MT

2017____ ____ 1100 **Seat No.** [ ][ ][ ][ ][ ][ ][ ]

MT- GENERAL MATHEMATICS (71)GEOMETRY- SEMI PRELIM II- PAPER- II (E)

Time : 2½ Hours **Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) Radius of the circle $= \frac{1}{2} \times$ diameter ½

$= \frac{1}{2} \times 10$

$= \boxed{5 \text{ cm}}$ ½

(ii) Class mark $= \dfrac{\text{Lower limit} + \text{Upper limit}}{2}$ ½

$= \dfrac{60 + 65}{2}$

$= \dfrac{125}{2}$

$= \boxed{62.5}$ ½

(iii) O is the centre of the circle

radius = 7 cm

OP = 4.5 cm

y OP < radius ½

y Point P lies in the interior of the circle. ½

(iv) $\sin^2 90° - \tan^2 45°$

$= (1)^2 - (1)^2$ ½

$= 1 - 1$

$= 0$ ½

y $\boxed{\sin^2 90° - \tan^2 45° = 0}$

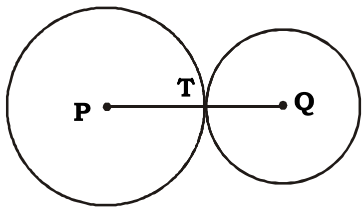
(v) Two circles are touching externally at point T.

y PQ = PT + QT [P - T - Q]

= 8 + 5

= 13

y $\boxed{PQ = 13 \text{ cm}}$ 1



(vi) 199.5 - 249.5
249.5 - 299.5
299.5 - 349.5
349.5 - 399.5 **1**

A.2. Solve ANY FOUR of the following :

(i) Draw seg AF ½

In right angled aAPR,

$AP^2 = AR^2 + PR^2$ [By Pythagoras theorem]

y $(25)^2 = (7)^2 + PR^2$ ½

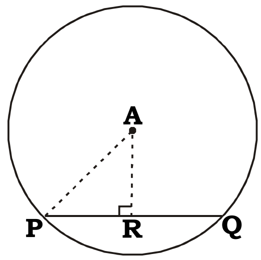
y $625 = 49 + PR^2$

y $625 - 49 = PR^2$ ½

y $576 = PR^2$

y $PR = 24$

y **Length of chord PQ is 24 cm.** ½



(ii)

$\sin B = \dfrac{AC}{BC} = \dfrac{1}{\sqrt{2}}$

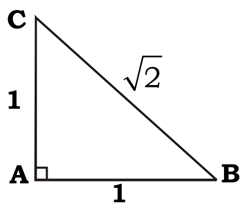
$\cos B = \dfrac{AB}{BC} = \dfrac{1}{\sqrt{2}}$

$\tan B = \dfrac{AC}{AB} = \dfrac{1}{1} = 1$ **1**

$\cot B = \dfrac{AB}{AC} = \dfrac{1}{1} = 1$

$\sec B = \dfrac{BC}{AB} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

$\text{cosec } B = \dfrac{BC}{AB} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$ **1**



(iii) Let S be the sample space

y S = { HH, HT, TH, TT }

y n (S) = 4 ½

(a) A is event of getting at least one head

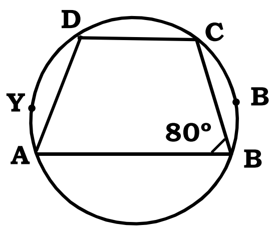
y A = { HH, HT, TH }

y n (A) = 3

y $P(A) = \dfrac{n(A)}{n(S)}$

y $\boxed{P(A) = \dfrac{3}{4}}$ ½

	(b) Let B be event of getting no head	½
	y B = { TT }	
	y n (B) = 3	
	y $P(B) = \frac{n(B)}{n(S)}$	
	y $\boxed{P(B) = \frac{1}{4}}$	½
(iv)	(a) seg TZ ì chord	½
	(b) seg LM ì diameter	½
	(c) seg CL ì radius	½
	(d) seg CM ì radius	½
(v)	L.H.S. = $\sin^2 45^\circ + \cos^2 45^\circ$	
	$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$	1
	$= \frac{1}{2} + \frac{1}{2}$	
	$= \frac{2}{2}$	
	= 1	
	= R.H.S.	
	y $\sin^2 45^\circ + \cos^2 45^\circ = 1$	1
(vi)	Midpoint of class 21 - 25	
	$\frac{21+25}{2} = 23$	1
	Exclusive form continuous classes	
	10.5 - 15.5	
	15.5 - 20.5	
	20.5 - 25.5	
	25.5 - 30.5	1
A.3.	**Solve ANY THREE of the following :**	
(i)	(a) ĎABC $= \frac{1}{2}$ m(arc ADC) [Inscribed angle theorem]	
	$80 = \frac{1}{2}$ m(arc ADC)	
	$80 \times 2 =$ m(arc ADC)	
	y $\boxed{m(\text{arc ADC}) = 160^\circ}$	1



