MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 1 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY Five of the following : **5**

(i) Write the first five terms of the following Arithmetic Progressions where, the common difference 'd' and the first term 'a' is given :
a = 2, d = 2.5

(ii) Determine whether the given value of 'x' is a roots of given quadratic equation.
$x^2 - 2x + 1 = 0$, x = 1

(iii) Find the value of discriminant of the following equation.
$x^2 - 3x + 2 = 0$

(iv) If $D_y = -15$ and $D = -5$ is the value of the determinant for simultaneous equation in x and y, find y.

(v) If A = 40, $\bar{d} = 1.08$ and h = 3 then find mean.

(vi) For a pie diagram, $\theta = 75°$, Total = 54000 find the data.

Q.2. Solve ANY FOUR of the following : **8**

(i) Find the first three terms of the sequence for which S_n is given below :
$S_n = n^2(n + 1)$

(ii) Form the quadratic equation if its roots are :
5 and – 7

(iii) What is the equation of Y - axis? Hence, find the point of intersection of Y - axis. and the line y = 3x + 2.

(iv) In the following experiment write the sample space S, number of sample points n (S), events P, Q, R using set and n (P), n (Q) and n (R).
Find among the events defined above which are : complementary events, mutually exclusive events and exhaustive events.
A coin is tossed and a die is thrown simultaneously :
P is the event of getting head and odd number.
Q is the event of getting either H or T and an even number.
R is the event of getting a number on die greater than 7 and a tail.

(v) Which term of an A.P. is 93, if a = 150 and d = -3.

(vi) If two coins are tossed then find the probability of the events :
(a) at least one tail turns up
(b) no head turns up

Q.3. Solve ANY THREE of the following : 9

(i) Mary got a job with a starting salary of Rs. 15000/- per month. She will get an incentive of Rs. 100/- per month. What will be her salary after 20 months?

(ii) Solve the given quadratic equation by completing square.
$x^2 + 8x + 9 = 0$

(iii) A card is drawn at random from well shuffled pack of 52 cards. Find the probability that the card drawn is :
(a) a spade
(b) not of diamond

(iv) Find the probability of a four turning up at least once in two tosses of a fair die.

(v) Following is the component wise expenditure per article. Draw a pie chart :

Component	Expenditure (in Rs.)	Labour	Transportation	Packing	Taxes
Raw material	800	300	100	100	140

Q.4. Solve ANY TWO of the following : 8

(i) Solve the given simultaneous equation using graphical method.
$x + 2y = 5$; $y = -2x - 2$

(ii) Draw frequency polygon for the following data on land holding :

Area in hecters	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80
No. of farmers	58	103	208	392	112	34	12

(iii) Find the sum of all odd natural numbers from 1 to 150.

Q.5. Solve ANY TWO of the following : 10

(i) Solve the following equations :
$9\left[x^2 + \frac{1}{x^2}\right] - 3\left[x - \frac{1}{x}\right] - 20 = 0$

(ii) For the data given find median number of packages received per day by a post office.
Below is given frequency distribution of no. of packages received at a post office per day.

No. of packages	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
No. of days	2	8	16	24	30	20

(iii) A boat takes 6 hours to travel 8 km upstream and 32 km downstream, and it takes 7 hours to travel 20 km upstream and 16 km downstream. Find the speed of the boat in still water and the speed of the stream.

Best Of Luck 

MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 1 (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) $a = 2, d = 2.5$

Here, $t_1 = a = 2$

$t_2 = t_1 + d = 2 + 2.5 = 4.5$

$t_3 = t_2 + d = 4.5 + 2.5 = 7$

$t_4 = t_3 + d = 7 + 2.5 = 9.5$

$t_5 = t_4 + d = 9.5 + 2.5 = 12$

$\therefore$ The first five terms of the A.P. are 2, 4.5, 7, 9.5 and 12. **1**

(ii) $x^2 - 2x + 1 = 0, x = 1$

Putting $x = 1$ in L.H.S. we get,

L.H.S. $= (1)^2 - 2(1) + 1$

$= 1 - 2(1) + 1$

$= 2 - 2$

$= 0$

$= $ R.H.S.

$\therefore$ L.H.S. = R.H.S.

Thus equation is satisfied.

