MT

2017____ ____ 1100 **Seat No.** | | | | | | | |

MT- GENERAL MATHEMATICS (71)GEOMETRY- SEMI PRELIM II- PAPER- I (E)

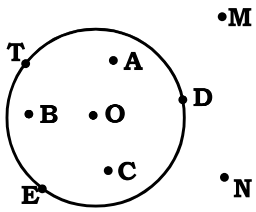
Time : 2½ Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) Diameter of the circle = 2 × radius
= 2 × 18
= $\boxed{36 \text{ cm}}$ **1**

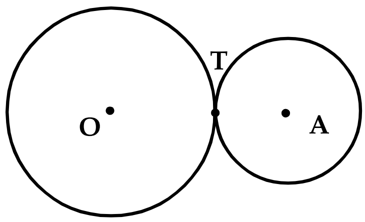
(ii) Class mark $= \dfrac{\text{Lower limit} + \text{Upper limit}}{2}$
$= \dfrac{30 + 34}{2}$ ½
$= \boxed{32}$ ½

(iii) Points in the interior part is O, A, B, C.
Points in the exterior points is M, N. ½
Points on the circle is T, E, D. ½



(iv) $\sin 60^\circ \times \cos 30^\circ = \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2}$
$= \boxed{\dfrac{3}{4}}$ **1**

(v) Two circles are touching externally at point T.

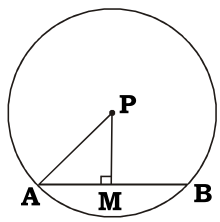


y PQ = PT + QT [P – T – Q] ½
= 8 + 5
= 13
y $\boxed{PQ = 13 \text{ cm}}$ ½

(vi) Exclusive form (Continuous form)
10.5 - 15.5
15.5 - 20.5
20.5 - 25.5
25.5 - 30.5 **1**

A.2. Solve ANY FOUR of the following :

(i) Let P be the centre and AB be the chord of the circle.



Draw seg PM ⊥ seg AB, A - M - B, ½

y AM $= \frac{1}{2}$ AB [Perpendicular drawn from the centre of the circle to the chord bisects the chord] ½

$= \frac{1}{2} \times 10$

$= 5$

∴ AM = 5 cm

In right angled ΔPMA

$AM^2 + PM^2 = AP^2$ [By Pythagoras theorem] ½

$5^2 + PM^2 = 13^2$

$25 + PM^2 = 169$

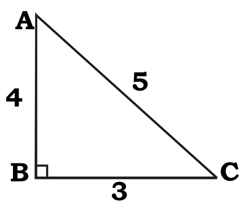
$PM^2 = 169 - 25$

$PM^2 = 144$

PM = 12 ½

y The distance of the chord from the centre of the circle is 12 cm.

(ii)



$\sin A = \frac{BC}{AC} = \frac{3}{5}$

$\cos A = \frac{AB}{AC} = \frac{4}{5}$

$\tan A = \frac{BC}{AB} = \frac{3}{4}$ 1

$\cot A = \frac{AB}{BC} = \frac{4}{3}$

$\text{cosec } A = \frac{AC}{BC} = \frac{5}{3}$

$\sec A = \frac{AC}{AB} = \frac{5}{4}$ 1

(iii) Let S be the sample space

y S = { H, T } ½

y n (S) = 2

Let A be the event that neither head nor tail turns up

y A = { } ½

y n (A) = 0

y $P(A) = \frac{n(A)}{n(S)}$ ½

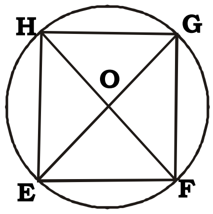
y $P(A) = \frac{0}{2}$

y $\boxed{P(A) = 0}$ ½

(iv) (a) Radii : seg OH, seg OE, seg OG, seg OF ½

(b) Diameter : seg HF, seg EG ½

(c) Chords : seg HG, seg HE, seg EF, seg GF, seg HF, seg EG 1



(v) L.H.S. = sin 2A

= sin 2(30°) ½

= sin 60°

$= \frac{\sqrt{3}}{2}$ ½

R.H.S. = 2sin A × cos A

= 2 (sin30°) × cos30°

$= 2\left(\frac{1}{2}\right) \times \frac{\sqrt{3}}{2}$ ½

$= 1 \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$ ½

y **sin 2A = 2sin A × cos A**

(vi) Class size is 2 1

Classes

151 - 153

153 - 155

155 - 157

157 - 159 1

A.3. Solve ANY THREE of the following :