(b) m(arc ABC) + m(arc ADC) = 360º [Measure of circle]

∴ m(arc ABC) + 160 = 360º

∴ m(arc ABC) = 360 – 160

∴ **m(arc ABC) = 200º** **1**

(c) seg AB || seg DC and transversal BC

ĎABC + ĎBCD = 180º [Interiror angle formed by transversal are supplementary]

∴ 80 + ĎBCD = 180º

∴ ĎBCD = 180º – 80º

∴ **ĎBCD = 100º** **1**

(ii) $4\sin^2 60^\circ + 3\tan^2 30^\circ - 8\sin 45^\circ \times \cos 45^\circ$

$= 4\left(\frac{\sqrt{3}}{2}\right)^2 + 3\left(\frac{1}{\sqrt{3}}\right)^2 - 8\left(\frac{1}{\sqrt{2}}\right) \times \frac{1}{\sqrt{2}}$ **1**

$= 4 \times \frac{3}{4} + 3 \times \frac{1}{3} - \frac{8}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$ ½

$= 3 + 1 - \frac{8}{2}$ ½

$= 4 - 4$ ½

$= \boxed{0}$ ½

(iii) Class width (h) = 4

Age in years	**Class Mark (x_i)**	**No. of people (f_i)**	**$f_i x_i$**
7 - 11	9	5	45
11 - 15	13	9	117
15 - 19	17	13	221
19 - 23	21	21	441
23 - 27	25	16	400
27 - 31	29	15	435
31 - 35	33	12	396
35 - 39	37	9	333
Total		**100**	**2388**

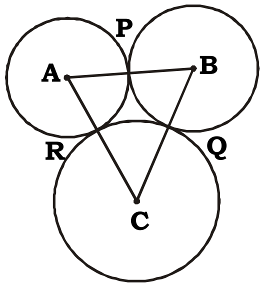
1

Mean $= \frac{\text{p}\, f_i x_i}{\text{p}\, f_i}$ **1**

y Mean $= \frac{2388}{100}$

y **Mean = 23.88 years.** **1**

(iv) Let r_1, r_2, r_3 be the radii of the three externally touching circles with centres A, B and C respectively.



AB = AP + PB [A - P - B] ½

$5 = r_1 + r_2$ ......(i)

BC = BQ + QC [B - Q - C] ½

y $7 = r_2 + r_3$ ......(ii)

AC = AR + RC [A - R - C] ½

y $6 = r_1 + r_3$ ......(iii)

Adding (i), (ii) and (iii)

$5 + 7 + 6 = r_1 + r_2 + r_2 + r_3 + r_1 + r_3$

y $18 = 2r_1 + 2r_2 + 2r_3$

y $18 = 2 (r_1 + r_2 + r_3)$

y $9 = r_1 + r_2 + r_3$ ......(iv) [Dividing by 2]

Substituting (i) in (iv),

y $9 = 5 + r_3$

y $r_3 = 9 - 5$

y $r_3 = 4$ ½

Substituting (ii) in (iv),

y $9 = r_1 + 7$

y $r_1 = 9 - 7$

y $r_1 = 2$ ½

Substituting (iii) in (iv),

y $9 = r_2 + 6$

y $r_2 = 9 - 6$

y $r_2 = 3$ ½

y **Radii of the circles with centres A, B and C are 2 cm, 3 cm and 4 cm respectively.**

(v) The sample space is

$S = \{ B_1, B_2, B_3, W_1, W_2, W_3, R_1, R_2, R_3 \}$

y n (S) = 9 ½

(a) Let A be the event that the marble picked up is white.