So 1 is the root of the given quadratic equation. **1**

(iii) $x^2 - 3x + 2 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 1, b = -3, c = 2$

$\Delta = b^2 - 4ac$

$= (-3)^2 - 4(1)(2)$

$= 9 - 8$

$= 1$

$\therefore$ $\Delta = 1$ **1**

(iv) $D_y = -15$ and $D = -5$

By Cramer's rule,

$$y = \frac{D_y}{D}$$

$\therefore \quad y = \frac{-15}{-5}$

$\therefore \quad \boxed{y = 3}$ **1**

(v) mean $(\bar{x}) = A + \bar{d}$

$= 40 + 1.08$

$= 41.08$

$\therefore$ mean is 41.08 units **1**

(vi) $\theta = \frac{\text{Data}}{\text{Total}} \times 360$

$\therefore \quad 75 = \frac{\text{Data}}{54000} \times 360$

$\therefore \quad \text{Data} = \frac{75 \times 54000}{360}$

$\therefore$ Data = 11250 **1**

A.2. Solve ANY Four of the following :

(i) $S_n = n^2 (n + 1)$

$\therefore \quad S_1 = 1^2 (1 + 1) = 1 (2) = 2$

$\therefore \quad S_2 = 2^2 (2 + 1) = 4 (3) = 12$

$\therefore \quad S_3 = 3^2 (3 + 1) = 9 (4) = 36$

We know that,

$t_1 = S_1 = 2$ **1**

$t_2 = S_2 - S_1 = 12 - 2 = 10$

$t_3 = S_3 - S_2 = 36 - 12 = 24$

$\therefore$ The first three terms of the sequence are 2, 10 and 24. **1**

(ii) The roots of the quadratic equation are 5 and – 7.

Let $\alpha = 5$ and $\beta = 7$

$\therefore \quad \alpha + \beta = 5 + (-7) = 5 - 7 = -2$

and $\alpha.\beta = 5 \times -7 = -35$

We know that,

$x^2 - (\alpha + \beta)x + \alpha.\beta = 0$ **1**

$\therefore \quad x^2 - (-2)x + (-35) = 0$

$\therefore \quad x^2 + 2x - 35 = 0$

$\therefore$ The required quadratic equation is $x^2 + 2x - 35 = 0$ **1**

(iii) The equation of Y-axis is x = 0

Let the point of intersection of the line y = 3x + 2 with Y-axis be (0, k) **1**

$\because$ (0, k) lies on the line it satisfies the equation
$\therefore$ Substituting x = 0 and y = k in the equation we get,
k = 3 (0) + 2
$\therefore$ k = 2
$\therefore$ The point of intersection of the line y = 3x + 2 with Y-axis is (0, 2). **1**

(iv) A coin is tossed and a die is thrown
S = { H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6 }
$\therefore$ n (S) = 12
P is the event of getting head and an odd number
P = { H1, H3, H5 }
$\therefore$ n (P) = 3
Q is the event of getting either H or T and an even number
Q = { H2, H4, H6, T2, T4, T6 }
$\therefore$ n (Q) = 6
R is the event of getting a number greater than 7 and a tail. **1**
R = { }
$\therefore$ n (R) = 0
$P \cap Q = \phi$
$\therefore$ P and Q are mutually exclusive events.
$Q \cap R = \phi$
$\therefore$ Q and R are mutually exclusive events. **1**
$P \cap R = \phi$
$\therefore$ P and R are mutually exclusive events.

(v) For an A.P. a = 150, d = -3, t_n = 93

$t_n = a + (n-1)d$
$\therefore$ 93 = 150 + (n-1) (-3) **1**
$\therefore$ -57– 3 = -3n
$\therefore$ – 60 = -3n
$\therefore$ n = 20
$\therefore$ 20^{th} term of an A.P. is 93 **1**

(vi) When two coins are tossed
S = { HH, HT, TH, TT }
n (S) = 4
(a) Let A be the event that atleast one tail turns up
A = { HT, TH, TT }
n (A) = 3
$P(A) = \frac{n(A)}{n(S)}$

$\therefore$ $\boxed{P(A) = \frac{3}{4}}$ **1**

(b) Let B be event that no head turns up

$B = \{TT\}$

$n(B) = 1$

$P(B) = \frac{n(B)}{n(S)}$

$\therefore$ $\boxed{P(B) = \frac{1}{4}}$ **1**

A.3. Solve ANY THREE of the following :

