Q.P. SET CODE
C

MT - y

Seat No. | | | | | | | |

2013 ___ ___ 1100 - MT - y - MATHEMATICS (71) ALGEBRA - SET - C (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Note :

(i) All questions are compulsory.
(ii) Use of calculator is not allowed.

Q.1. Solve ANY Five of the following : **5**

(i) 3 coins are tossed simultaneously, write sample space and n(s).

(ii) Is following list of number an Arithmetic Progression? Justify.
1, 4, 7, 10, ...

(iii) Write D_x for the following simultaneous equation. :
$3x + 4y = 8$; $x - 2y = 5$

(iv) Write the quadratic equation in standard from $ax^2 + bx + c = 0$
$m(m - 7) = 0$

(v) If the point (3, 2)lies on the graph of the equation $5x + ay = 19$, then find a.

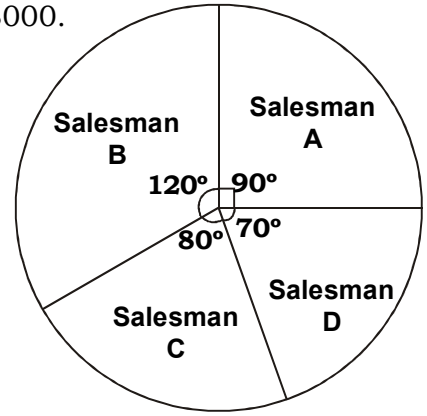
(vi) If $\Sigma f_i x_i = 255$ and $\Sigma f_i = 30$, find $\bar{x}$

Q.2. Solve ANY FOUR of the following : **8**

(i) Solve the following quadratic equation by factorization method.
$3x^2 - 11x + 6 = 0$

(ii) The taxi fare is Rs. 14 for the first kilometer and Rs. 2 for each additional kilometer. What will be fare for 10 kilometers ?

(iii) The sales due to salesmen in week are given below by the pie diagram. Study the diagram and answer the following questions if the total sale due to salesman A is Rs. 18000.
(a) Find the salesman with highest sale.
(b) Find the total sale.



(iv) If a card is drawn from a pack of 52 cards, find the probability of getting a card bearing number between 2 to 5 including 2 and 5

(v) Find t_{11} from the following A.P. 4, 9, 14, ...

(vi) Without actually solving the simultaneous equations given below, decide whether simultaneous equations have unique solution, no solution or infinitely many solutions.
$3x - 7y = 15$; $6x = 14y + 10$

Q.3. Solve ANY THREE of the following : **9**

(i) Solve the following simultaneous equations using Cramer's rule :

$y = \frac{5x - 10}{2}$; $4x + 5 = -y$

(ii) Given the following sequence, determine whether it is arithmetic or not. If it is an Arithmetic Progression, find its general term :
– 5, 2, 9, 16, 23, 30, ...

(iii) In the following experiment, write the sample space S, number of sample point n (S), event A, B, C and n (A), n (B), n (C). Also find complementary events, mutually exclusive events and exhaustive events.
Two coins are tossed, A is the event of getting at most one head, B is the event getting both heads, C is the event of getting same face on both the coins.

(iv) Area under different crops in a certain village is given below. Represent it by pie diagram

Crop	Jowar	Wheat	Sugarcane	Vegetables
Area in hectare	8000	6000	2000	2000

(v) Represent the following data using histogram.

Height of students (cm.)	140-144	145-149	150-154	155-159
Number of students	2	12	10	4

Q.4. Solve ANY TWO of the following : **8**

(i) Two dice are thrown. Find the probability of the events.
(a) The product of numbers on their upper faces is 12.
(b) The sum of the numbers on their faces is multiple of 7.

(ii) Second and fourth term of on A.P. is 12 and 20 respectively. Find the sum of first 25 terms of that A.P.

(iii) Solve the following equation :
$(p^2 + p)(p^2 + p - 3) = 28$

Q.5. Solve ANY TWO of the following : **10**

(i) A man starts his job with a certain monthly salary and a fixed increment every year. If his salary will be Rs. 11000 after 2 years and Rs. 14000 after 4 years of his service. What is his starting salary and what is the annual increment ?

