are touching circles then the common point lies on the line joining their centres] ½

y **OA = OT + AT** [$\because$ O - T - A] ½

(v) $\frac{\text{A (aABE)}}{\text{A (aABD)}} = \frac{\text{BE}}{\text{AD}}$ [Triangles with common base] ½



∴ $\frac{\text{A (aABE)}}{\text{A (aABD)}} = \frac{6}{9}$

∴ $\boxed{\frac{\text{A (aABE)}}{\text{A (aABD)}} = \frac{2}{3}}$ ½

(vi) A cylinder and cone have equal height and equal radii

y Volume of cone $= \frac{1}{3} \times$ volume of cylinder ½

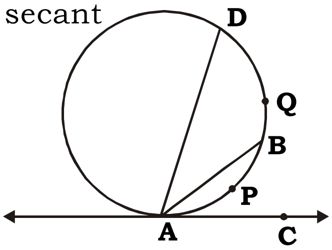
$= \frac{1}{3} \times 300$

$= 100 \text{ cm}^3$

y Volume of the cone is 100 cm^3. ½

A.2. Solve ANY FOUR of the following :

(i) m ĎBAC $= \frac{1}{2}$ m(arc APB) [Tangent secant theorem] ½



y m ĎBAC $= \frac{1}{2} \times 80$

y m ĎBAC = 40° ½

m ĎBAD $= \frac{1}{2}$ m (arc BQD) [Inscribed angle theorem] ½

y 30 $= \frac{1}{2}$ m (arc BQD)

y m (arc BQD) = 30 × 2

y m (arc BQD) = 60° ½

(ii) Mark point X as shown in the figure

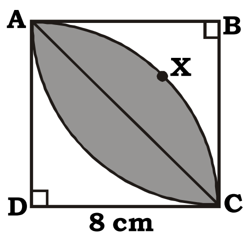
□ABCD is a square [Given]

side = 8 cm

Radius (r) = side of a square

y r = 8 cm

Measure of arc (̄) = 90° [Angle of a square] ½



Area of the segment AXC $= r^2 \left[\frac{\text{®}^-}{360} - \frac{\sin{}^-}{2}\right]$ ½

$= 8^2 \left[\frac{3.14 \times 90}{360} - \frac{\sin 90}{2}\right]$

$= 64\left[\frac{1.57}{2} - \frac{1}{2}\right]$

$= 64\left[\frac{1.57 - 1}{2}\right]$

$= \frac{64 \times 0.57}{2}$

$= \frac{36.48}{2}$ cm²

Area of shaded region $= 2 \times$ Area of segment AXC ½

$= 2 \times \frac{36.48}{2}$

$= 36.48$ cm²

∴ **Area of shaded region is 36.48 cm².** ½

(iii) $(37)^2 = 1369$ ......(i) ½

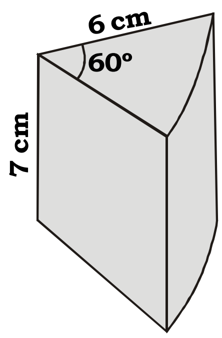
$(12)^2 + (35)^2 = 144 + 1225$

$= 1369$ ......(ii) ½

∴ $(37)^2 = (12)^2 + (35)^2$ [From (i) and (ii)]

∴ The given sides form a right angled triangle. [By Converse of Pythagoras theorem] **1**

(iv)



For a sector,

Measure of arc (θ) = 60°

Radius (r) = 6 cm

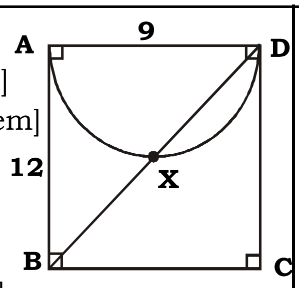
(a) Curved surface area of the cheese = Length of arc × height ½

$= \frac{\theta}{360} \times 2\pi r \times h$ ½

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6 \times 7$ ½

$= 44$ cm²

∴ **The curved surface area of the cheese is 44 cm².** ½

(v) In aABD 

m ĎBAD = 90° [Angle of a rectangle] ½

$BD^2 = AB^2 + AD^2$ [By pythagoras theorem]

y $BD^2 = 12^2 + 9^2$ [Given]

y $BD^2 = 144 + 81$

y $BD^2 = 225$

y BD = 15 cm [Taking square roots] ½

m ĎBAD = 90° [Angle of a rectangle]