(i) Since Mary's salary increases by Rs. 100 every month the successive salaries are in A.P. **1**

Starting salary of Mary (a) = Rs. 15000

Monthly incentive in salary (d) = 100

No. of months (n) = 20

Salary after twenty months = t_{20} = ? **1**

$\therefore$ $t_n = a + (n - 1)\,d$

$\therefore$ $t_{20} = a + (20 - 1)\,d$

$\therefore$ $t_{20} = 15000 + 19\,(100)$

$\therefore$ $t_{20} = 15000 + 1900$

$\therefore$ $t_{20} = 16900$

$\therefore$ Mary salary after twenty months is Rs. 16900. **1**

(ii) $x^2 + 8x + 9 = 0$

$\therefore$ $x^2 + 8x = -9$ .....(i)

Third term $= \left(\frac{1}{2} \times \text{coefficient of } x\right)^2$

$= \left(\frac{1}{2} \times 8\right)^2$ **1**

$= (4)^2$

$= 16$

Adding 16 to both sides of (i) we get,

$x^2 + 8x + 16 = -9 + 16$

$\therefore$ $(x + 4)^2 = 7$

$\therefore$ $(x + 4)^2 = \left(\sqrt{7}\right)^2$

Taking square root on both the sides we get, **1**

$x + 4 = \pm\sqrt{7}$

$\therefore$ $x = -4 \pm \sqrt{7}$

$\therefore$ $x = -4 + \sqrt{7}$ or $x = -4 - \sqrt{7}$

$\boxed{-4 + \sqrt{7} \text{ and } -4 - \sqrt{7} \text{ are the roots of the given quadratic equations.}}$ **1**

(iii) There are 52 cards in a pack

$\therefore$ n (S) = 52

(a) Let A be event that the card drawn is a spade card

$\therefore$ There are 13 spade cards

$\therefore$ n (A) = 13

$$P(A) = \frac{n(A)}{n(S)} \qquad \text{½}$$

$$\therefore P(A) = \frac{13}{52}$$

$$\therefore \boxed{P(A) = \frac{1}{4}} \qquad \mathbf{1}$$

(b) Let B be event that the card drawn is not a diamond

There are 13 diamond cards

$\therefore$ There are 39 cards which are not of diamond

$\therefore$ n (B) = 39

$$P(B) = \frac{n(B)}{n(S)} \qquad \text{½}$$

$$\therefore P(B) = \frac{39}{52}$$

$$\therefore \boxed{P(B) = \frac{3}{4}} \qquad \mathbf{1}$$

(iv) The sample space when a fair die is tossed twice.

S = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), **1**
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

n (S) = 36

Let A be the event of getting 4 at least one time when two dice are thrown.

A = { (1, 4), (2, 4), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), **1**
(4, 5), (4, 6), (5, 4), (6, 4) }

n (A) = 11

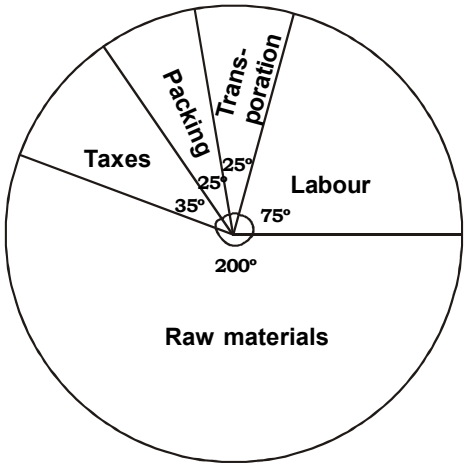
$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$\therefore \boxed{P(A) = \frac{11}{36}} \qquad \mathbf{1}$$

(v)

Component	Expenditure	Measure of central angle
Raw material	800	$\frac{800}{1440} \times 360^\circ = 200^\circ$
Labour	300	$\frac{300}{1440} \times 360^\circ = 75^\circ$
Transportation	100	$\frac{100}{1440} \times 360^\circ = 25^\circ$
Packing	100	$\frac{100}{1440} \times 360^\circ = 25^\circ$
Taxes	140	$\frac{140}{1440} \times 360^\circ = 35^\circ$
Total	**1440**	**360°**

