MT

2017 ___ ___ 1100

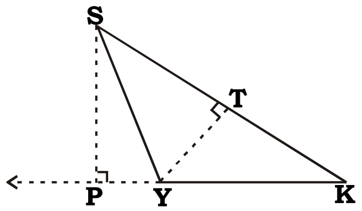
MT - GEOMETRY - SEMI PRELIM - II : PAPER - 6

Time : 2 Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) $\frac{A\ (aSYK)}{A\ (aYTK)} = \frac{YK \times SP}{TK \times YT}$ [The ratio of the areas of two triangles is equal to the ratio of the products of their bases and corresponding heights] ½

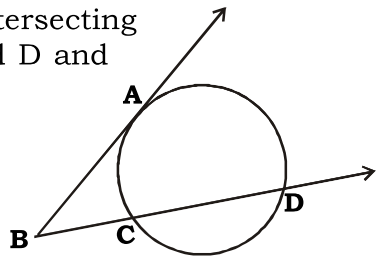
y $\frac{A\ (aSYK)}{A\ (aYTK)} = \frac{13 \times 6}{12 \times 5}$ [Given]



y $\frac{A\ (aSYK)}{A\ (aYTK)} = \frac{13}{10}$

y $\boxed{A\ (aSYK) : A\ (aYTK) = 13 : 10}$ ½

(ii) Line BCD is a secant intersecting the circle at points C and D and line BA is a tangent at A ½



y $AB^2 = BC \times BD$

y $6^2 = 4 \times BD$

y $36 = 4 \times BD$

y $BD = \frac{36}{4}$

y $\boxed{BD = 9 \text{ units}}$ ½

(iii) F + V = E + 2 ½

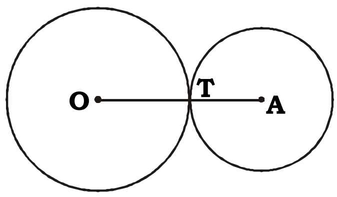
y F + 6 = 12 + 2

y F + 6 = 14

y F = 14 – 6

y F = 8 ½

(iv) **Given :** Two circles with centres O and A touch each other externally at point T.

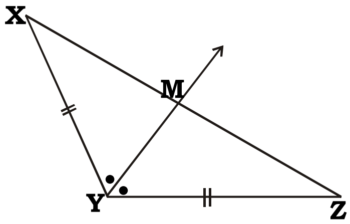


To Prove : OA = OT + AT

Proof : O - T - A [If two circles are touching circles then the common point lies on the line joining their centres] ½

y **OA = OT + AT** [∵ O - T - A] ½

(v)



In ΔXYZ,
ray YM bisects $\angle XYZ$ [Given]

$\therefore \dfrac{XY}{YZ} = \dfrac{XM}{MZ}$ [Property of angle bisector of a triangle] ½

$\therefore 1 = \dfrac{XM}{MZ}$ [$\because$ XY = YZ]

$\therefore$ **XM = MZ** ½

(vi) Volume of cuboid = Volume of cube [Given] ½

y $3 \times 9 \times x = (6)^3$

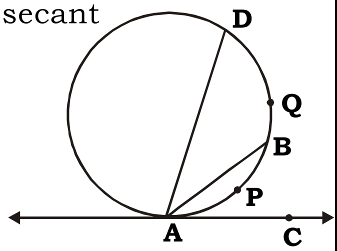
y $3 \times 9 \times x = 6 \times 6 \times 6$

y $x = \dfrac{6 \times 6 \times 6}{3 \times 9}$

y $\boxed{x = 8}$ ½

A.2. Solve ANY FOUR of the following :

(i) m ĎBAC $= \dfrac{1}{2}$ m(arc APB) [Tangent secant theorem] ½



y m ĎBAC $= \dfrac{1}{2} \times 80$

y $\boxed{\text{m ĎBAC} = 40^\circ}$ ½

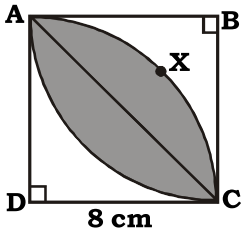
m ĎBAD $= \dfrac{1}{2}$ m (arc BQD) [Inscribed angle theorem] ½

y $30 = \dfrac{1}{2}$ m (arc BQD)

y m (arc BQD) = 30×2

y $\boxed{\text{m (arc BQD)} = 60^\circ}$ ½

(ii) Mark point X as shown in the figure



☐ABCD is a square [Given]
side = 8 cm
Radius (r) = side of a square
y r = 8 cm
Measure of arc (‾) = 90° [Angle of a square] ½

