

MT

2014 ___ ___ 1100

Seat No.

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MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 2 (E)

Time : 2 Hours

(Pages 3)

Max. Marks : 40

Note :

- (i) All questions are compulsory.
- (ii) Use of calculator is not allowed.

Q.1. Solve ANY Five of the following :

5

- (i) Write the first five terms of the following Arithmetic Progressions where, the common difference 'd' and the first term 'a' is given :
 $a = 10, d = -3$
- (ii) Determine whether the given values of 'x' is a roots of given quadratic equation.
 $x^2 + 2x + 1 = 0, x = -1$
- (iii) Find the value of discriminant of the following equation.
 $2x^2 + x + 1 = 0$
- (iv) If $D_x = -18$ and $D = 3$ are the values of the determinants for certain simultaneous equations in x and y, find x.
- (v) If $A = 55, \bar{d} = -3.25$ and $h = 10$, then, find mean.
- (vi) For a pie diagram, $\theta = 75^\circ$, Total = 54000, find the data.

Q.2. Solve ANY FOUR of the following :

8

- (i) Find the twenty fifth term of the A. P. : 12, 16, 20, 24,
- (ii) Form the quadratic equation if its roots are :
3 and -11

- (iii) If $12x + 13y = 29$ and $13x + 12y = 21$, Find $x + y$.
- (iv) In each of the following experiments write the sample space S , number of sample points $n(S)$, events P, Q, R using set and $n(P), n(Q)$ and $n(R)$. Find the events among the events defined above which are : complementary events, mutually exclusive events and exhaustive events.
A die is thrown :
- (a) P is the event of getting an odd number.
(b) Q is the event of getting an even number.
(c) R is the event of getting a prime number.
- (v) Which term of an A.P. is 55, if $a = 3$ and $d = 1.3$.
- (vi) Two coins are tossed. Find the probability of the events.
(i) head appears on both the coins.
(ii) head does not appear.

Q.3. Solve ANY THREE of the following :**9**

- (i) The taxi fare is Rs. 14 for the first kilometer and Rs. 2 for each additional kilometer. What will be fare for 10 kilometers ?
- (ii) Solve the given quadratic equations by completing square.
 $z^2 + 6z - 8 = 0$
- (iii) A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that the card drawn is
(a) bears a number between 4 and 7 both inclusive.
(b) bears a number between 3 and 8 both inclusive.
- (iv) What is the probability that an ordinary year has 53 Sundays ?
- (v) Area under different crops in a certain village is given below. Represent it by pie diagram

Crop	Jowar	Wheat	Sugarcane	Vegetables
Area in hectare	8000	6000	2000	2000

Q.4. Solve ANY TWO of the following :**8**

- (i) Solve the given simultaneous equations using graphical method :
 $4x = y - 5$; $y = 2x + 1$

- (ii) Represent the following data using frequency curve :

Electricity bill in a month (in Rs.)	200 - 400	400-600	600 - 800	800 - 1000
No. of families	362	490	185	63

Draw histogram and hence draw frequency curve.

- (iii) Find the sum of all numbers from 1 to 140 which are divisible by 4.

Q.5. Solve ANY TWO of the following :

10

- (i) The sum of the squares of five consecutive natural numbers is 1455. Find the numbers.
- (ii) For the data given, find median calories consumed daily by a boy. Number of calories (in' 00) consumed daily by a sample of 15 years old boys are given below.

Calories	1000 – 1500	1500 – 2000	2000 – 2500	2500 – 3000	3000 – 3500	3500 – 4000	4000 – 4500
No. of boys	5	13	16	18	27	10	4

- (iii) A bus covers a certain distance with uniform speed. If the speed of the bus would have been increased by 15 km/h, it would have taken two hours less to cover the same distance and if the speed of the bus would have been decreased by 5 km/h, it would have taken one hour more to cover the same distance. Find the distance covered by the bus.