Then $A = \{ W_1, W_2 \}$

y n (A) = 2 ½

$$P (A) = \frac{n (A)}{n (S)}$$

y $\boxed{P (A) = \frac{2}{9}}$ ½

(b) Let C be the event that the marble picked up is blue

Then $C = \{ B_1, B_2, B_3 \}$

y n (C) = 3 ½

$P(C) = \frac{n(C)}{n(S)}$ ½

y $P(C) = \frac{3}{9}$

y $\boxed{P(C) = \frac{1}{3}}$ ½

(c) Let D be the event that the marble picked up is red

Then D = { R_1, R_2, R_3, R_4 }

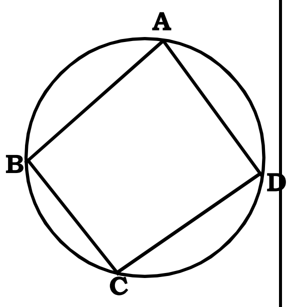
y n (D) = 4

$P(D) = \frac{n(D)}{n(S)}$

y $\boxed{P(D) = \frac{4}{9}}$

A.4. **Solve ANY TWO of the following :**

(i)



Given : ☐ABCD is a cyclic

To Prove : m ĎABC + m ĎADC = 180°

m ĎBAD + m ĎBCD = 180° 1

Proof : m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii)

[Inscribed angle theorem] 1

Adding (i) and (ii), we get

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC) ½

y m ĎABC + m ĎADC = $\frac{1}{2}$[m (arc ADC) + m (arc ABC)]

y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y m ĎABC + m ĎADC = 180° ....(iii)

In ☐ABCD,

m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360°

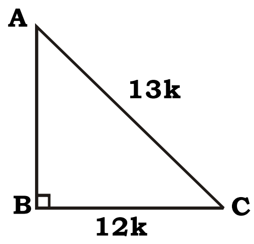
[∵ Sum of measure of angles of a quadrilateral is 360°] ½

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y **m ĎBAD + m ĎBCD = 180°** ½

(iii) In aABC,

ĎABC = 90°



$\sin A = \dfrac{12}{13}$ [Given]

y $\sin A = \dfrac{BC}{AC}$ [From the figure]

y $\dfrac{BC}{AC} = \dfrac{12}{13}$ **1**

y BC = 12k, AC = 13 k [k is constant, k > 0]

$AB^2 + BC^2 = AC^2$ [By Pythagoras theorem]

y $AB^2 = AC^2 - BC^2$

y $AB^2 = (13k)^2 - (12k)^2$ **1**

y $AB^2 = 169k^2 - 144k^2$

y $AB^2 = 25k^2$

y AB = 5k [Taking square root] **1**

$\cos A = \dfrac{AB}{AC} = \dfrac{5k}{13k} = \dfrac{5}{13}$

$\tan A = \dfrac{BC}{AB} = \dfrac{12k}{5k} = \dfrac{12}{5}$ **1**

(iii)

Weight (in gms)	No. of packets
200 - 201	12 ì f_1
201 - 202	26 ì f_m
202 - 203	20 ì f_2
203 - 204	9
204 - 205	2
205 - 206	1

1

Here the maximum frequency f_m = 26.

The corresponding class 201 - 202 is the modal class.

L = 201, f_m = 26, f_1 = 12, f_2 = 20, h = 1 ½

$$\text{Mode} = L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right)h$$ ½

$$= 201 + \left(\frac{26 - 12}{2\,(26) - 12 - 20}\right)1$$

$$= 201 + \left(\frac{14}{52 - 32}\right)$$ ½

$= 201 + \left(\frac{14}{20}\right)$

$= 201 + \left(\frac{7}{10}\right)$ ½

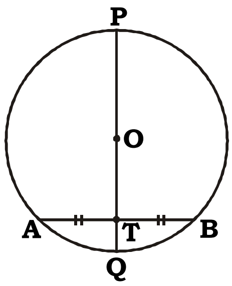
$= 201 + 0.7$

$= 201.7$

y **Mode of weight of coffee is 201.7 gms.** 1

^.5. **Solve ANY TWO of the following :**

(i) Draw seg OB and seg PB ½



radius $= \frac{1}{2} \times$ diameter

$\therefore$ OP = OQ = OB $= \frac{1}{2}$PQ $= \frac{1}{2} \times 30$

$\therefore$ OP = OQ = OB = 15 ½

AT = TB [$\because$ T is the midpoint of seg AB] ½

$\therefore$ seg OT $\perp$ chord AB [Segment joining the centre of the circle to the midpoint of the chord is perpendicular to the chord) ½