1



2

A.4. **Solve ANY TWO of the following :**

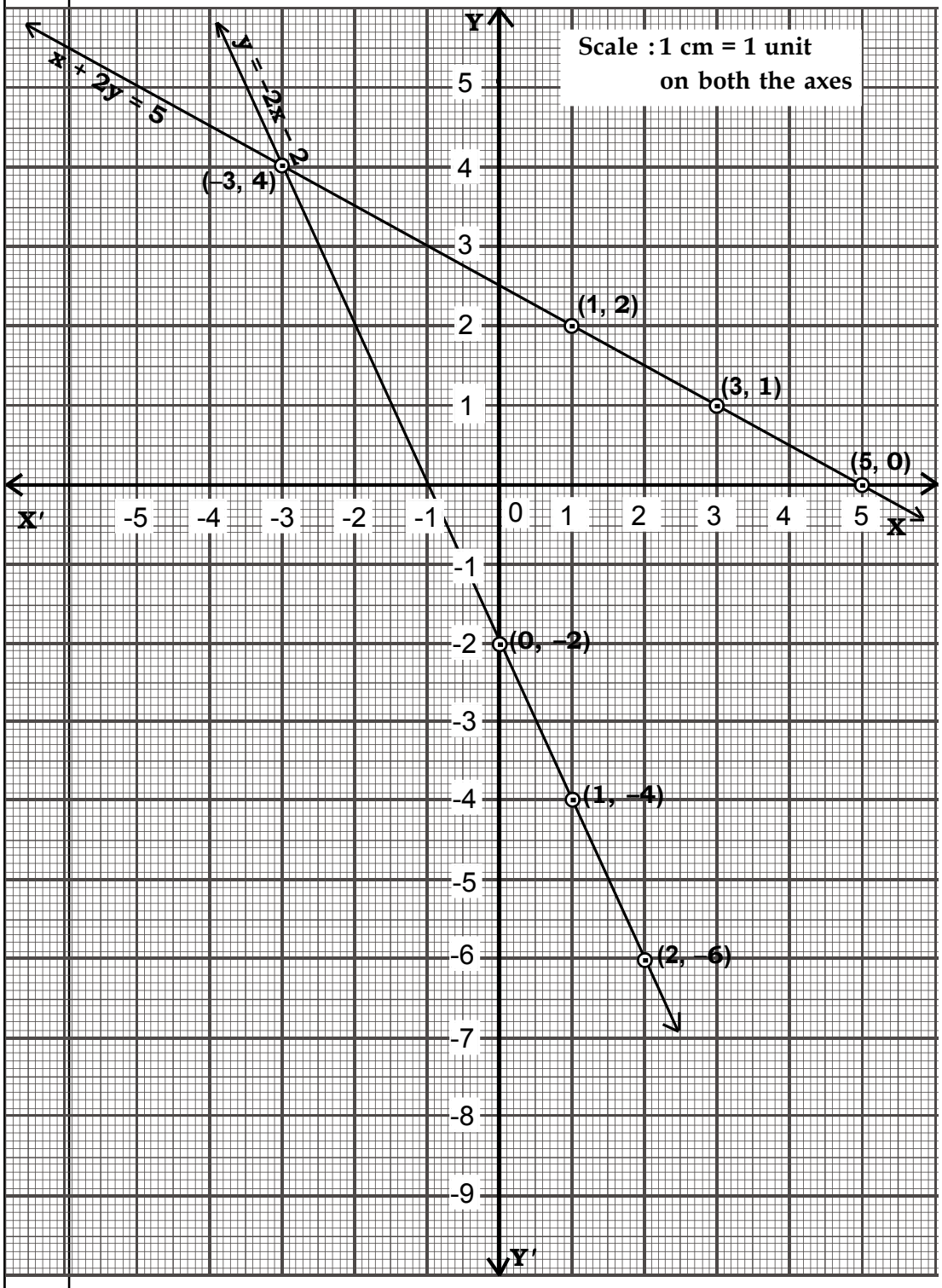
(i) $x + 2y = 5 \quad y = -2x - 2$

$\therefore \quad x = 5 - 2y$

x	5	3	1
y	0	1	2
(x, y)	(5, 0)	(3, 1)	(1, 2)

x	0	1	2
y	–2	–4	–6
(x, y)	(0, –2)	(1, –4)	(2, –6)