Best Of Luck 🍀

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Preliminary Model Answer Paper

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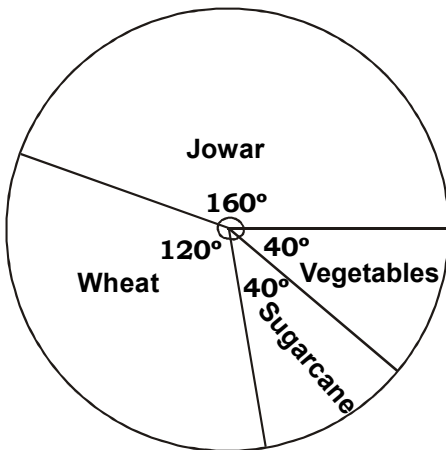
<p>A.1.</p> <p>(i)</p> <p>(ii)</p> <p>(iii)</p> <p>(iv)</p>	<p>Attempt ANY FIVE of the following :</p> <p>$a = 10, d = -3$</p> <p>Here, $t_1 = a = 10$</p> $t_2 = t_1 + d = 10 + (-3) = 10 - 3 = 7$ $t_3 = t_2 + d = 7 + (-3) = 7 - 3 = 4$ $t_4 = t_3 + d = 4 + (-3) = 4 - 3 = 1$ $t_5 = t_4 + d = 1 + (-3) = 1 - 3 = -2$ <p>\therefore The first five terms of the A.P. are 10, 7, 4, 1 and -2.</p> <p>Putting $x = -1$ in L.H.S. we get,</p> $\begin{aligned} \text{L.H.S.} &= (-1)^2 + 2(-1) + 1 \\ &= 1 - 2 + 1 \\ &= 2 - 2 \\ &= 0 \\ &= \text{R.H.S.} \end{aligned}$ <p>\therefore L.H.S. = R.H.S.</p> <p>Thus equation is satisfied.</p> <p>So -1 is the root of the given quadratic equation.</p> $2x^2 + x + 1 = 0$ <p>Comparing with $ax^2 + bx + c = 0$ we have $a = 2, b = 1, c = 1$</p> $\begin{aligned} \Delta &= b^2 - 4ac \\ &= (1)^2 - 4(2)(1) \\ &= 1 - 8 \\ &= -7 \end{aligned}$ <p>\therefore $\Delta = -7$</p> <p>$D_x = -18$ and $D = 3$</p> <p>By Cramer's rule,</p> $x = \frac{D_x}{D}$	<p>1</p> <p>1</p> <p>1</p>
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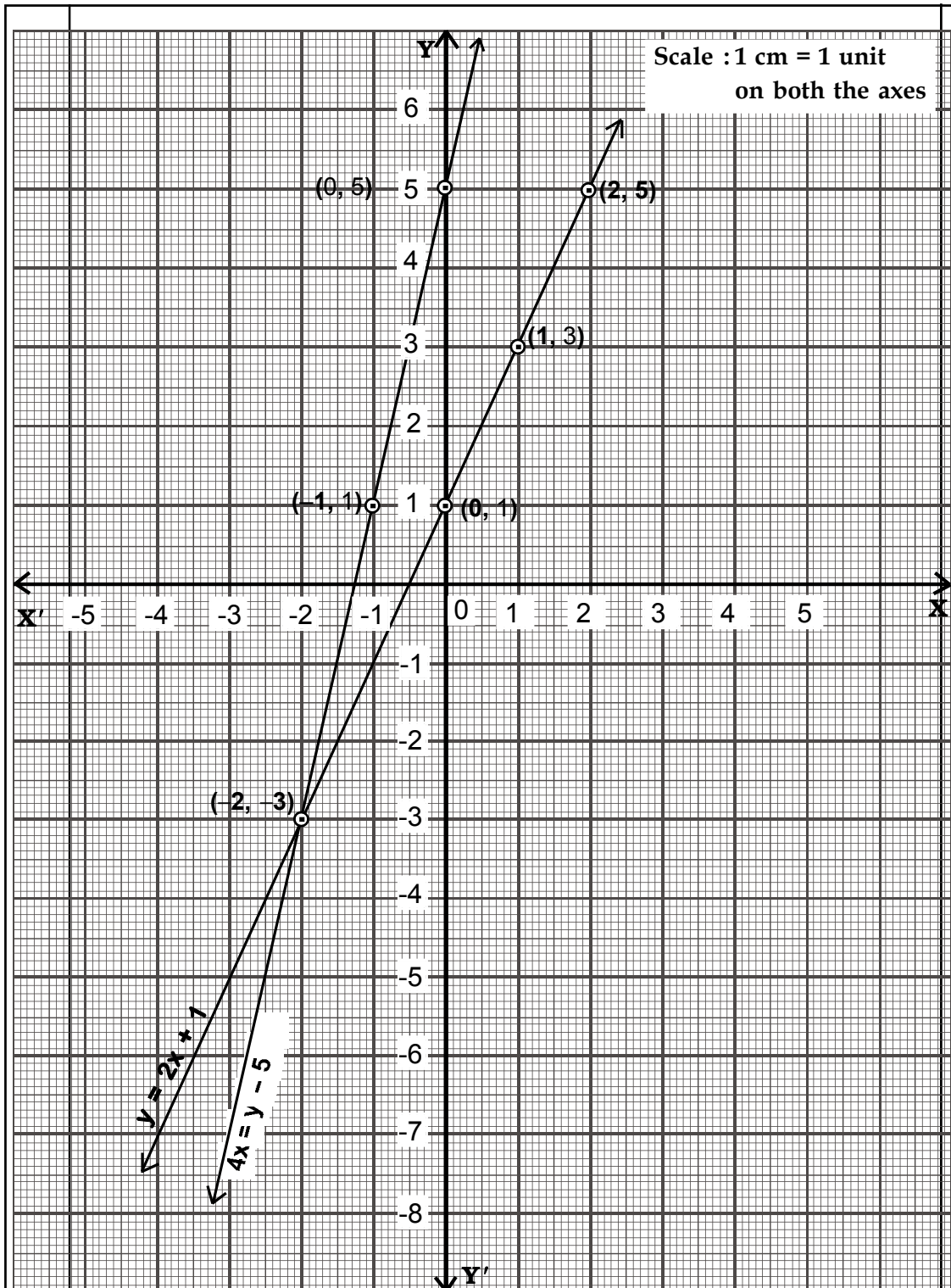
	$\therefore x = \frac{-18}{3}$ $\therefore \boxed{x = -6}$	1
(v)	$\begin{aligned} \text{Mean } (\bar{x}) &= A + \bar{d} \\ &= 55 + (-3.25) \\ &= 55 - 3.25 \\ &= 51.75 \end{aligned}$ $\therefore \boxed{\text{Mean is 51.75 units.}}$	1
(vi)	$\theta = \frac{\text{Data}}{\text{Total}} \times 360$ $\therefore 50 = \frac{\text{Data}}{54000} \times 360$ $\therefore \text{Data} = \frac{50 \times 54000}{360}$ $\therefore \boxed{\text{Data} = 7500}$	1
A.2.	Solve ANY Four of the following :	
(i)	<p>For the given A.P. 12, 16, 20, 24,</p> <p>Here, $a = t_1 = 12$ $d = t_2 - t_1 = 16 - 12 = 4$</p> <p>We know,</p> $t_n = a + (n - 1) d$ $\therefore t_{25} = a + (25 - 1) d$ $\therefore t_{25} = 12 + 24 (4)$ $\therefore t_{25} = 12 + 96$ $\therefore t_{25} = 108$ $\therefore \boxed{\text{The twenty fifth term of A.P. is 108.}}$	1
(ii)	<p>The roots of the quadratic equation are -3 and -11.</p> <p>Let $\alpha = -3$ and $\beta = -11$</p> $\therefore \alpha + \beta = -3 + (-11) = -3 - 11 = -14$ <p>and $\alpha \cdot \beta = -3 \times -11 = 33$</p> <p>We know that,</p> $x^2 - (\alpha + \beta)x + \alpha \cdot \beta = 0$ $\therefore x^2 - (-14)x + 33 = 0$ $\therefore x^2 + 14x + 33 = 0$ $\therefore \boxed{\text{The required quadratic equation is } x^2 + 14x + 33 = 0}$	1