Area of the segment AXC $= r^2 \left[\frac{®¯}{360} - \frac{\sin ¯}{2}\right]$ ½

$= 8^2 \left[\frac{3.14 \times 90}{360} - \frac{\sin 90}{2}\right]$

$= 64 \left[\frac{1.57}{2} - \frac{1}{2}\right]$

$= 64 \left[\frac{1.57 - 1}{2}\right]$

$= \frac{64 \times 0.57}{2}$

$= \frac{36.48}{2}$ cm²

Area of shaded region = 2 × Area of segment AXC ½

$= 2 \times \frac{36.48}{2}$

= 36.48 cm²

y Area of shaded region is 36.48 cm². ½

(iii) In aABC and aDAC,

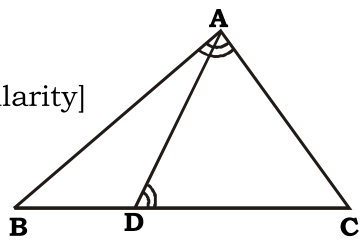
ĎBAC] ĎADC [Given]

ĎACB] ĎACD [Common angle] 1

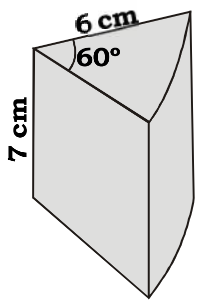
y aABC ~ aDAC [By AA test of similarity] ½

y $\frac{AC}{DC} = \frac{BC}{AC}$ [c.s.s.t.]

y **AC² = BC × DC** ½



(iv)



For a sector,

Measure of arc (¯) = 60°

Radius (r) = 6 cm

(a) Curved surface area of the cheese = Length of arc × height ½

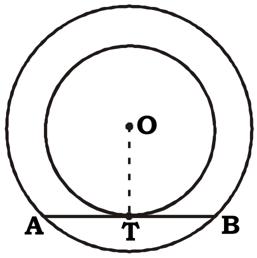
$= \frac{¯}{360} \times 2®r \times h$ ½

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6 \times 7$ ½

$= 44 \text{ cm}^2$

y The curved surface area of the cheese is 44 cm². ½

(v)



Construction : Let O be the centre of the concentric circles and draw seg OT. ½

Proof : Line AB is tangent to the smaller circle at T. [Given]

y seg OT { line AB ....(i) [Radius is perpendicular to the tangent] ½

With respect to the bigger circle,

seg OT { chord AB [From (i)]

y AT = BT [Perpendicular from the centre of a circle to a chord bisects the chord]

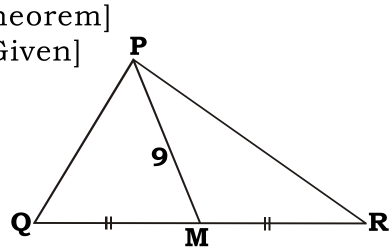
y **T is the mid-point of seg AB.** 1

(vi) In aPQR,

seg PM is the median [Given]

y $PQ^2 + PR^2 = 2PM^2 + 2QM^2$ [By Appollonius theorem] ½

y $290 = 2(9)^2 + 2QM^2$ [Given]



y $290 = 2(81) + 2QM^2$

y $290 = 162 + 2QM^2$ ½

y $290 - 162 = 2QM^2$

y $128 = 2QM^2$

y $QM^2 = \frac{128}{2}$

y $QM^2 = 64$

y QM = 8 units [Taking square roots] ½

y $QM = \frac{1}{2} QR$ [∵ M is midpoint of side QR]

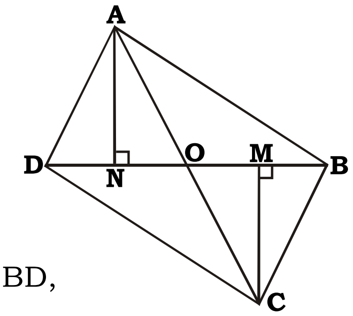
y $8 = \frac{1}{2} QR$

y $8 \times 2 = QR$

y QR = 16 units ½

A.3. **Solve ANY THREE of the following :**

(i)



aADB and aCDB have a common base BD,

y $\frac{A(\Delta ADB)}{A(\Delta CDB)} = \frac{AN}{CM}$ ....(i) [Triangles with common base] **1**