1



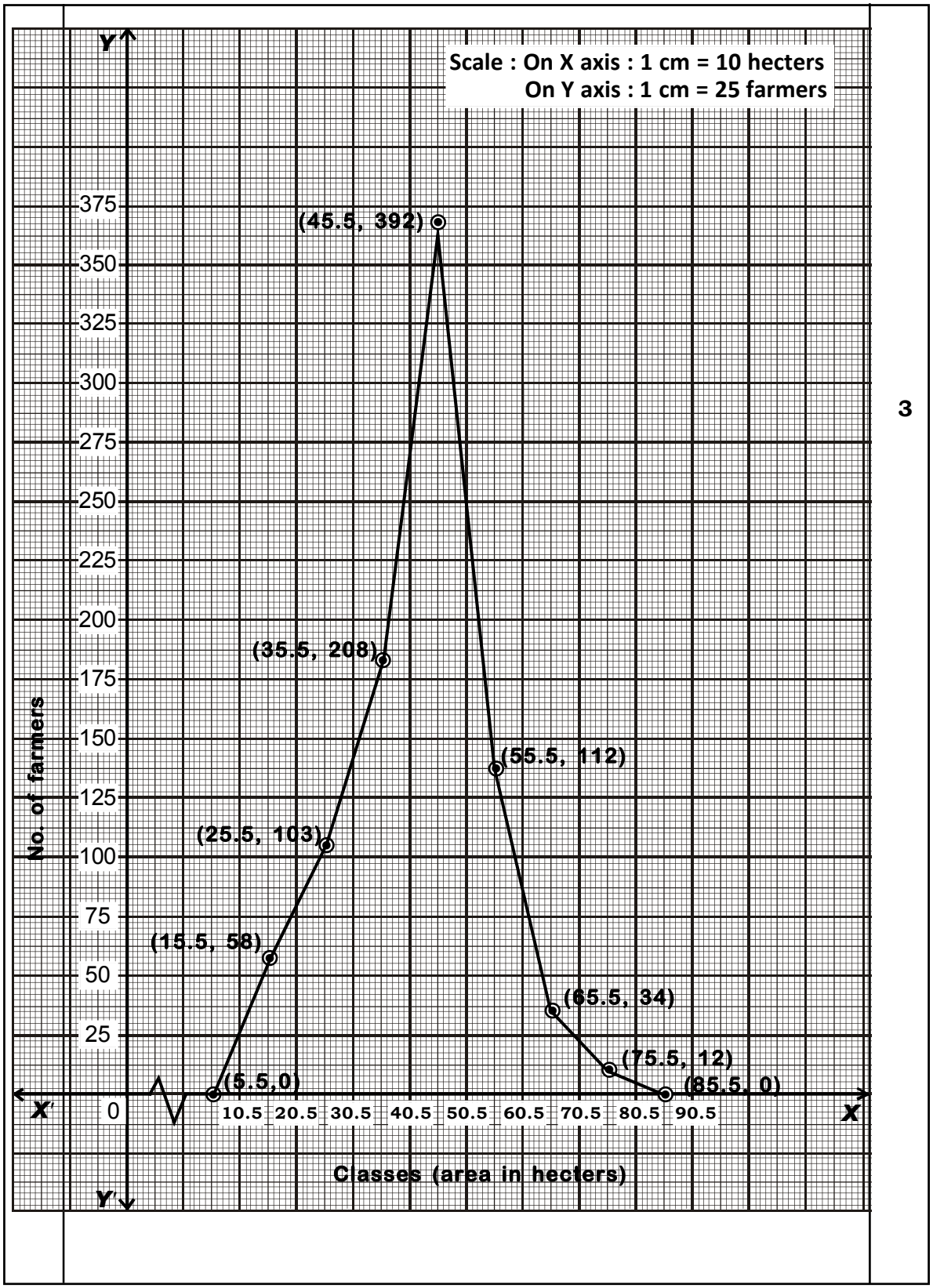
2

∴ x = – 3 and y = 4 is the solution of given simultaneous equations. 1

(ii)

Area in hecters	Continuous classes	Class mark	No. of farmers
11 - 20	10.5 - 20.5	15.5	58
21 - 30	20.5 - 30.5	25.5	103
31 - 40	30.5 - 40.5	35.5	208
41 - 50	40.5 - 50.5	45.5	392
51 - 60	50.5 - 60.5	55.5	112
61 - 70	60.5 - 70.5	65.5	34
71 - 80	70.5 - 80.5	75.5	12

1



3

(iii) The odd natural numbers from 1 to 150 are as follows
1, 3, 5, 7, 9, ..., 149.
These numbers form an A.P. with a = 1, d = 2
Let, 149 be n^{th} term of an A.P.