<p>(iii)</p>	$12x + 13y = 29 \quad \dots(i)$ $13x + 12y = 21 \quad \dots(ii)$ <p>Adding (i) and (ii),</p> $12x + 13y = 29$ $13x + 12y = 21$ <hr style="width: 50%; margin-left: 0;"/> $25x + 25y = 50$	<p>1</p>
	<p>Dividing throughout by 25 we get,</p> $x + y = \frac{50}{25}$ <p>\therefore $x + y = 2$</p>	
<p>(iv)</p>	<p>When a die is thrown</p> $S = \{1, 2, 3, 4, 5, 6\}$ $n(S) = 6$ <p>P is the event of getting an odd number</p> $P = \{1, 3, 5\}$ <p>$\therefore n(P) = 3$</p> <p>Q is the event of getting an even number</p> $Q = \{2, 4, 6\}$ <p>$\therefore n(Q) = 3$</p> <p>R is the event of getting a prime number</p> $R = \{2, 3, 5\}$ <p>$\therefore n(R) = 3$</p> <p>Here $P \cap Q = \phi$ and $P \cup Q = S$</p>	<p>1</p>
	<p>\therefore P and Q are complementary events.</p>	
<p>(v)</p>	<p>For an A.P. $a = 3, d = 1.3, t_n = 55$</p> <p>$\therefore t_n = a + (n - 1)d$</p> <p>$\therefore 55 = 3 + (n-1)(1.3)$</p> <p>$\therefore 55 - 3 = 1.3n - 1.3$</p> <p>$\therefore 52 + 1.3 = 1.3n$</p> <p>$\therefore 53 - 3 = 1.3n$</p> <p>$\therefore n = 41$</p> <p>41st term of an A.P. is 55</p>	<p>1</p>
<p>(vi)</p>	<p>Two coins are tossed</p> $S = \{HH, HT, TH, TT\}$ $n(S) = 4$	<p>1</p>

