Q.P. SET CODE
A

MT - w

Seat No. | | | | | | | |

2013 ___ ___ 1100 - MT - w - MATHEMATICS (71) ALGEBRA - SET - A (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY Five of the following : **5**

(i) A die is thrown, write sample space and n(S).

(ii) Is the following list of numbers an Arithmetic Progression? Justify.
1, 3, 6, 10, ...

(iii) If $D_x = -18$ and $D = 3$ are the values of the determinants for certain simultaneous equations in x and y, find x.

(iv) Write the quadratic equation in standard from $ax^2 + bx + c = 0$
$8 - 3x - 4x^2 = 0$

(v) Examine whether the point (2, 5) lies on the graph of the equation
$3x - y = 1$.

(vi) If $\Sigma f_i d_i = 295$ and $\Sigma f_i = 25$, find $\bar{d}$.

Q.2. Solve ANY FOUR of the following : **8**

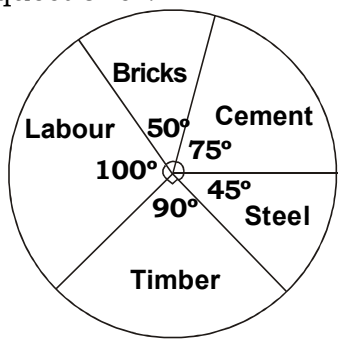
(i) Solve the following quadratic equation by factorization method.
$3x^2 + 34x + 11 = 0$

(ii) The taxi fare is Rs. 14 for the first kilometer and Rs. 2 for each additional kilometer. What will be fare for 10 kilometers ?

(iii) The following pie diagram represents expenditure on different items in constructing a building. Answer the following questions :

(a) Which is the item with the maximum expenditure ?

(a) Which is the item with the minimum expenditure ?



(iv) If a card is drawn from a pack of 52 cards, find the probability of getting a black card

(v) Find the twenty fifth term of the A. P. : 12, 16, 20, 24, ...

(vi) Without actually solving the simultaneous equations given below, decide whether they have unique solution, no solution or infinitely many solutions.

$3x + 5y = 16; 4x - y = 6$

Q.3. Solve ANY THREE of the following : 9

(i) Solve the following simultaneous equations using Cramer's rule :

$3x - y = 7; x + 4y = 11$

(ii) Find t_n for an Arithmetic Progression where $t_3 = 22$, $t_{17} = -20$.

(iii) In the following experiment write the sample space S, number of sample points n (S), events P, Q, R using set and n (P), n (Q) and n (R).

Find among the events defined above which are : complementary events, mutually exclusive events and exhaustive events.

A die is thrown :

P is the event of getting an odd number.

Q is the event of getting an even number.

R is the event of getting a prime number.

(iv) The following data gives the number of students using different modes of transport :

Mode of transport	Bicycle	Bus	Walk	Train	Car
Number of students	140	100	70	40	10

Represent the above data using pie diagram.

(v) Draw the histogram to represent the following data.

Daily sales of a store in (₹)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	Total
Number of days in a month	2	12	10	4	2	30

Q.4. Solve ANY TWO of the following : 8

(i) A coin is tossed three times then find the probability of
(a) getting head on middle coin
(b) getting exactly one tail
(c) getting no tail

(ii) Find the sum of the first n odd natural numbers.
Hence find 1 + 3 + 5 + ... + 101.

(iii) Solve the following equation :
$2 (y^2 - 6y)^2 - 8 (y^2 - 6y + 3) - 40 = 0$

Q.5. Solve ANY TWO of the following : 10

(i) A boat takes 6 hours to travel 8 km upstream and 32 km downstream, and it takes 7 hours to travel 20 km upstream and 16 km downstream. Find the speed of the boat in still water and the speed of the stream.

(ii) Following table gives frequency distribution of trees planted by different housing societies in a particular locality.