∴ line BA is a tangent to the circle at point A

[A line perpendicular to the radius at its outer end is a tangent to the circle]

y Line AB is a tangent and line BXD is a secant intersecting at points X and D

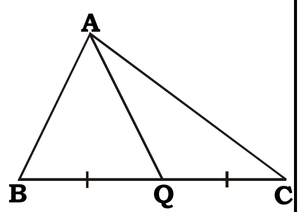
y $AB^2 = BX . BD$ [Tangent secant property] ½

y $12^2 = BX . 15$

y $144 = BX . 15$

y $BX = \frac{144}{15}$

y **BX = 9.6 cm** ½

(vi) In aABC, 

seg AQ is the median [Given]

y $BQ = QC = \frac{1}{2} \times BC$ ½

y $BQ = QC = \frac{1}{2} \times 10$ [Given]

y BQ = QC = 5 units ......(i) ½

$AB^2 + AC^2 = 2AQ^2 + 2BQ^2$ [By Appollonius theorem]

y $122 = 2AQ^2 + 2\,(5)^2$ [From (i) and given] ½

y $122 = 2AQ^2 + 2\,(25)$

y $122 = 2AQ^2 + 50$

y $2AQ^2 = 122 - 50$

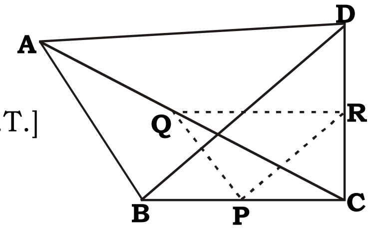
y $2AQ^2 = 72$

y $AQ^2 = 36$ [Taking square roots] ½

y **AQ = 6 units**

A.3. Solve ANY THREE of the following :

(i)



In aABC,
seg PQ || side AB [Given]

y $\frac{CP}{BP} = \frac{CQ}{AQ}$ ....(i) [By B.P.T.] **1**

In aBCD,
seg PR || side BD [Given]

y $\frac{CP}{BP} = \frac{CR}{DR}$ ....(ii) [By B.P.T.] **1**

In aACD,

y $\frac{CQ}{AQ} = \frac{CR}{DR}$ [From (i) and (ii)]

y **seg QR || side AD** [By converse of B.P.T.] **1**

(ii)

Curved surface area of the frustum of a cone = 180 cm²
Perimeters of circular bases are 18 cm and 6 cm

y $2®r_1 = 18$ ....(i) **½**

$2®r_2 = 6$ ....(ii) **½**

Adding (i) and (ii), we get

$2®r_1 + 2®r_2 = 18 + 6$

y $2® (r_1 + r_2) = 24$

y $® (r_1 + r_2) = \frac{24}{2}$

y $® (r_1 + r_2) = 12$ ....(iii) **1**

Curved surface area of the frustum of a cone = $®(r_1 + r_2)\ l$

y $180 = ®(r_1 + r_2)\ l$

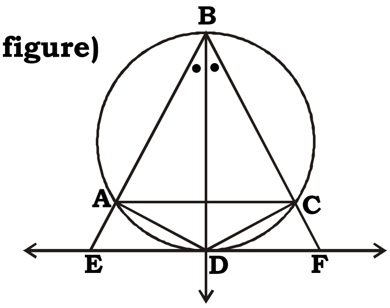
y $180 = 12 \times l$ [From (iii)] **1**

y $l = 15$ cm

y Slant height of the frustum of a cone is 15 cm.

(iii)

(1 mark for figure)

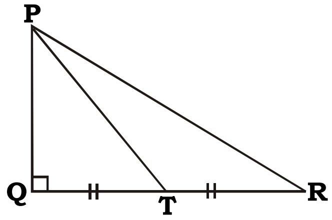


ĎEDA] ĎABD ....(i)
ĎFDC] ĎCBD ....(ii) } [Angles in alternate segment] **1**

But, ĎABD] ĎCBD ....(iii) [∵ Ray BD bisects ĎABC]

y **ĎEDA**] **ĎFDC** [From (i), (ii) and (iii)] **1**

(iv)