(ii) The measurements (in mm) of the diameters of the head of screws are given below. Find mean by step deviation method.

Diameter (in mm)	33 - 35	36 - 38	39 - 41	42 - 44	45 - 47
No. of screws	10	19	23	21	27

(iii) For doing some work Ganesh takes 10 days more than John. If both work together they complete the work in 12 days. Find the number of days if Ganesh worked alone?

Best Of Luck 👍

A.P. SET CODE
C

MT - y

2013 ___ __ 1100 - MT - y - MATHEMATICS (71) ALGEBRA - SET - C (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. Solve ANY FIVE of the following :

(i) When 3 coins are tossed simultaneously
S = { HHH, HTH, THH, TTH, HHT, HTT, THT, TTT }
n(S) = 8 **1**

(ii) $t_1 = 1, t_2 = 4, t_3 = 7, t_4 = 10$
$t_2 - t_1 = 4 - 1 = 3$
$t_3 - t_2 = 7 - 4 = 3$
$t_4 - t_3 = 10 - 7 = 3$
$\because$ The difference between any two consecutive terms 3 which is constant.
$\therefore$ The sequence is an A.P. **1**

(iii) $3x + 4y = 8$
$x - 2y = 5$
$D_x = \begin{vmatrix} 8 & 4 \\ 5 & -2 \end{vmatrix}$ **1**

(iv) $m(m - 7) = 0$
$\therefore \quad m^2 - 7m = 0$
$\therefore \quad m^2 - 7m + 0 = 0$ **1**

(v) $\because$ (3, 2) lies on the graph of the equation $5x + ay = 19$.
It satisfies the equation,
$\therefore$ Substituting $x = 3$ and $y = 2$ in the equation we get,
$5(3) + a(2) = 19$
$\therefore \quad 15 + 2a = 19$
$\therefore \quad 2a = 19 - 15$
$\therefore \quad 2a = 4$
$\therefore \quad a = \frac{4}{2}$
$\therefore \quad a = 2$ **1**

(vi) $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{255}{30}$

$= 8.5$ **1**

A.2. Solve ANY Four of the following :

(i) $3x^2 - 11x + 6 = 0$

$\therefore \quad 3x^2 - 9x - 2x + 6 = 0$

$\therefore \quad 3x(x - 3) - 2(x - 3) = 0$

$\therefore \quad (x - 3)(3x - 2) = 0$

$\therefore \quad x - 3 = 0 \quad \text{or} \quad 3x - 2 = 0$ **1**

$\therefore \quad x = 3 \quad \text{or} \quad 3x = 2$

$\therefore \quad x = 3 \quad \text{or} \quad x = \frac{2}{3}$ **1**

$\therefore$ 3 and $\frac{2}{3}$ are the roots of given quadratic equation.

(ii) Since the taxi fare increases by Rs. 2 every kilometer after the first, the successive taxi fares form an A.P.
The taxi fare for first kilometer (a) = Rs. 14
Increase in taxi fare in every kilometer after first kilometer (d) = 2
No. of kilometers covered by taxi (n) = 10
Taxi fare for 10 kilometers = t_{10} = ?

$t_n = a + (n + 1) d$

$\therefore \quad t_{10} = a + (10 - 1) d$ **1**

$\therefore \quad t_{10} = 14 + 9 (2)$

$\therefore \quad t_{10} = 14 + 18$

$\therefore \quad t_{10} = 32$

$\therefore$ Taxi fare for ten kilometers is Rs. 32. **1**

(iii) (a) The highest sale is done by salesman B. **1**

(b) Let total sales done be Rs. x
Sales done due to salesman A = Rs. 18000
Measure of central angle for salesman A = 90°
As per the given condition,

$\frac{90}{360} \times x = 18000$

$\therefore \quad x = \frac{18000 \times 360}{90}$

$\therefore$ $x = 18000 \times 4$

$\therefore$ $x = 72000$

$\therefore$ Total sales done is Rs. 72000 **1**

(iv) There are 52 cards in a pack of playing card

$\therefore$ $n(S) = 52$

Let C be the event that card drawn bears number between 2 to 5 including 2 and 5

No. of cards between 2 to 5 including 2 and 5 is 4.