No. of trees	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40
No. of societies	2	7	9	8	6	4

Find the mean number of trees planted by housing society by using 'step deviation method'.

(iii) One tank can be filled up by two taps in 6 hours. The smaller tap alone takes 5 hours more than the bigger tap alone. Find the time required by each tap to fill the tank separately.

Best Of Luck

A.P. SET CODE
A

MT - w

2013 ___ __ 1100 - MT - w - MATHEMATICS (71) ALGEBRA - SET - A (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1.	**Attempt ANY FIVE of the following :**	
(i)	When a die is thrown S = {1,2,3,4,5,6} n(S) = 6	**1**
(ii)	$t_1 = 1,\ t_2 = 3,\ t_3 = 6,\ t_4 = 10$ $t_2 - t_1 = 3 - 1 = 2$ $t_3 - t_2 = 6 - 3 = 3$ $t_4 - t_3 = 10 - 6 = 4$ $\because$ The difference between any two consecutive terms is not constant. $\therefore$ The sequence is not an A.P.	**1**
(iii)	$D_x = -18$ and $D = 3$ By Cramer's rule, $x = \frac{D_x}{D}$ $\therefore\ x = \frac{-18}{3}$ $\therefore\ x = -6$	**1**
(iv)	$8 - 3x - 4x^2 = 0$ $0 = 4x^2 + 3x - 8$ $\therefore\ 4x^2 + 3x - 8 = 0$	**1**
(v)	Substituting x = 2 and y = 5 in the L.H.S. of the equation 3x – y =1 L.H.S. = 3x – y = 3 (2) – 5 = 6 – 5 = 1 = R.H.S. $\therefore$ x = 2 and y = 5 satisfies the equation 3x – y = 1 Hence (2, 5) lies on the graph of the equation 3x – y = 1.	**1**

(vi)	$\bar{d} = \frac{\Sigma f_i d_i}{\Sigma f_i}$	
	$= \frac{295}{25}$	
	$\bar{d} = 11.8$	**1**
A.2.	**Solve ANY Four of the following :**	
(i)	$3x^2 + 34x + 11 = 0$	
	$\therefore\ 3x^2 + 33x + x + 11 = 0$	
	$\therefore\ 3x(x + 11) + 1(x + 11) = 0$	
	$\therefore\ (x + 11)(3x + 1) = 0$	**1**
	$\therefore\ x + 11 = 0$ or $3x + 1 = 0$	
	$\therefore\ x = -11$ or $3x = -1$	
	$\therefore\ x = -11$ or $x = \frac{-1}{3}$	**1**
	$\therefore$ -11 and $\frac{-1}{3}$ are the roots of given quadratic equation.	
(ii)	Since the taxi fare increases by Rs. 2 every kilometer after the first, the successive taxi fares form an A.P.	
	The taxi fare for first kilometer (a) = Rs. 14	
	Increase in taxi fare in every kilometer after first kilometer (d) = 2	
	No. of kilometers covered by taxi (n) = 10	
	Taxi fare for 10 kilometers = t_{10} = ?	**1**
	$t_n = a + (n + 1)d$	
	$\therefore\ t_{10} = a + (10 - 1)d$	
	$\therefore\ t_{10} = 14 + 9(2)$	
	$\therefore\ t_{10} = 14 + 18$	
	$\therefore\ t_{10} = 32$	
	$\therefore$ Taxi fare for ten kilometers is Rs. 32.	**1**
(iii)	(a) The item with maximum expenditure is labour.	**1**
	(b) The item with minimum expenditure is steel.	**1**
(iv)	There are 52 cards in a pack	
	$\therefore\ n(S) = 52$	
	Let A be event that the card drawn is a black card	
	Total no. of black cards = 26	
	$\therefore\ n(A) = 26$	**1**