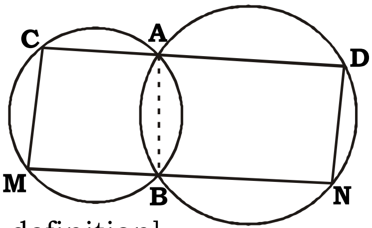
In $\triangle PQR$,
seg PT is the median [Given]

$\therefore PQ^2 + PR^2 = 2PT^2 + 2QT^2$ ......(i) [By Appollonius theorem] ½

In $\triangle PQT$,
$m\angle PQT = 90^\circ$ [Given]
$\therefore PT^2 = PQ^2 + QT^2$ [By Pythagoras theorem]
$\therefore QT^2 = PT^2 - PQ^2$ .....(ii) 1

$\therefore PQ^2 + PR^2 = 2PT^2 + 2(PT^2 - PQ^2)$ [From (i) and (ii)] ½
$\therefore PQ^2 + PR^2 = 2PT^2 + 2PT^2 - 2PQ^2$ ½
$\therefore PR^2 = 4PT^2 - 2PQ^2 - PQ^2$ ½
$\therefore$ **$PR^2 = 4PT^2 - 3PQ^2$**

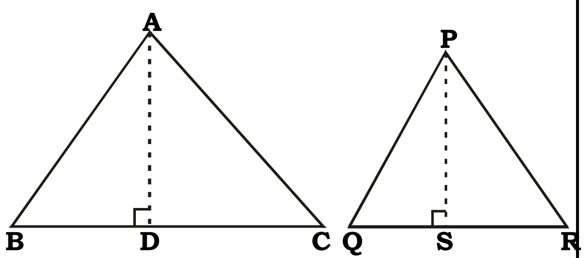
(v)



Construction : Draw seg AB.
Proof : $\square$ABMC is cyclic [By definition]
$\therefore m\angle MCA + m\angle MBA = 180^\circ$ ......(i) [Opposite angles of a cyclic quadrilateral are supplementary] 1
$\square$ABND is cyclic [By definition]
$\therefore \angle MBA \cong \angle ADN$ .....(ii) [The exterior angle of a cyclic quadrilateral is equal to its interior opposite angle] 1
$\therefore m\angle MCA + m\angle ADN = 180^\circ$ [From (i) and (ii)]
$\therefore m\angle MCD + m\angle CDN = 180^\circ$ [C - A - D]
$\therefore$ **seg CM || seg DN** [By Interior angles test] 1

A.4. **Solve ANY TWO of the following :**

(i)



Given : $\triangle ABC \sim \triangle PQR$.

To Prove : $\dfrac{A(\triangle ABC)}{A(\triangle PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$

Construction :

(i)Draw seg AD⊥{ side BC, B - D - C ½

(ii)Draw seg PS { side QR, Q - S - R

Proof : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC \times AD}{QR \times PS}$ [The ratio of the areas of two triangles is equal to ratio of the products of a base and its corresponding height] ½

$\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ ......(i)

aABC ~ aPQR [Given]

y $\frac{AB}{PQ} = \frac{BC}{QR}$ ......(ii) [c.s.s.t.] ½

Also, ĎB]≅ĎQ .....(iii) [c.a.s.t.] ½

In aADB and aPSQ,

ĎADB]≅ĎPSQ [Each is a right angle]

ĎB] ĎQ [From (ii)]

y aADB ~ aPSQ [By A-A test of similarity] ½

y $\frac{AD}{PS} = \frac{AB}{PQ}$ .....(iv) [c.s.s.t.] ½

y $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (i), (ii) and (iv)] ½

y $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB^2}{PQ^2}$ ....(vi)

Similarly we can prove

$\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ ....(vii)

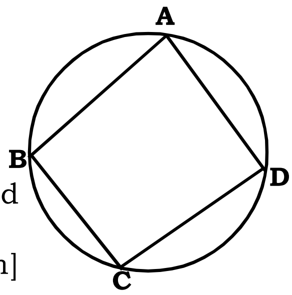
y $\mathbf{\frac{A(aABC)}{A(aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}}$ [From (vi) and (viii)] ½

(ii) **Given :** ☐ABCD is a cyclic

To Prove : m ĎABC + m ĎADC = 180° ½

m ĎBAD + m ĎBCD = 180°

Proof :