	<p>(a) head appears on both the coins. Let A be the event that head appears on both the coins $A = \{HH\}$ $n(A) = 1$ $P(A) = \frac{n(A)}{n(S)}$</p> <p>$\therefore P(A) = \frac{1}{4}$</p> <p>(b) head does not appear. Let B be the event that head does not appear $B = \{TT\}$ $n(B) = 1$ $P(B) = \frac{n(B)}{n(S)}$</p> <p>$\therefore P(B) = \frac{1}{4}$</p> <p>A.3. Solve ANY THREE of the following :</p> <p>(i) Since the taxi fare increases by Rs. 2 every kilometer after the first, the successive taxi fares form an A.P. The taxi fare for first kilometer (a) = Rs. 14 Increase in taxi fare in every kilometer after first kilometer (d) = 2 No. of kilometers covered by taxi (n) = 10 Taxi fare for 10 kilometers = $t_{10} = ?$ $t_n = a + (n - 1) d$ $\therefore t_{10} = a + (10 - 1) d$ $\therefore t_{10} = 14 + 9(2)$ $\therefore t_{10} = 14 + 18$ $\therefore t_{10} = 32$</p> <p>\therefore Taxi fare for ten kilometers is Rs. 32.</p> <p>(ii) $z^2 + 6z - 8 = 0$ $\therefore z^2 + 6z = 8$ (i) Third term = $\left(\frac{1}{2} \times \text{coefficient of } z\right)^2$ $= \left(\frac{1}{2} \times 6\right)^2$ $= (3)^2$ $= 9$ Adding 9 to both sides of (i), we get,</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
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	$z^2 + 6z + 9 = 8 + 9$ $\therefore (z + 3)^2 = 17$ <p>Taking square root on both the sides we get,</p> $z + 3 = \pm\sqrt{17}$ $\therefore z = -3 \pm \sqrt{17}$ $\therefore z = -3 + \sqrt{17} \quad \text{or} \quad z = -3 - \sqrt{17}$	1
	$\therefore -3 + \sqrt{17} \text{ and } -3 - \sqrt{17} \text{ are the roots of the given quadratic equations.}$	1
(iii)	<p>There are 52 cards in a pack of cards</p> $\therefore n(S) = 52$ <p>(i) Let A be the event that card drawn bears a number between 4 and 7 both inclusive</p> <p>\therefore There are 4 numbers from 4 to 7 inclusive of both and there are 4 types of cards</p> <p>\therefore Total no. of cards bearing numbers from 4 to 7 both inclusive in the pack of cards are $4 \times 4 = 16$</p> $n(A) = 16$ $P(A) = \frac{n(A)}{n(S)}$ $\therefore P(A) = \frac{16}{52}$ $\therefore P(A) = \frac{4}{13}$	$\frac{1}{2}$
	<p>(ii) Let B be the event that card drawn bears a number between 3 and 8 both inclusive.</p> <p>\therefore There are 6 numbers from 3 to 8 both inclusive and there are 4 types of cards</p> <p>\therefore Total no. of cards bearing numbers from 3 to 8 both inclusive in the pack of cards is $6 \times 4 = 24$</p> $n(B) = 24$ $\therefore P(B) = \frac{n(B)}{n(S)}$ $\therefore P(B) = \frac{24}{52}$ $\therefore P(B) = \frac{6}{13}$	$\frac{1}{2}$
(iv)	<p>An ordinary year has 365 days 52 weeks and 1 extra day 52 weeks will have 52 Sundays The sample space for one extra day is</p>	1