	$P(A) = \frac{n(A)}{n(S)}$	
	$\therefore P(A) = \frac{26}{52}$	
	$\therefore P(A) = \frac{1}{2}$	1
(v)	For the given A.P. 12, 16, 20, 24, ...	
	Here, $a = t_1 = 12$	
	$d = t_2 - t_1 = 16 - 12 = 4$	
	We know,	
	$t_n = a + (n - 1)\,d$	1
	$\therefore t_{25} = a + (25 - 1)\,d$	
	$\therefore t_{25} = 12 + 24\,(4)$	
	$\therefore t_{25} = 12 + 96$	
	$\therefore t_{25} = 108$	
	$\therefore$ The twenty fifth term of A.P. is 108.	1
(vi)	$3x + 5y = 16$	
	Comparing with $a_1x + b_1y = c_1$ we get, $a_1 = 3$, $b_1 = 5$, $c_1 = 16$	
	$4x - y = 6$	
	Comparing with $a_2x + b_2y = c_2$ we get, $a_2 = 4$, $b_2 = -1$, $c_2 = 6$	1
	$\therefore \frac{a_1}{a_2} = \frac{3}{4}$	
	$\therefore \frac{b_1}{b_2} = \frac{5}{-1} = -5$	
	$\therefore \frac{c_1}{c_2} = \frac{16}{6} = \frac{8}{3}$	
	$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	
	$\therefore$ The simultaneous equations $3x + 5y = 16$ and $4x - y = 6$ have unique solution.	1
A.3.	**Solve ANY THREE of the following :**	
(i)	$3x - y = 7$	
	$x + 4y = 11$	
	$D = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - (-1 \times 1) = 12 + 1 = 13$	

$D_x = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = (7 \times 4) - (-1 \times 11) = 28 + 11 = 39$

$D_y = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = (3 \times 11) - (7 \times 1) = 33 - 7 = 26$ **1**

By Cramer's rule,

$x = \frac{D_x}{D} = \frac{39}{13} = 3$ **1**

$y = \frac{D_y}{D} = \frac{26}{13} = 2$

$\therefore$ x = 3 and y = 2 is the solution of given simultaneous equations. **1**

(ii) Given :For an A.P. $t_3 = 22$ and $t_{17} = -20$

Find : t_n.

Sol.

$t_n = a + (n - 1) d$

$t_3 = a + (3 - 1) d$

$22 = a + 2d$

$\therefore \quad a + 2d = 22 \quad \ldots\ldots(i)$

$t_{17} = a + (17 - 1) d$

$-20 = a + 16d$

$\therefore \quad a + 16d = -20 \quad \ldots\ldots(ii)$ **1**

Subtracting (ii) from (i),

$$\begin{array}{rcrcr} a & + & 2d & = & 22 \\ a & + & 16d & = & -20 \\ (-) & (-) & & & (+) \\ \hline & & -14d & = & 42 \end{array}$$

$\therefore \quad d = \frac{42}{-14}$

$\therefore \quad d = -3$

Substituting d = – 3 in (i),

$a + 2(-3) = 22$

$a - 6 = 22$

$\therefore \quad a = 22 + 6$

$\therefore \quad a = 28$ **1**

$t_n = a + (n - 1) d$

$\therefore \quad t_n = 28 + (n - 1)(-3)$

$\therefore \quad t_n = 28 - 3n + 3$

$\therefore \quad t_n = 31 - 3n$ **1**

(iii) When a die is thrown

S = { 1, 2, 3, 4, 5, 6 }

n (S) = 6

P is the event of getting an odd number

P = { 1, 3, 5 }

∴ n (P) = 3

Q is the event of getting an even number

Q = { 2, 4, 6 }

∴ n (Q) = 3

R is the event of getting a prime number

R = { 2, 3, 5 } **2**

∴ n (R) = 3

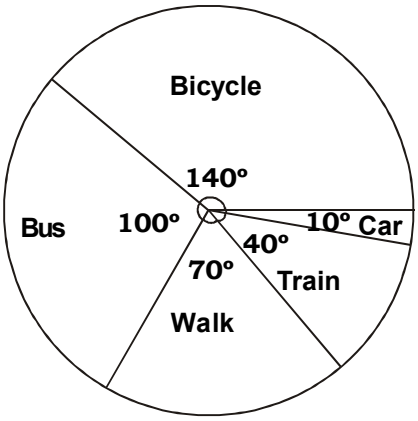
Here $P \cap Q = \phi$ and $P \cup Q = S$