(½ mark for figure)

m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii)

} [Inscribed angle theorem] 1

Adding (i) and (ii), we get

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC)

y m ĎABC + m ĎADC = $\frac{1}{2}$[m (arc ADC) + m (arc ABC)] ½ y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y **m ĎABC + m ĎADC = 180° ....(iii)**

In ☐ABCD,

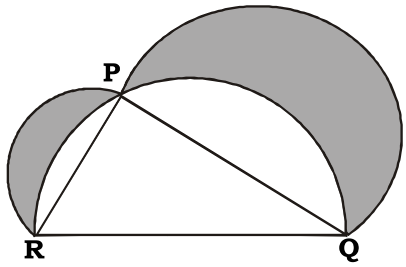
m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°

[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(iii)



Diameter PR = 6 units

y Its radius (r_1) = 3 units ½

Diameter PQ = 8 units

y Its radius (r_2) = 4 units

In aPQR,

m ĎRPQ = 90° ......(i) [Angle subtended by a semicircle] ½

$QR^2 = PR^2 + PQ^2$ [By Pythagoras theorem]

y $QR^2 = 6^2 + 8^2$

y $QR^2 = 36 + 64$

y QR = 100

y QR = 10 units [Taking square roots] ½

Diameter QR = 10 units

y Its radius (r_3) = 5 units

aPQR is a right angled triangle [From (i)]

A (aPQR) = $\frac{1}{2}$ × product of perpendicular sides

= $\frac{1}{2}$ × PR × PQ

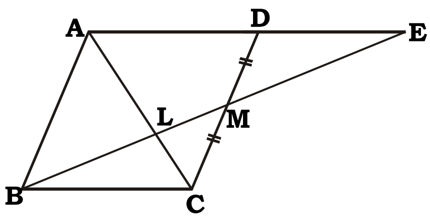
= $\frac{1}{2}$ × 6 × 8

= 24 sq. units. ½

Area of shaded portion = Area of semicircle with diameter PR + Area of semicircle with diameter PQ + Area of aPQR – Area of semicircle with diameter QR

= $\frac{1}{2}$®r_1^2 + $\frac{1}{2}$®r_2^2 + 24 – $\frac{1}{2}$®r_3^2 ½

= $\left(\frac{1}{2}\right.$®r_1^2 + $\frac{1}{2}$®r_2^2 – $\frac{1}{2}$®$\left.r_3^2\right)$ + 24

	$= \frac{1}{2}$® $(r_1^2 + r_2^2 - r_3^2) + 24$		½
	$= \frac{1}{2} \times 3.14\ (3^2 + 4^2 - 5^2) + 24$		
	$= \frac{1}{2} \times 3.14 \times (9 + 16 - 25) + 24$		½
	$= \frac{1}{2} \times 3.14\ (0) + 24$		
	$= 0 + 24$		
	$= 24$ sq. units		
	y Area of shaded portion is 24 sq.units		½
A.5. (i)	**Solve ANY TWO of the following :**		
			
	In aELA and aBLC,		
	ĎAEL]⁼ĎCBL	[From (i) and E - L - B]	
	ĎALE]⁼ĎCLB	[Vertically opposite angles]	
	y aELA ~ aBLC	[By AA test of similarity]	**1**
	y $\frac{EL}{BL} = \frac{EA}{BC}$ .....(i)	[c.s.s.t.]	½
	☐ABCD is a parallelogram	[Given]	
	y seg AD || seg BC	[By definition]	
	y seg AE || seg BC	[∵ A - D - E]	
	y On transversal BE,		
	ĎAEB]⁼ĎCBE .....(ii)	[Converse of alternate angles test]	½
	In aDME and aCMB,		
	side DM] side CM	[Given]	
	ĎDME]⁼ĎCMB	[Vertically opposite angles]	
	ĎDEM]⁼ĎCBM	[From (ii) and A - D - E, B - M - E]	
	y aDME]⁼aCMB	[By SAA test of congruence]	**1**
	y DE = BC .....(iii)	[c.s.c.t.]	
	But, AD = BC .....(iv)	[Opposite sides of a parallelogram]	
	y DE = BC = AD .....(v)	[From (iii) and (iv)]	
	y $\frac{EL}{BL} = \frac{ED + DA}{BC}$	[From (i) and E - D - A]	**1**