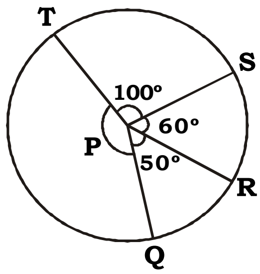
(i) (a) m (arc QRS) = m (arc QR) + m (arc RS) ½

[Addition of arcs]

= ĎQPR + ĎRPS

[Central angle]

= 50 + 60



m (arc QRS) = 110° ½

(b) m (arc QST) = m (arc QRS) + m (arc CST) ½

[Addition of arcs]

= ĎQPS + ĎSPT [Central angle]

= 110 + 100

m (arc QST) = 210° ½

(c) m (arc RTS) = 360 – m (arc SR) ½

= 360 – 60

m (arc RTS) = 300° ½

(ii) cosec A = $\frac{3}{\sqrt{2}}$

sin A = $\frac{1}{\text{cosec A}}$ **1**

y sin A = 1 ÷ cosec A

y sin A = 1 ÷ $\frac{3}{\sqrt{2}}$ **1**

y sin A = 1 × $\frac{\sqrt{2}}{3}$

y sin A = $\frac{\sqrt{2}}{3}$ **1**

(iii) Class width (h) = 10

Money (in Rs.)	**Class Mark (x_i)**	**No. of students (f_i)**	$f_i x_i$
0 - 10	5	5	25
10 - 20	15	7	105
20 - 30	25	5	125
30 - 40	35	2	70
40 - 50	45	6	270
Total		**25**	**595**

1

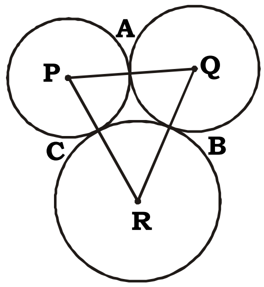
Mean $= \frac{p\ f_i x_i}{p\ f_i}$ 1

y Mean $= \frac{595}{25}$

y Mean = Rs. 23.8

y Mean of money collected is Rs. 23.8 1

(iv) Let r_1, r_2 and r_3 be the radius of if the circles with centres P, Q, R respectively. ½



PQ = PA + AQ = $r_1 + r_2 = 3$ ......(i)
QR = QB + BR = $r_2 + r_3 = 5$ .....(ii)
PR = PC + CR = $r_1 + r_3 = 4$ .....(iii)
By addition equations (i), (ii) and (iii)
$2r_1 + 2r_2 + 2r_3 = 12$
y $2(r_1 + r_2 + r_3) = 12$
y $r_1 + r_2 + r_3 = 6$ ......(iv)
By subtracting equation (i) from (iv)
$r_1 + r_2 + r_3 - (r_1 + r_2) = (6 - 3)$ ½
y $r_3 = 3$
By subtracting equation (ii) from (iv)
$r_1 + r_2 + r_3 - (r_2 + r_3) = (6 - 5)$ ½
y $r_1 = 1$ ½
By subtracting equation (iii) from (iv)
$r_1 + r_2 + r_3 - (r_1 + r_3) = (6 - 4)$ ½
y $r_2 = 2$

y Radii of the circles with centre P, Q, R are 1, 2 and 3 respectively. ½

(v) Let S be the sample space
y S = { 12, 13, 21, 23, 31, 32 } ½
y n (S) = 6
(a) Let A be the event of getting even number
y A = { 12, 32 } ½
y n (A) = 2

P (A) $= \frac{n(A)}{n(S)}$

y P (A) $= \frac{2}{6}$

y P (A) $= \frac{1}{3}$ ½

(b) Let B be the event of getting prime number

B = { 13, 23, 31 }

y n (B) = 3

P (B) = $\frac{n(B)}{n(S)}$

y P (B) = $\frac{3}{6}$

y **P (B) = $\frac{1}{2}$** ½

(c) Let C be the event of getting number greater than 30

y C = { 31, 32 } ½

y n (C) = 2

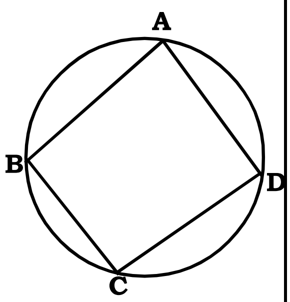
P (C) = $\frac{n(C)}{n(S)}$

y P (C) = $\frac{2}{6}$

y **P (C) = $\frac{1}{3}$** ½

A.4. **Solve ANY TWO of the following :**

(i)