∴ P and Q are complementary events. **1**

(iv)

Mode of transport	**No. of Students**	**Measure of central angle (θ)**
Bicycle	140	$\frac{140}{360} \times 360^\circ = 140^\circ$
Bus	100	$\frac{100}{360} \times 360^\circ = 100^\circ$
Walk	70	$\frac{70}{360} \times 360^\circ = 70^\circ$
Train	40	$\frac{40}{360} \times 360^\circ = 40^\circ$
Car	10	$\frac{10}{360} \times 360^\circ = 10^\circ$
Total	**360**	**360°**

1



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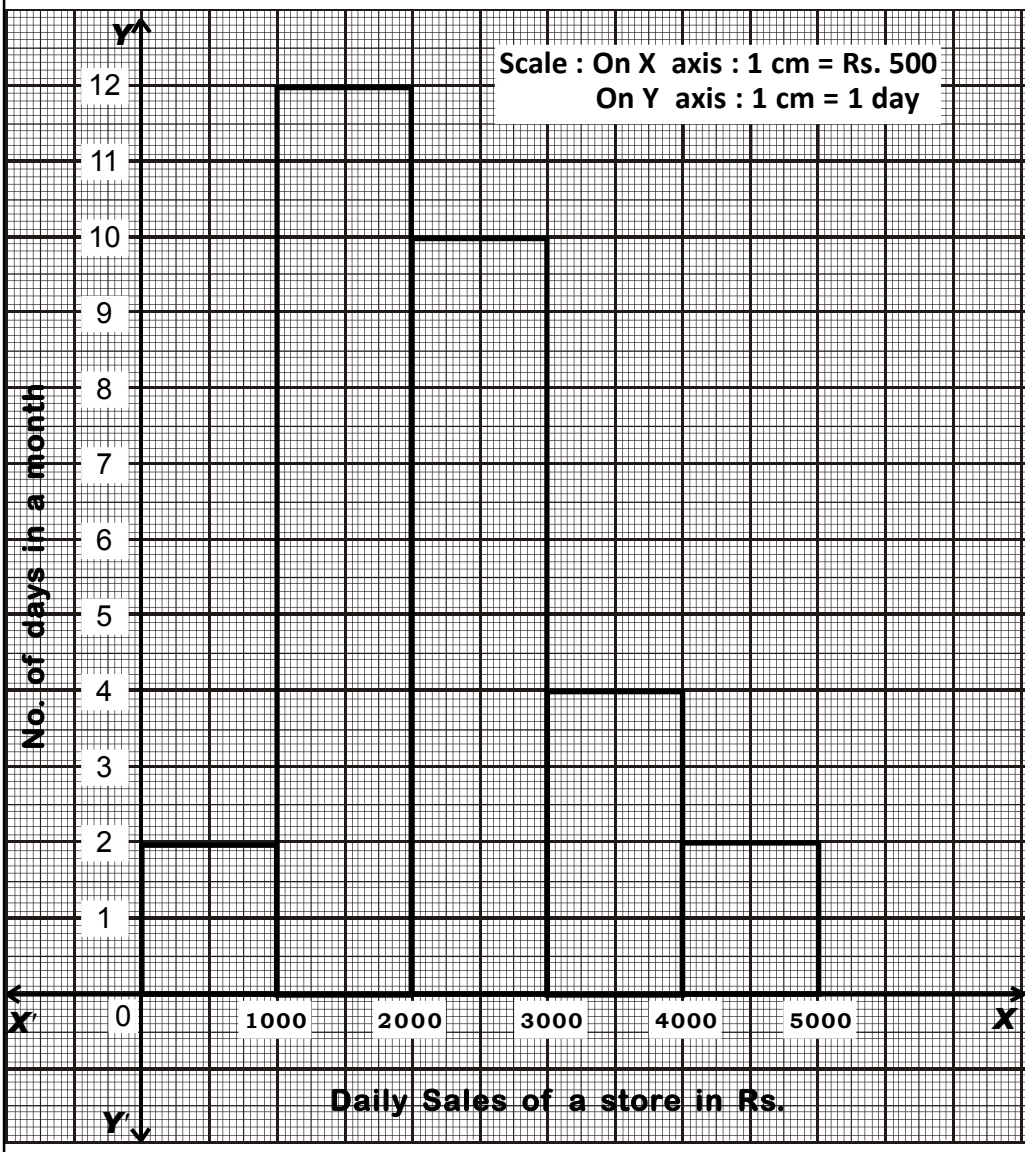
(v)