y $\frac{EL}{BL} = \frac{BC + BC}{BC}$ [From (v)] ½

y $\frac{EL}{BL} = \frac{2BC}{BC}$

y $\frac{EL}{BL} = \frac{2}{1}$

y **EL = 2BL** ½

(ii) Height of the cylindrical container (h) = 14cm

Its radius (r) = 6 cm

Volume of cylindrical container = ® r^2h ½

= ® × 6 × 6 × 14

= 504® cm^3 ½

But, volume of ink filled in the cylindrical container = 91% of 504 ®

= $\frac{91}{100}$ × 504® cm^3 ½

Length of ball pen refill (h_1) = 12m

its inner diameter = 2 mm

y Its radius (r_1) = 1 mm

= $\frac{1}{10}$ cm ½

Volume of the refill = ®r_1^2 h_1 ½

= ® × $\frac{1}{10}$ × $\frac{1}{10}$ × 12

= $\frac{12 ®}{100}$ cm^3 ½

y But, volume of ink filled = 84% of $\frac{12 ®}{100}$ ½

= $\frac{84}{100}$ × $\frac{12 ®}{100}$ cm^3

Number of refills that can be filled with ink

= $\frac{\text{Volume of ink filled in the cylindrical container}}{\text{Volume of ink filled in each refill}}$ ½

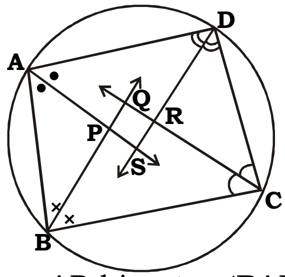
= $\frac{\frac{91 \times 504®}{100}}{\frac{84 \times 12®}{100 \times 100}}$ ½

= $\frac{91 \times 504 ®}{100} \times \frac{100 \times 100}{84 \times 12 ®}$

= 4550

y Number of refills that can be filled with this ink is 4550. ½

(iii)



Given : (i) □ABCD is cyclic.
(ii) Ray AP, ray BQ, ray CR and ray DS are the bisectors of ĎA, ĎB, ĎC and ĎD respectively. 1

To Prove : □PQRS is cyclic.

Proof : ĎDAP ≅ ∠BAP [∵ ray AP bisects ∠BAD]

Let, m ĎDAP = m ĎBAP = a° ....(i)

Similarly,

m ĎABP = m ĎCBP = b° ....(ii)

m ĎBCR = m ĎDCR = c° ....(iii)

m ĎADR = m ĎCDR = d° ....(iv) ½

In aBQC,

m ĎBQC + m ĎQBC + m ĎQCB = 180° [∵ Sum of the measures of angles of a triangle is 180°]

y m ĎBQC + b + c = 180 [From (ii) and (iii)]

y m ĎBQC = (180 – b – c)°

y m ĎPQR = (180 – b – c)° ....(v) ½

[∵ B - P - Q and C - R - Q]

Similarly, we can prove

m ĎPSR = (180 - a - d)° ......(vi) ½

Adding (v) and (vi),

m ĎPQR + m ĎPSR = 180 – b – c + 180 – a – d

y m ĎPQR + m ĎPSR = 360 – a – b – c – d

y m ĎPQR + m ĎPSR = 360 – (a + b + c + d) .....(vii) ½

In □ABCD,

m ĎBAD + m ĎABC + m ĎBCD + m ĎADC = 360°

[∵ Sum of the measures of angles of a quadrilateral is 360°]

m ĎBAP + m ĎDAP + m ĎABP + m ĎPBC + m ĎBCQ + m ĎDCQ + m ĎCDR + m ĎADR = 360 ½

[Angle addition property]

y a + a + b + b + c + c + d + d = 360

[From (i), (ii), (iii) and (iv)]

y 2a + 2b + 2c + 2d = 360

y 2 (a + b + c + d) = 360 ½

y a + b + c + d = 180 ......(viii)

y m ĎPQR + m ĎPSR = 360 – 180

[From (vii) and (viii)] ½

y m ĎPQR + m ĎPSR = 180°

y **□PQRS is cyclic.** [If opposite angles of a quadrilateral are supplementary, then quadrilateral is cyclic] ½

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