Given : □ABCD is a cyclic

To Prove : m ĎABC + m ĎADC = 180° 1

m ĎBAD + m ĎBCD = 180°

Proof : m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii)

} [Inscribed angle theorem] 1

Adding (i) and (ii), we get

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC) ½

y m ĎABC + m ĎADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)]

y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y m ĎABC + m ĎADC = 180° ....(iii)

In □ABCD,

m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°

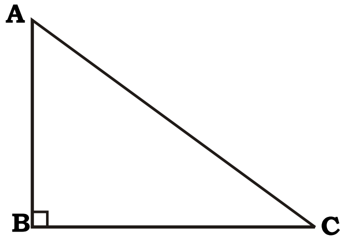
[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(ii)

$16 \cot A = 12$



$\cot A = \frac{12}{16}$

∴ $\cot A = \frac{3}{4}$ ½

In ΔABC,

$\angle ABC = 90^\circ$

$\cot A = \frac{3}{4}$ [Given]

∴ $\cot A = \frac{AB}{BC}$ [By definition]

∴ $\frac{AB}{BC} = \frac{3}{4}$ ½

Let the common multiple be k.

∴ $AB = 3k, BC = 4k$ [k is constant, k > 0]

$AC^2 = AB^2 + BC^2$ [By pythagoras theorem] ½

y $AC^2 = (3k)^2 + (4k)^2$

y $AC^2 = 9k^2 + 16k^2$

y $AC^2 = 25k^2$

y $AC = \sqrt{25k^2}$

y $AC = 5k$

$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$

$\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$ **1**

y $\frac{\sin A + \cos A}{\sin A - \cos A} = \frac{\frac{4}{5} + \frac{3}{5}}{\frac{4}{5} - \frac{3}{5}}$ ½

$= \frac{\frac{7}{5}}{\frac{1}{5}}$

$= \frac{7}{5} \div \frac{1}{5}$ ½

$= \frac{7}{5} \times \frac{5}{1}$

$= 7$

y $\boxed{\frac{\sin A + \cos A}{\sin A - \cos A} = 7}$ ½

(iii)

Size of form (in acres)	Frequency (f_i) (No. of farms)	Cumulative frequency less than type
5 - 15	7	7
15 - 25	12	19
25 - 35	17	36 ì *c.f.*
35 - 45	25 ì *f*	61 ← Median Class
45 - 55	31	92
55 - 65	5	97
65 - 75	3	100
Total	**100 ì N**	

1

½

Here total frequency = pf_i = N = 100

y $\frac{N}{2} = \frac{100}{2} = 50$

Cumulative frequency (less than type) which is just greater than 50 is 61. Therefore corresponding class 35 - 45 is median class.

L = 35, N = 100, *c.f.* = 36, f = 25, h = 10 ½

Median $= L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$ 1

$= 35 + \left(\frac{100}{2} - 36\right)\frac{10}{25}$ ½

$= 35 + (50 - 36)\frac{10}{25}$

$= 35 + (14)\frac{10}{25}$

$= 35 + \frac{140}{25}$

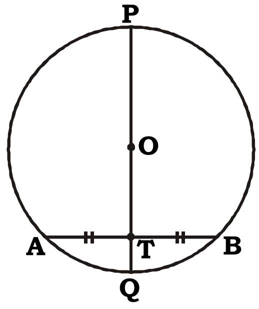
$= 35 + 5.6$

$= 40.6$

y **Median of size of farm is 40.6 acres.** ½

A.5. Solve ANY TWO of the following :

(i)