Daily sales of a store in (₹)	Number of days in a month
0 - 1000	2
1000 - 2000	12
2000 - 3000	10
3000 - 4000	4
4000 - 5000	2
Total	**30**



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A.4.	**Solve ANY TWO of the following :**	
(i)	When a coin tossed three times S = { HHH, HTH, THH, TTH, HHT, HTT, THT, TTT } n (S) = 8	1
	(a) Let A be the event of getting head on middle coin A = { HHH, THH, HHT, THT } n (A) = 4 $P(A) = \frac{n(A)}{n(S)}$ $\therefore P(A) = \frac{4}{8}$ $\therefore P(A) = \frac{1}{2}$	1
	(b) Let B be the event of getting exactly one tail B = { HTH, THH, HHT } n (B) = 3 $P(B) = \frac{n(B)}{n(S)}$ $\therefore P(B) = \frac{3}{8}$	1
	(c) Let C be the event of getting no tail C = { HHH } n (C) = 1 $P(C) = \frac{n(C)}{n(S)}$ $\therefore P(C) = \frac{1}{8}$	1
(ii)	The first n odd natural numbers are as follows : 1, 3, 5, 7, ..., n $a = 1, d = t_2 - t_1 = 3 - 1 = 2$ $S_n = \frac{n}{2}[2a + (n-1)d]$	1
	$\therefore S_n = \frac{n}{2}[2(1) + (n-1)2]$ $\therefore S_n = \frac{n}{2}[2 + 2n - 2]$	

	$= \frac{n}{2}[2n]$	
	$\therefore S_n = n^2$ .....(i)	1
	$1 + 3 + 5 + ... + 101$	
	Let, 101 be the nth term of A.P.	
	$t_n = 101$	
	$t_n = a + (n - 1) d$	
	$\therefore 101 = a + (n - 1) d$	
	$\therefore 101 = 1 + (n - 1) 2$	
	$\therefore 101 = 1 + 2n - 2$	
	$\therefore 101 = 2n - 1$	
	$\therefore 101 + 1 = 2n$	
	$\therefore 2n = 102$	
	$\therefore n = 51$	1
	$\therefore$ 101 is the 51st term of A.P.,	
	$\therefore$ We have to find sum of 51 terms i.e. S_{51},	
	$S_n = n^2$ [From (i)]	
	$\therefore S_{51} = (51)^2$	
	$\therefore S_{51} = 2601$	1
(iii)	$2 (y^2 - 6y)^2 - 8 (y^2 - 6y + 3) - 40 = 0$	
	Substituting $y^2 - 6y = m$ we get,	
	$\therefore 2m^2 - 8 (m + 3) - 40 = 0$	1
	$\therefore 2m^2 - 8m - 24 - 40 = 0$	
	$\therefore 2m^2 - 8m - 64 = 0$	
	Dividing throughout by 2 we get,	
	$m^2 - 4m - 32 = 0$	
	$\therefore m^2 - 8m + 4m - 32 = 0$	
	$\therefore m (m - 8) + 4 (m - 8) = 0$	
	$\therefore (m - 8) (m + 4) = 0$	
	$\therefore m - 8 = 0$ or $m + 4 = 0$	
	$\therefore m = 8$ or $m = - 4$	1
	Resubstituting $m = y^2 - 6y$ we get,	
	$y^2 - 6y = 8$ .....(i) or $y^2 - 6y = - 4$ .....(ii)	
	From (i), $y^2 - 6y = 8$	
	$\therefore y^2 - 6y - 8 = 0$	
	Comparing with $ax^2 + bx + c = 0$ we have $a = 1, b = - 6, c = - 8$	
	$b^2 - 4ac = (- 6)^2 - 4 (1) (- 8)$	
	$= 36 + 32$	
	$= 68$	
	$y = \frac{- b \pm \sqrt{b^2 - ac}}{2a}$	