In aANO and aCMO,

ĎANO] ĎCMO [∵ Each is 90°]

ĎAON] ĎCOM [Vertically opposite angles]

y aANO ~ aCMO [By AA test of similarity] **1**

y $\frac{AN}{CM} = \frac{AO}{CO}$ ......(ii) [c.s.s.t.] **½**

y $\mathbf{\frac{A\ (aADB)}{A\ (aCDB)} = \frac{AO}{CO}}$ [From (i) and (ii)] **½**

(ii) Diameters of circular ends of frustum are 18 cm and 8 cm

y $r_1 = \frac{18}{2} = 9$ cm and $r_2 = \frac{8}{2} = 4$ cm

Slant height (l) = 13 cm

Curved surface area of frustum of frustum = ® ($r_1 + r_2$) l

= ® (9 + 4) × 13

= ® × 13 × 13

= 169® cm² **1**

Radius of a cylinder (r_2) = 4 cm

Its height (h) = 10 cm

Curved surface area of a cylinder = 2®rh

= 2 × ® × 4 × 10

= 80® cm² **1**

Surface area of tin required to make the funnel

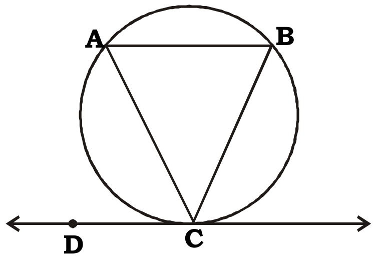
= Curved surface area of frustum + curved surface area of cylinder

= 169® + 80® **½**

= 249 ®cm²

y The surface area of the tin required to make the funnel is 249 cm². **½**

(iii)



Take a point D on the tangent at C as shown in the figure.

seg AB || line CD. [Given]

y On transversal AC,

ĎBAC] ĎACD ....(i) [Converse of alternate angles test] **1**

But, ĎACD] ĎABC ....(ii) [Angles in alternate segment] **1**

In aABC,

ĎBAC] ĎABC [From (i) and (ii)] ½

y seg AC] seg BC [Converse of Isosceles triangle theorem] ½

y **AC = BC**

(iv) In aABC,

AB = 5 units

BC = 6 units [Given]

AC = 7 units

Perimeter of aPQR = 360 units [Given]

y PQ + QR + PR = 360 ......(i) ½

aABC ~ aPQR [Given]

y $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ [c.s.s.t.] ½

y $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR}$

y $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{5 + 6 + 7}{PQ + QR + PR}$ [By theorem on equal ratios] ½

y $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{18}{360}$ [From (i)]

y $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{1}{20}$ .....(ii)

$\frac{5}{PQ} = \frac{1}{20}$ [From (ii)]

y PQ = 100 units ½

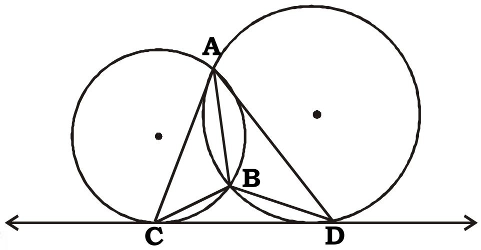
$\frac{6}{QR} = \frac{1}{20}$ [From (ii)]

y QR = 120 units ½

$\frac{7}{PR} = \frac{1}{20}$ [From (ii)]

y PR = 140 units ½

(iii)



Proof :

m ĎBAC = m ĎBCD ......(i) ⎫ [Angles in alternate segments] ½

m ĎBAD = m ∠BDC ......(ii) ⎭ ½

In aBCD,

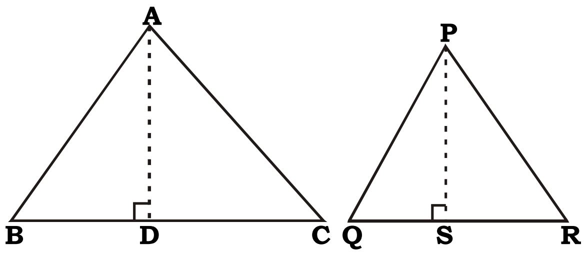
m ĎBCD + m ĎBDC + m ĎCBD = 180° [Sum of the measures of angles of a triangle is 180°] ½

y m ĎBAC + m ĎBAD + m ĎCBD = 180° [From (i) and (ii)] ½

y **m ĎCAD + m ĎCBD = 180°** [Angle addition property] **1**

A.4. **Solve ANY TWO of the following :**

(i)



Given : aABC ~ aPQR.