$t_n = 149$

$t_n = a + (n - 1)\,d$ **1**

$149 = 1 + (n - 1)\,2$

$149 = 1 + 2n - 2$

$149 = 2n - 1$

$149 + 1 = 2n$

$\therefore\ 2n = 150$

$\therefore\ n = 75$

$\therefore$ 149 is 75^{th} term of A.P. **1**

$\therefore$ We have to find sum of 75 terms i.e. S_{75}

$S_n = \frac{n}{2}[2a + (n - 1)\,d]$

$\therefore\ S_{75} = \frac{75}{2}[2\,(1) + (75 - 1)\,2]$

$\therefore\ S_{75} = \frac{75}{2}[2 + 74\,[2)]$ **1**

$= \frac{75}{2}[2 + 148]$

$= \frac{75}{2}(150)$

$= 75\,(75)$

$\therefore\ S_{75} = 5625$

Alternative method :

$S_n = \frac{n}{2}[t_1 + t_n]$

$S_{75} = \frac{75}{2}[t_1 + t_{75}]$

$= \frac{75}{2}[1 + 149]$

$= \frac{75}{2}[150]$

$= 75\,[75]$

$\therefore\ S_{75} = 5625$

$\therefore$ Sum of all odd natural numbers from 1 to 150 is 5625. **1**

A.5. Solve ANY TWO of the following :

(i) $9\left[x^2 + \frac{1}{x^2}\right] - 3\left[x - \frac{1}{x}\right] - 20 = 0$ .....(i)

Substituting $x - \frac{1}{x} = m$

Squaring both the sides we get,

$\left(x - \frac{1}{x}\right)^2 = m^2$ **1**

$\therefore\ x^2 - 2 + \frac{1}{x^2} = m^2$

$\therefore\ x^2 + \frac{1}{x^2} = m^2 + 2$

$\therefore$ Equation (i) becomes,

$9(m^2 + 2) - 3m - 20 = 0$

$\therefore\ 9m^2 + 18 - 3m - 20 = 0$

$\therefore \quad 9m^2 - 3m - 2 = 0$

$\therefore \quad 9m^2 + 3m - 6m - 2 = 0$

$\therefore \quad 3m(3m+1) - 2(3m+1) = 0$

$\therefore \quad (3m+1)(3m-2) = 0$

$\therefore \quad 3m + 1 = 0 \text{ or } 3m - 2 = 0$ **1**

$\therefore \quad 3m = -1 \text{ or } 3m = 2$

$\therefore \quad m = -\frac{1}{3} \text{ or } m = \frac{2}{3}$

Resubstituting $m = x - \frac{1}{x}$ we get,

$x - \frac{1}{x} = -\frac{1}{3}$ .....(ii) or $x - \frac{1}{x} = \frac{2}{3}$ .....(iii)

From (ii), $x - \frac{1}{x} = -\frac{1}{3}$

Multiplying throughout by 3x we get,

$\therefore \quad 3x^2 - 3 = -x$

$\therefore \quad 3x^2 + x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 3, b = 1, c = -3$

$b^2 - 4ac = (1)^2 - 4(3)(-3)$

$= 1 + 36$

$= 37$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **1**

$= \frac{-1 \pm \sqrt{37}}{2(3)}$

$= \frac{-1 \pm \sqrt{37}}{6}$

$\therefore \quad x = \frac{1 + \sqrt{37}}{6} \text{ or } x = \frac{-1 - \sqrt{37}}{6}$

From (iii), $x - \frac{1}{x} = \frac{2}{3}$

Multiplying throughout by 3x, we get;

$3x^2 - 3 = 2x$

$\therefore \quad 3x^2 - 2x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 3, b = -2, c = -3$