$\because$ There are 4 types of cards

$\therefore$ The total no. of cards between 2 to 5 including 2 and 5 is $4 \times 4 = 16$

$n(C) = 16$ **1**

$$P(C) = \frac{n(C)}{n(S)}$$

$$\therefore P(C) = \frac{16}{52}$$

$$\therefore P(C) = \frac{4}{13}$$ **1**

(v) For the A.P. 4, 9, 14, ...

$a = 4, d = 5$

$t_n = a + (n - 1)\,d$ **1**

$\therefore$ $t_{11} = 4 + (11 - 1)\,5$

$\therefore$ $t_{11} = 4 + 50$

$\therefore$ $t_{11} = 54$ **1**

(vi) $3x - 7y = 15$

Comparing with $a_1x + b_1y = c_1$ we get, $a_1 = 3$, $b_1 = -7$, $c_1 = 15$

$6x = 14y + 10$

$\therefore$ $6x - 14y = 10$

Comparing with $a_2x + b_2y = c_2$ we get, $a_2 = 6$, $b_2 = -14$, $c_2 = 10$ **1**

$$\therefore \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$$

$$\therefore \frac{b_1}{b_2} = \frac{-7}{-14} = \frac{1}{2}$$

$$\therefore \frac{c_1}{c_2} = \frac{15}{10} = \frac{3}{2}$$

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ The simultaneous equations $3x - 7y = 15$ and $6x = 14y + 10$ have no solution. **1**

A.3. Solve ANY THREE of the following :

(i) $y = \frac{5x - 10}{2}$

$\therefore \quad 2y = 5x - 10$

$\therefore \quad -5x + 2y = -10$

$4x + 5 = -y$

$\therefore \quad 4x + y = -5$

$D = \begin{vmatrix} -5 & 2 \\ 4 & 1 \end{vmatrix} = (-5 \times 1) - (2 \times 4) = -5 - 8 = -13$

$D_x = \begin{vmatrix} -10 & 2 \\ -5 & 1 \end{vmatrix} = (-10 \times 1) - (2 \times -5) = -10 + 10 = 0$

$D_y = \begin{vmatrix} -5 & -10 \\ 4 & -5 \end{vmatrix} = (-5 \times -5) - (-10 \times 4) = 25 + 40 = 65$ **1**

By Cramer's rule,

$x = \frac{D_x}{D} = \frac{0}{-13} = 0$ **1**

$y = \frac{D_y}{D} = \frac{65}{-13} = -5$

$\therefore$ $x = 0$ and $y = -5$ is the solution of given simultaneous equations. **1**

(ii) $-5, 2, 9, 16, 23, 30, \ldots\ldots$

$t_1 = -5, t_2 = 2, t_3 = 9, t_4 = 16, t_5 = 23, t_6 = 30$

$t_2 - t_1 = 2 - (-5) = 2 + 5 = 7$

$t_3 - t_2 = 9 - 2 = 7$

$t_4 - t_3 = 16 - 9 = 7$

$t_5 - t_4 = 23 - 16 = 7$

$t_6 - t_5 = 30 - 23 = 7$

$\because$ The difference between any two consecutive terms is 7 which is a constant. **1**

$\therefore$ The sequence is an A.P. with $a = t_1 = -5$.

Common difference (d) = 7

$t_n = a + (n - 1)\, d$

$t_n = -5 + (n - 1)\, 7$ **1**

$t_n = -5 + 7n - 7$

$t_n = 7n - 12$

$\therefore$ The general term of A.P. is $7n - 12$. **1**

(iii) Two coins are tossed

$\therefore$ S = { HH, HT, TH, TT }

$\therefore$ n (S) = 4

A is the event of getting at the most one head

$\therefore$ A = { HT, TH, TT }

$\therefore$ n (A) = 3

B is the event of getting both heads.

$\therefore$ B = { HH }

$\therefore$ n (B) = 1

C is the event of getting same face on both the coins

$\therefore$ C = { HH, TT }

$\therefore$ n (C) = 2 **2**

$A \cap B = \phi$ and $A \cup B = S$

$\therefore$ A and B are complementary and mutually exclusive events.