To Prove : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Construction :

(i)Draw seg AD⁼{ side BC, B - D - C ½

(ii)Draw seg PS { side QR, Q - S - R

Proof : $\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC \times AD}{QR \times PS}$ [The ratio of the areas of two triangles is equal to ratio of the products of a base and its corresponding height] ½

$\frac{A\ (aABC)}{A\ (aPQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ ......(i)

aABC ~ aPQR [Given]

y $\frac{AB}{PQ} = \frac{BC}{QR}$ ......(ii) [c.s.s.t.] ½

Also, ĎB]⁼ĎQ .....(iii) [c.a.s.t.] ½

In aADB and aPSQ,

ĎADB]⁼ĎPSQ [Each is a right angle]

ĎB] ĎQ [From (ii)]

y aADB ~ aPSQ [By A-A test of similarity] ½

y $\frac{AD}{PS} = \frac{AB}{PQ}$ .....(iv) [c.s.s.t.] ½

y $\frac{\text{A (aABC)}}{\text{A (aPQR)}} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (i), (ii) and (iv)] ½

y $\frac{\text{A (aABC)}}{\text{A (aPQR)}} = \frac{AB^2}{PQ^2}$ ....(vi)

Similarly we can prove

$\frac{\text{A (aABC)}}{\text{A (aPQR)}} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ ....(vii)

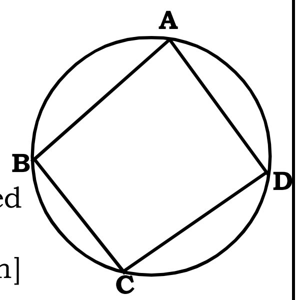
y $\mathbf{\frac{A(aABC)}{A(aPQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}}$ [From (vi) and (viii)] ½

(ii) **Given :** □ABCD is a cyclic

To Prove : m ĎABC + m ĎADC = 180° ½

m ĎBAD + m ĎBCD = 180°

Proof :



(½ mark for figure)

m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii)

} [Inscribed angle theorem] 1

Adding (i) and (ii), we get

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC)

y m ĎABC + m ĎADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)] ½

y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y **m ĎABC + m ĎADC = 180° ....(iii)**

In □ABCD,

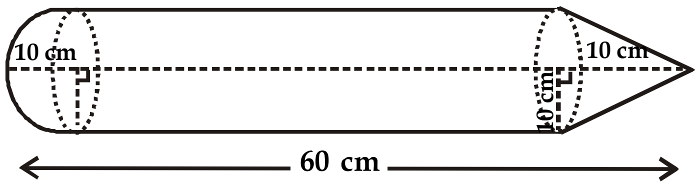
m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°

[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(iii)



A toy is a combination of cylinder, hemisphere and cone, each with radius 10 cm ½

y r = 10 cm

y Height of the conical part (h) = 10 cm

Height of the hemispherical part = its radius = 10cm

Total height of the toy = 60cm

y Height of the cylindrical part (h_1) = 60 – 10 – 10

= 60 – 20

= 40 cm 1

l^2 = $r^2 + h^2$

y l^2 = $10^2 + 10^2$

y l^2 = 100 + 100

l^2 = 200

y l = $\sqrt{200}$

[Taking square roots]

l = $10\sqrt{2}$ cm ½

Slant height of the conical part (l) = $10\sqrt{2}$

= 10 × 1.41

= 14.1 cm ½

Total surface area of the toy

= Curved surface area of the conical part + Curved surface area of the cylindrical part + Curved surface area of the hemispherical part ½

= ®rl + 2®rh_1 + 2®r^2

= ®r (l + 2h_1 + 2r) ½

= 3.14 × 10 (14.1 + 2 × 40 + 2 × 10)

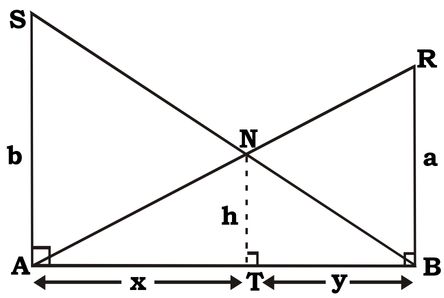
= 31.4 (14.1 + 80 + 20)