$$= \frac{-(-6) \pm \sqrt{68}}{2\,(1)}$$

$$= \frac{6 \pm \sqrt{4 \times 17}}{2}$$

$$= \frac{6 \pm 2\sqrt{17}}{2}$$

$$= \frac{2\left(3 \pm \sqrt{17}\right)}{2}$$

$$= 3 \pm \sqrt{17}$$

$\therefore$ $y = 3 + \sqrt{17}$ or $y = 3 - \sqrt{17}$ **1**

From (ii), $y^2 - 6y = -4$

$\therefore$ $y^2 - 6y + 4 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 1$, $b = -6$, $c = 4$

$$b^2 - 4ac = (-6)^2 - 4\,(1)\,(4)$$
$$= 36 - 16$$
$$= 20$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{20}}{2\,(1)}$$

$$= \frac{6 \pm \sqrt{4 \times 5}}{2}$$

$$= \frac{6 \pm 2\sqrt{5}}{2}$$

$$= \frac{2\left(3 \pm \sqrt{5}\right)}{2}$$

$$= 3 \pm \sqrt{5}$$

$\therefore$ $y = 3 + \sqrt{5}$ or $y = 3 - \sqrt{5}$

$\therefore$ $y = 3 + \sqrt{17}$ or $y = 3 - \sqrt{17}$ or $y = 3 + \sqrt{5}$ or $y = 3 - \sqrt{5}$ **1**

A.5. **Solve ANY TWO of the following :**

(i) Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

$\therefore$ Speed of the boat upstream = $(x - y)$ km/hr

and speed of the boat downstream = $(x + y)$ km/hr

We know that, Time = $\dfrac{\text{Distance}}{\text{Speed}}$

As per the first condition,

$$\frac{8}{x-y} + \frac{32}{x+y} = 6 \quad \text{.....(i)}$$

As per the second condition,

$$\frac{20}{x-y} + \frac{16}{x+y} = 7 \quad \text{.....(ii)}$$ **1**

Substituting $\frac{1}{x-y} = m$ and $\frac{1}{x+y} = n$ in (i) and (ii) we get,

$$8m + 32n = 6 \quad \text{.....(iii)}$$

$$20m + 16n = 7 \quad \text{.....(iv)}$$ **1**

Multiplying (iv) by 2 we get,

$$40m + 32n = 14 \quad \text{.....(v)}$$

Subtracting (v) from (iii),

$$\begin{array}{rcl} 8m + 32n & = & 6 \\ 40m + 32n & = & 14 \\ (-) \quad (-) & & (-) \\ \hline -32m & = & -8 \end{array}$$