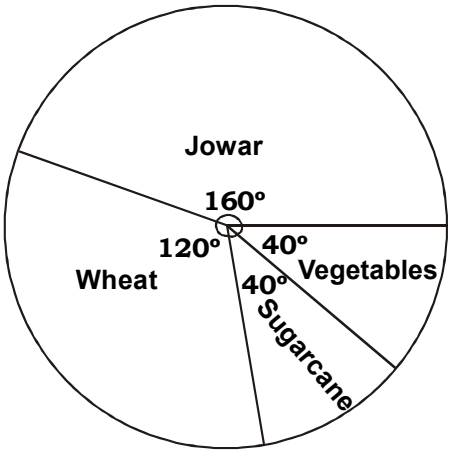
$\therefore$ $A \cup C = S$

$\therefore$ A and C are exhaustive events. **1**

(iv)

Crop	**Area in hectare**	**Measure of central angle**
Jowar	8000	$\frac{8000}{18000} \times 360^\circ = 160^\circ$
Wheat	6000	$\frac{6000}{18000} \times 360^\circ = 120^\circ$
Sugarcane	2000	$\frac{2000}{18000} \times 360^\circ = 40^\circ$
Vegetables	2000	$\frac{2000}{18000} \times 360^\circ = 40^\circ$
Total	**18000**	**360°**

1



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v.

Marks obtained	Continuous classes	Frequency No. of students
140 - 144	139.5 - 144.5	2
145 - 149	144.5 - 149.5	12
150 - 154	149.5 - 154.5	10
155 - 159	154.5 - 159.5	4

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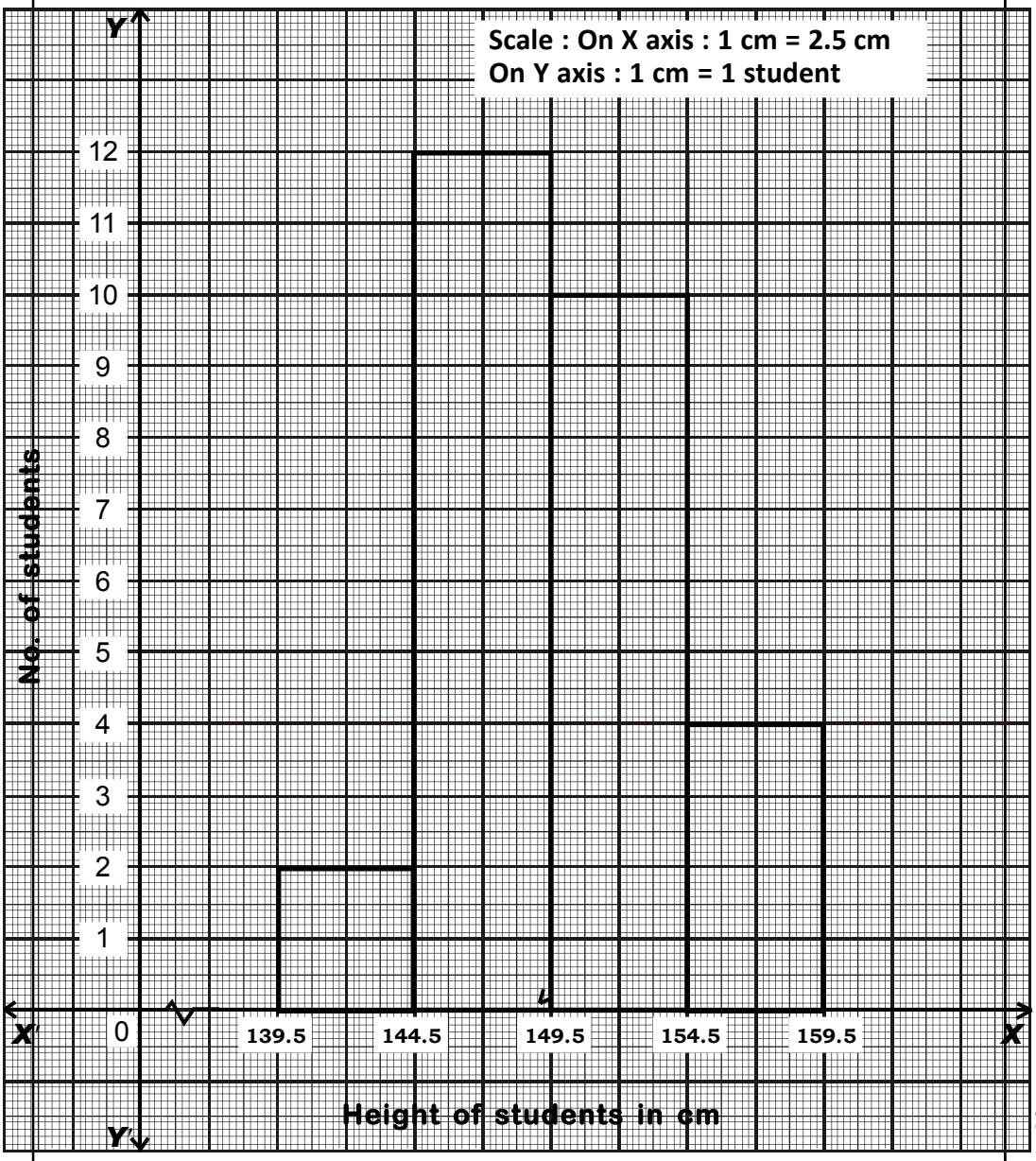