In right angled ΔOTB,

$OB^2 = OT^2 + TB^2$ [By Pythagoras theorem] ½

$(15)^2 = (9)^2 + (TB)^2$

$\therefore$ $225 = 81 + TB^2$

$\therefore$ $225 - 81 = TB^2$

$\therefore$ $144 = TB^2$

$\therefore$ TB = 12 units [Taking square roots]

TB $= \frac{1}{2}$AB [$\because$ T is the midpoint of seg AB]

$\therefore$ 12 $= \frac{1}{2}$AB

$\therefore$ **AB = 24 units** ½

PT = PO + OQ [P - O - T]

$\therefore$ PT = 15 + 9

$\therefore$ **PT = 24 units**

In right angled ΔPTB,

$PB^2 = PT^2 + TB^2$ [By Pythagoras theorem] ½

$\therefore$ $PB^2 = (24)^2 + (12)^2$

$\therefore$ $PB^2 = 576 + 144$

$\therefore$ $PB^2 = 720$

$\therefore$ $PB = \sqrt{720}$

$\therefore$ $PB = \sqrt{144 \times 5}$

y $\boxed{PB = 12\sqrt{5} \text{ units}}$

$OQ = OT + TQ$ [O - T - Q]

$\therefore$ $15 = 9 + TQ$

$\therefore$ $15 - 9 = TQ$

y $TQ = 6$ units ½

In right angled ΔTBQ,

$BQ^2 = TQ^2 + TB^2$ [By Pythagoras theorem] ½

$\therefore$ $BQ^2 = (6)^2 + (12)^2$

$\therefore$ $BQ^2 = 36 + 144$

$\therefore$ $BQ^2 = 180$

$\therefore$ $BQ = \sqrt{180}$

$\therefore$ $BQ = \sqrt{36 \times 5}$

y $\boxed{BQ = 6\sqrt{5} \text{ units}}$ ½

(ii) Let S be the sample space

y S = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) } ½

y n (S) = 36 ½

(a) Let A be the event of getting doublet

y A = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }

y n (A) = 6 ½

$P(A) = \frac{n(A)}{n(S)}$

y $P(A) = \frac{6}{36}$

y $\boxed{P(A) = \frac{1}{6}}$ ½

(b) Let B be the event of getting even number as the sum

y B = { (1, 1), (1, 3), (2, 2), (3, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (4, 6), (5, 5), (6, 4), (6, 6) }

y n (B) = 18 ½

$P(B) = \frac{n(B)}{n(S)}$

y $P(B) = \frac{18}{36}$

y $\boxed{P(B) = \frac{1}{2}}$ ½

(c) Let C be the event of getting prime number as sum

y C = { (1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (5, 6), (6, 5) }

y n (C) = 15 ½

$P(C) = \frac{n(C)}{n(S)}$

y $P(C) = \frac{15}{36}$

y $\boxed{P(C) = \frac{5}{12}}$ ½

(d) Let D be the event of getting multiple of 3 as sum

D = { (1, 2), (2, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (3, 6), (4, 5), (5, 4), (6, 3), (6, 6) }

y n (D) = 12 ½

$P(D) = \frac{n(D)}{n(S)}$

y $P(D) = \frac{12}{36}$

y $\boxed{P(D) = \frac{1}{3}}$ ½

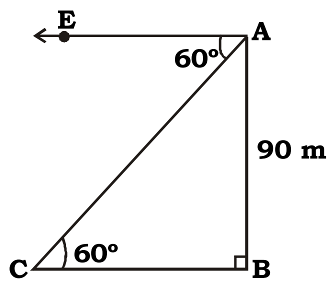
(iii) Seg AB represents the lighthouse ½

C represents the position 90m of ship.

∠EAC is the angle of depression.

AB = 90 m

m ∠EAC = 60° ½



	$\angle EAC \cong \angle ACB$ [Converse of alternate angle test]	½
	$m\angle ACB = 60°$	
	In right angled ΔABC	
	$\tan 60° = \frac{AB}{BC}$ [By definition]	½
	y $\sqrt{3} = \frac{90}{BC}$	½
	y $BC = \frac{90}{\sqrt{3}}$	½
	$\therefore BC = \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	1
	$\therefore BC = \frac{90 \times \sqrt{3}}{3}$	½
	$\therefore BC = 30\sqrt{3}$	
	$\therefore$ The ship is $30\sqrt{3}$ m far from the lighthouse.	½

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