$$\therefore \quad m = \frac{-8}{-32}$$

$$\therefore \quad m = \frac{1}{4}$$

Substituting $m = \frac{1}{4}$ in (iii),

$$8\left(\frac{1}{4}\right) + 32n = 6$$

$$\therefore \quad 2 + 32n = 6$$

$$\therefore \quad 32n = 6 - 2$$

$$\therefore \quad 32n = 4$$

$$\therefore \quad n = \frac{4}{32}$$

$$\therefore \quad n = \frac{1}{8}$$ **1**

Resubstituting the values of m and n we get,

$$m = \frac{1}{x-y}$$

$$\therefore \quad \frac{1}{4} = \frac{1}{x-y}$$

$$\therefore \quad x - y = 4 \quad \text{.....(vi)}$$

$$n = \frac{1}{x+y}$$

$$\therefore \quad \frac{1}{8} = \frac{1}{x+y}$$

$\therefore \quad x + y = 8 \quad \ldots\ldots(vii)$ **1**

Adding (vi) and (vii),

$$\begin{array}{rcl} x - y & = & 4 \\ x + y & = & 8 \\ \hline 2x & = & 12 \end{array}$$

$\therefore \quad x = \frac{12}{2}$

$\therefore \quad x = 6$

Substituting x = 6 in (vii)

$6 + y = 8$

$\therefore \quad y = 8 - 6$

$\therefore \quad y = 2$

$\therefore$ The speed of boat in still water is 6 km/hr and speed of stream is 2 km/ hr. **1**

(ii) Class width (h) = 5, Assumed mean (A) = 22.5

No. of trees	**Class Mark (x_i)**	$d_i = x_i - A$	$u_i = \frac{d_i}{h}$	**No. of societies (f_i)**	$f_i u_i$
10 - 15	12.5	– 10	– 2	2	– 4
15 - 20	17.5	– 5	– 1	7	– 7
20 - 25	22.5 → A	0	0	9	0
25 - 30	27.5	5	1	8	8
30 - 35	32.5	10	2	6	12
35 - 40	37.5	15	3	4	12
Total				**36**	**21**

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$\bar{u} = \frac{\Sigma f_i u_i}{\Sigma f_i}$

$\therefore \quad \bar{u} = \frac{21}{36}$

$\therefore \quad \bar{u} = 0.583$ **1**

Mean $(\bar{x}) = A + h\bar{u}$

$= 22.5 + 5\ (0.583)$

$= 22.5 + 2.92$

$= 25.42$

$\therefore$ Mean of trees planted by societies 25.42 trees. **1**

(iii) Let the time taken to fill a tank by a bigger tap alone be x hrs.

$\therefore$ The time taken by smaller tap alone is (x + 5) hrs.

Time taken by both the taps together to fill the same tank is 6 hrs.

$\therefore$ Portion of tank filled in 1 hr by bigger tap = $\frac{1}{x}$

Portion of tank filled in 1 hr by smaller tap = $\frac{1}{x+5}$ **1**

Portion of tank filled in 1 hr by both taps together = $\frac{1}{6}$

As per the given condition,

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$ **1**

$$\therefore \quad \frac{x+5+x}{x(x+5)} = \frac{1}{6}$$

$$\therefore \quad \frac{2x+5}{x^2+5x} = \frac{1}{6}$$

$$\therefore \quad 6(2x+5) = 1(x^2+5x)$$

$$\therefore \quad 12x+30 = x^2+5x$$

$$\therefore \quad 0 = x^2+5x-12x-30$$ **1**

$$x^2-7x-30 = 0$$

$$\therefore \quad x^2+3x-10x-30 = 0$$

$$\therefore \quad x(x+3)-10(x+3) = 0$$

$$\therefore \quad (x+3)(x-10) = 0$$

$$\therefore \quad x+3=0 \text{ or } x-10=0$$

$$\therefore \quad x=-3 \text{ or } x=10$$ **1**

$\because$ x is the time taken by bigger tap

$\therefore$ $x \neq -3$

Hence $x = 10$

$\therefore$ $x + 5 = 10 + 5 = 15$

$\therefore$ Time taken by bigger tap alone is 10 hrs and smaller tap alone is 15 hrs. **1**

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