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A.4 **Solve ANY TWO of the following :**

(i) Two dice are thrown

S = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

$\therefore$ n (S) = 36 **1**

(a) Let A be the event that product of numbers on their upper faces is 12.

A = { (2, 6), (3, 4), (4, 3), (6, 2) }

n (A) = 4

$$P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{4}{36}$$

$$\therefore P(A) = \frac{1}{9}$$ **1½**

(b) Let B be the event that sum of the numbers on their upper faces is multiple of 7.

B = { (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) }

n (B) = 6

$$P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{6}{36}$$

$$\therefore P(B) = \frac{1}{6}$$ **1½**

(ii) $t_2 = 12, \quad t_4 = 20$

$t_n = a + (n - 1)d$

$t_2 = a + (2 - 1)d$

$12 = a + d$

$\therefore a + d = 12$ .....(i)

$t_4 = a + (4 - 1)d$

$20 = a + 3d$

$\therefore a + 3d = 20$ .....(ii) **1**

Subtracting (ii) from (i),

$a + d = 12$
$a + 3d = 20$
(–) (–) (–)
$-2d = -8$
$\therefore \quad d = 4$
Substituting d = 4 in (i),
$a + 4 = 12$
$\therefore \quad a = 12 - 4$
$\therefore \quad a = 8$ **1**

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{25} = \frac{25}{2}[2a + (25 - 1)d]$ **1**

$= \frac{25}{2}[2(8) + 24\ (4)]$

$= \frac{25}{2}[16 + 96]$

$= \frac{25}{2}[112]$

$S_{25} = 1400$

$\therefore$ Sum of 25 terms of the A.P is 1400. **1**

(iii) $(p^2 + p)(p^2 + p - 3) = 28$
Substituting $p^2 + p = m$ we get,
$\therefore \quad m\ (m - 3) = 28$
$\therefore \quad m^2 - 3m - 28 = 0$
$\therefore \quad m^2 - 7m + 4m - 28 = 0$
$\therefore \quad m\ (m - 7) + 4\ (m - 7) = 0$
$\therefore \quad (m - 7)\ (m + 4) = 0$
$\therefore \quad m - 7 = 0 \quad \text{or} \quad m + 4 = 0$
$\therefore \quad m = 7 \quad \text{or} \quad m = -4$ **1**
Resubstituting $m = p^2 + p$ we get,
$p^2 + p = 7 \quad \text{or} \quad p^2 + p = -4$
$\therefore \quad p^2 + p - 7 = 0$.....(i) or $p^2 + p + 4 = 0$ .....(ii)
From (i), $p^2 + p - 7 = 0$
Comparing with $ax^2 + bx + c = 0$ we have a =1, b = 1, c = – 7
$b^2 - 4ac = (1)^2 - 4\ (1)\ (-7)$
$= 1 + 28$
$= 29$ **1**