Draw seg OB and seg PB

radius = $\frac{1}{2}$ × diameter

y OP = OQ = OB = $\frac{1}{2}$PQ = $\frac{1}{2}$ × 30 ½

y OP = OQ = OB = 15

	AT = TB	[$\because$ T is the midpoint of seg AB]	
y	seg OT $\perp$ chord AB	[Segment joining the centre of the circle to the midpoint of the chord is perpendicular to the chord)	½
	In right angled ΔOTB,		
	$OB^2 = OT^2 + TB^2$	[By Pythagoras theorem]	½
	$(15)^2 = (9)^2 + (TB)^2$		
y	$225 = 81 + TB^2$		
y	$225 - 81 = TB^2$		
y	$144 = TB^2$		
y	TB = 12 units	[Taking square roots]	½
	$TB = \frac{1}{2}AB$	[$\because$ T is the midpoint of seg AB]	
y	$12 = \frac{1}{2}AB$		
y	$\boxed{AB = 24 \text{ units}}$		½
	PT = PO + OQ	[P - O - T]	
y	PT = 15 + 9		
y	$\boxed{PT = 24 \text{ units}}$		½
	In right angled ΔPTB,		
	$PB^2 = PT^2 + TB^2$	[By Pythagoras theorem]	
y	$PB^2 = (24)^2 + (12)^2$		
y	$PB^2 = 576 + 144$		
y	$PB^2 = 720$		
y	$PB = \sqrt{720}$		
y	$PB = \sqrt{144 \times 5}$		
$\therefore$	$\boxed{PB = 12\sqrt{5} \text{ units}}$		1
	OQ = OT + TQ	[O - T - Q]	
y	15 = 9 + TQ		
y	15 – 9 = TQ		
$\therefore$	TQ = 6 units		
	In right angled ΔTBQ,		
	$BQ^2 = TQ^2 + TB^2$	[By Pythagoras theorem]	
y	$BQ^2 = (6)^2 + (12)^2$		
y	$BQ^2 = 36 + 144$		
y	$BQ^2 = 180$		
y	$BQ = \sqrt{180}$		
y	$BQ = \sqrt{36 \times 5}$		
y	$\boxed{BQ = 6\sqrt{5} \text{ units}}$		1

(ii) Let S be the sample space

y S = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), ½
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

y n (S) = 36 ½

(a) A is the event that sum of numbers on dice is atleast 11

y A = { (5, 6), (6, 5), (6, 6) }

y n (A) = 3 ½

$$P(A) = \frac{n(A)}{n(S)}$$

y $P(A) = \frac{3}{36}$

y $\boxed{P(A) = \frac{1}{12}}$ ½

(b) B is the event that sum of the numbers a dice is divisible by 9

B = { (3, 6), (4, 5), (5, 4), (6, 3) } ½

y n (B) = 4

$$P(B) = \frac{n(B)}{n(S)}$$

y $P(B) = \frac{4}{36}$

y $\boxed{P(B) = \frac{1}{9}}$ ½

(c) C is the event that number on upper face of first die is greater than the number or upper face of second die

y C = { (2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), ½
(5, 4), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) }

y n (C) = 15

$$P(C) = \frac{n(C)}{n(S)}$$

y $P(C) = \frac{15}{36}$

y $\boxed{P(C) = \frac{5}{12}}$ ½

(d) D is the event that sum of numbers on their upper faces is a perfect square

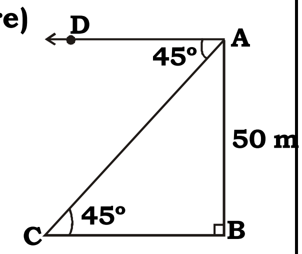
y D = { (1, 3), (2, 2), (3, 1), (3, 6), (4, 5), (5, 4), (6, 3) } ½

y n (D) = 7

y $P(D) = \frac{n(D)}{n(S)}$

y $\boxed{P(D) = \frac{7}{36}}$ ½

(iii) **(1 mark for figure)**



seg AB represents the height of the tower. ½

AB = 50 m ½

C is the position of the car.

$\angle$DAC is the angle of depression ½

m$\angle$DAC = 45° ½

$\angle$DAC = $\angle$ACB [Converse of alternate angles test] ½

∴ m$\angle$ACB = 45° ½

In right angled ΔABC

$\tan 45° = \frac{AB}{BC}$ [By definition] 1

$1 = \frac{50}{BC}$

∴ BC = 50 m

∴ Distance of the car from the tower is 50 m. 1

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