= 31.4 × 114.1

= 3582.74 cm^2

y Total surface area of the toy is 3582.74 cm^2. ½

A.5. (i)	**Solve ANY TWO of the following :**		



	AB = AT + TB		[∵ A - T - B]	
y	AB = (x + y) = p	……(i)		½
	In aATN and aABR,			
	ĎTAN ≅ ĎBAR		[Common angle]	
	ĎATN] ĎABR		[∵ each is 90°]	
y	aATN ¼ aABR		[By AA test of similarity]	½
y	$\frac{AT}{AB} = \frac{TN}{BR}$		[c.s.s.t]	
y	$\frac{x}{p} = \frac{h}{a}$	……(ii)	[Given and from (i)]	½
	In aBTN and aBAS,			
	∠TBN ≅ ĎABS		[Common angle]	
	ĎBTN] ĎBAS		[∵ each is 90°]	
y	aBTN ¼ aBAS		[By AA test of similarity]	½
y	$\frac{BT}{AB} = \frac{TN}{AS}$		[c.s.s.t]	
y	$\frac{y}{p} = \frac{h}{b}$	……(iii)	[Given and from (i)]	½
	Adding (ii) and (iii),			
	$\frac{x}{p} + \frac{y}{p} = \frac{h}{a} + \frac{h}{b}$			½
y	$\frac{x + y}{p} = h\left(\frac{1}{a} + \frac{1}{b}\right)$			½
y	$\frac{p}{p} = h\left(\frac{a + b}{ab}\right)$		[From (i)]	½
y	$1 = h\left(\frac{a + b}{ab}\right)$			
y	$\frac{ab}{a + b} = h$			
y	$\mathbf{h = \frac{ab}{a+b}}$ **metres**			1

(ii) Diameter of marble = 1.4 cm

y its radius (r) = $\frac{1.4}{2}$

= 0.7 cm ½

Volume of a marble = $\frac{4}{3}$®r^3 ½

= $\frac{4}{3}$ × ® × $\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$ cm^3 ½

∵ Marbles are submerged fully in the water, water level rises by 5.6 cm

y Height of water displaced (h) = 5.6 cm

Diameter of beaker = 7 cm

y Its radius (r_1) = $\frac{7}{2}$ cm

Volume of water displaced = ®r_1^2h ½

= ® × $\frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$ cm^3 ½

Number of marbles = $\frac{\text{Volume of water displaced}}{\text{Volume of marble}}$ ½

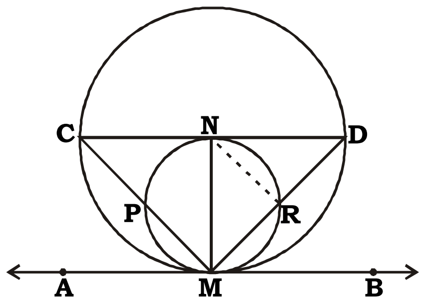
= ® × $\frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$ ÷ ($\frac{4}{3}$ × ® × $\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$) 1

= ® × $\frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} \times \frac{3}{4} \times \frac{1}{®} \times \frac{10}{7} \times \frac{10}{7} \times \frac{10}{7}$ 1

= 150

y **Number of marbles is 150.**

(iii)



Construction : Draw seg NR.

Proof :

ĎCMA] ∠CDM [Angles in alternate segments] ½

Let,

m ĎCMA= mĎCDM = x° ....(i)

ĎNMA] ĎNRM [Angles in alternate segments] ½

Let,
mĎNMA] ĎNRM = y° ....(ii)
m ĎNMC = m ĎNMA – m ĎCMA [Angle Addition property]
y m ĎNMC = (y – x)° ......(iii) [From (i) and (ii)] **1**
ĎNMR] ĎDNR ....(iv) [Angles in alternate segment]
ĎNRM is an exterior angle of a NDR
y m ĎNRM = m ĎNDR + mĎDNR [Remote interior angles] **1**
y m ĎNRM = ĎCDM + m ĎDNR [∵ C - N - D and D - R - M]
y y = x + m ĎDNR [From (i) and (ii)] **1**
y m ĎDNR = (y – x)° ......(v)
y m ĎNMR = (y – x)° [From (iv) and (v)]
y m ĎNMD = (y – x)° ......(vi) [D - R - M] **1**
y Ď**CMN**] Ď**DMN** [From (iii) and (vi)]

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