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{29}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{29}}{2}$$ **1**

From (ii), $p^2 + p + 4 = 0$

Comparing with $ax^2 + bx + c = 0$ we have a = 1, b =1, c = 4

$b^2 - 4ac = (1)^2 - 4\ (1)(4)$

$= 1 - 16$

$= -15$

$\because \quad b^2 - 4ac < 0$

$\therefore$ The roots of this equation are not real, hence not considered.

$$p = \frac{-1 \pm \sqrt{29}}{2}$$

$\therefore \quad p = \frac{-1 + \sqrt{29}}{2}$ or $p = \frac{-1 - \sqrt{29}}{2}$ **1**

A.5. Solve ANY TWO of the following :

(i) Let starting salary of man be Rs. x and the fixed annual increment be Rs. y.

As per the first given condition,

$x + 2y = 11000$ .....(i)

As per the second given condition,

$x + 4y = 14000$ .....(ii) **2**

Subtracting (ii) from (i),

$x + 2y = 11000$

$x + 4y = 14000$

(–) (–) (–)

$-2y = -3000$

$\therefore \quad y = \frac{-3000}{-2}$

$\therefore \quad y = 1500$ **1**

Substituting y = 1500 in (i),

$x + 2(1500) = 11000$

$\therefore \quad x + 3000 = 11000$

$\therefore \quad x = 11000 - 3000$

$\therefore$ $x = 8000$ **1**

$\therefore$ The starting salary of a man is Rs. 8000 and his fixed annual increment is Rs. 1500. **1**

(ii) Class width (h) = 3, Assumed mean (A) = 40

Diameter (in mm)	**Class Mark (x_i)**	**$d_i = x_i - A$**	**$u_i = \frac{d_i}{h}$**	**No. of screws (f_i)**	**$f_i u_i$**
33 - 35	34	– 6	– 2	10	– 20
36 - 38	37	– 3	– 1	19	– 19
39 - 41	40 $\rightarrow$ A	0	0	23	0
42 - 44	43	3	1	21	21
45 - 47	46	6	2	27	54
Total				**100**	**36**

3

$$\bar{u} = \frac{\Sigma f_i u_i}{\Sigma f_i}$$

$$\therefore \bar{u} = \frac{36}{100}$$

$\therefore$ $\bar{u} = 0.36$ **1**

$$\text{Mean } (\bar{x}) = A + h\bar{u}$$
$$= 40 + 3\,(0.36)$$
$$= 40 + 1.08$$
$$= 41.08$$

$\therefore$ Mean of diameter of the screws is 41.08 mm **1**

(iii) Let the number of days required by John alone to complete the work be x days

$\therefore$ No. of days requred by ganesh alone is (x + 10) days.
Also number of days required by both to complete the same work is 12 days

$\therefore$ Work done by John in 1 day = $\frac{1}{x}$

$\therefore$ Work done by Ganesh in 1 day = $\frac{1}{x+10}$

$\therefore$ Work done by both in 1 day = $\frac{1}{12}$ **1**

As per the given condition,

$$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$$ **1**

$\therefore \quad \frac{x + 10 + x}{x(x+10)} = \frac{1}{12}$

$\therefore \quad \frac{2x + 10}{x^2 + 10x} = \frac{1}{12}$

$\therefore \quad 12(2x + 10) = 1(x^2 + 10x)$

$\therefore \quad 24x + 120 = x^2 + 10x$

$\therefore \quad 0 = x^2 + 10x - 24x - 120$

$\therefore \quad x^2 - 14x - 120 = 0$

$\therefore \quad x^2 - 20x + 6x - 120 = 0$ **1**

$\therefore \quad x(x - 20) + 6(x - 20) = 0$

$\therefore \quad (x - 20)(x + 6) = 0$

$\therefore \quad x - 20 = 0 \text{ or } x + 6 = 0$

$\therefore \quad x = 20 \text{ or } x = -6$ **1**

$\because$ The numbers of days cannot be negative

$\therefore \quad x \neq -6$

Hence $x = 20$

$\therefore \quad x + 10 = 20 + 10 = 30$

$\therefore$ Ganesh alone worked for 30 days. **1**

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