$b^2 - 4ac = (-2)^2 - 4(3)(-3)$

$= 4 + 36$

$= 40$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **1**

$= \frac{-2 \pm \sqrt{40}}{2(3)}$

$= \dfrac{-2 \pm \sqrt{4 \times 10}}{6}$

$= \dfrac{-2 \pm 2\sqrt{10}}{6}$

$= \dfrac{-1 \pm \sqrt{10}}{3}$

$\therefore \quad x = \dfrac{-1+\sqrt{10}}{3}$ or $\dfrac{-1-\sqrt{10}}{3}$

$\therefore$ $x = \dfrac{1+\sqrt{37}}{6}$ or $x = \dfrac{-1-\sqrt{37}}{6}$ or $x = \dfrac{-1+\sqrt{10}}{3}$ or $x = \dfrac{-1-\sqrt{10}}{3}$ **1**

(ii)

Classes	**Frequency (f_i) (No. of days)**	**Cumulative frequency less than type**
10 - 20	2	2
20 - 30	8	10
30 - 40	16	26
40 - 50	24	50 → *c.f.*
50 - 60	30 → f	80
60 - 70	20	100
Total	**100 → N**	

2

Here total frequency = $\Sigma f_i = N = 100$

$\therefore \quad \dfrac{N}{2} = \dfrac{100}{2} = 50$ **1**

Cumulative frequency (less than type) which is just greater than 50 is 80. Therefore corresponding class 50 - 60 is median class.

L = 50, N = 100, *c.f.* = 50, f = 30, h = 10

Median $= L + \left(\dfrac{N}{2} - c.f.\right)\dfrac{h}{f}$

$= 50 + \left(\dfrac{100}{2} - 50\right)\dfrac{10}{30}$

$= 50 + (50 - 50)\dfrac{10}{30}$

$= 50 + (0)\dfrac{10}{30}$ **1**

$= 50 + 0$

$= 50$

$\therefore$ Median of package received by post office is 50. **1**

(iii) Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

$\therefore$ Speed of the boat upstream = (x – y) km/hr

and speed of the boat downstream = (x + y) km/hr

We know that, Time = $\frac{\text{Distance}}{\text{Speed}}$

As per the first condition,

$$\frac{8}{x-y}+\frac{32}{x+y} = 6 \quad \text{.....(i)}$$ **1**

As per the second condition,

$$\frac{20}{x-y}+\frac{16}{x+y} = 7 \quad \text{.....(ii)}$$

Substituting $\frac{1}{x-y} = m$ and $\frac{1}{x+y} = n$ in (i) and (ii) we get,

$$8m + 32n = 6 \quad \text{.....(iii)}$$ **1**

$$20m + 16n = 7 \quad \text{.....(iv)}$$

Multiplying (iv) by 2 we get,

$$40m + 32n = 14 \quad \text{.....(v)}$$

Subtracting (v) from (iii),

$$\begin{array}{rcl} 8m + 32n & = & 6 \\ 40m + 32n & = & 14 \\ \hline -32m & = & -8 \end{array}$$

$$\therefore \quad m = \frac{-8}{-32}$$ **1**

$$\therefore \quad m = \frac{1}{4}$$

Substituting $m = \frac{1}{4}$ in (iii),

$$8\left(\frac{1}{4}\right) + 32n = 6$$

$$\therefore \quad 2 + 32n = 6$$

$$\therefore \quad 32n = 6 - 2$$

$$\therefore \quad 32n = 4$$

$$\therefore \quad n = \frac{4}{32}$$

$$\therefore \quad n = \frac{1}{8}$$

Resubstituting the values of m and n we get,

$$m = \frac{1}{x-y}$$ **1**

$$\therefore \quad \frac{1}{4} = \frac{1}{x-y}$$

$\therefore \quad x - y = 4 \quad \ldots\ldots(vi)$

$n = \frac{1}{x + y}$

$\therefore \quad \frac{1}{8} = \frac{1}{x + y}$

$\therefore \quad x + y = 8 \quad \ldots\ldots(vii)$

Adding (vi) and (vii),

$$\begin{array}{rcl} x - y & = & 4 \\ x + y & = & 8 \\ \hline 2x & = & 12 \end{array}$$

$\therefore \quad x = \frac{12}{2}$

$\therefore \quad x = 6$

Substituting x = 6 in (vii)

$6 + y = 8$

$\therefore \quad y = 8 - 6$

$\therefore \quad y = 2$

$\therefore$ The speed of boat in still water is 6 km/hr and speed of stream is 2 km/ hr. **1**

❖❖❖❖