	$S = \{ \text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} \}$ $n(S) = 7$ <p>Let A be the event that the extra day is a Sunday</p> $\therefore A = \{ \text{Sunday} \}$ $P(A) = \frac{1}{7}$ $P(A) = \frac{n(A)}{n(S)}$ $\therefore P(A) = \frac{1}{7}$	1 1 1																								
(v)	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">Crop</th> <th style="text-align: center;">Area in hectare</th> <th style="text-align: center;">Measure of central angle</th> </tr> </thead> <tbody> <tr> <td>Jowar</td> <td style="text-align: center;">8000</td> <td style="text-align: center;">$\frac{8000}{18000} \times 360^\circ = 160^\circ$</td> </tr> <tr> <td>Wheat</td> <td style="text-align: center;">6000</td> <td style="text-align: center;">$\frac{6000}{18000} \times 360^\circ = 120^\circ$</td> </tr> <tr> <td>Sugarcane</td> <td style="text-align: center;">2000</td> <td style="text-align: center;">$\frac{2000}{18000} \times 360^\circ = 40^\circ$</td> </tr> <tr> <td>Vegetables</td> <td style="text-align: center;">2000</td> <td style="text-align: center;">$\frac{2000}{18000} \times 360^\circ = 40^\circ$</td> </tr> <tr> <td>Total</td> <td style="text-align: center;">18000</td> <td style="text-align: center;">360°</td> </tr> </tbody> </table> <div style="text-align: center;">  <p>A pie chart representing the distribution of crops. The largest sector is Jowar with a central angle of 160°. Wheat follows with 120°. Sugarcane and Vegetables each have a central angle of 40°.</p> </div>	Crop	Area in hectare	Measure of central angle	Jowar	8000	$\frac{8000}{18000} \times 360^\circ = 160^\circ$	Wheat	6000	$\frac{6000}{18000} \times 360^\circ = 120^\circ$	Sugarcane	2000	$\frac{2000}{18000} \times 360^\circ = 40^\circ$	Vegetables	2000	$\frac{2000}{18000} \times 360^\circ = 40^\circ$	Total	18000	360°	1 2						
Crop	Area in hectare	Measure of central angle																								
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Total	18000	360°																								
	<p>A.4. Solve ANY TWO of the following :</p> <p>(i) $4x = y - 5$ $y = 2x + 1$</p> <p>$\therefore 4x + 5 = y$</p> <p>$\therefore y = 4x + 5$</p> <table border="1" style="display: inline-table; margin-right: 50px; border-collapse: collapse;"> <tr><td style="text-align: center;">x</td><td style="text-align: center;">0</td><td style="text-align: center;">-1</td><td style="text-align: center;">-2</td></tr> <tr><td style="text-align: center;">y</td><td style="text-align: center;">5</td><td style="text-align: center;">1</td><td style="text-align: center;">-3</td></tr> <tr><td style="text-align: center;">(x, y)</td><td style="text-align: center;">(0, 5)</td><td style="text-align: center;">(-1, 1)</td><td style="text-align: center;">(-2, -3)</td></tr> </table> <table border="1" style="display: inline-table; border-collapse: collapse;"> <tr><td style="text-align: center;">x</td><td style="text-align: center;">0</td><td style="text-align: center;">1</td><td style="text-align: center;">2</td></tr> <tr><td style="text-align: center;">y</td><td style="text-align: center;">1</td><td style="text-align: center;">3</td><td style="text-align: center;">5</td></tr> <tr><td style="text-align: center;">(x, y)</td><td style="text-align: center;">(0, 1)</td><td style="text-align: center;">(1, 3)</td><td style="text-align: center;">(2, 5)</td></tr> </table>	x	0	-1	-2	y	5	1	-3	(x, y)	(0, 5)	(-1, 1)	(-2, -3)	x	0	1	2	y	1	3	5	(x, y)	(0, 1)	(1, 3)	(2, 5)	1
x	0	-1	-2																							
y	5	1	-3																							
(x, y)	(0, 5)	(-1, 1)	(-2, -3)																							
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y	1	3	5																							
(x, y)	(0, 1)	(1, 3)	(2, 5)																							



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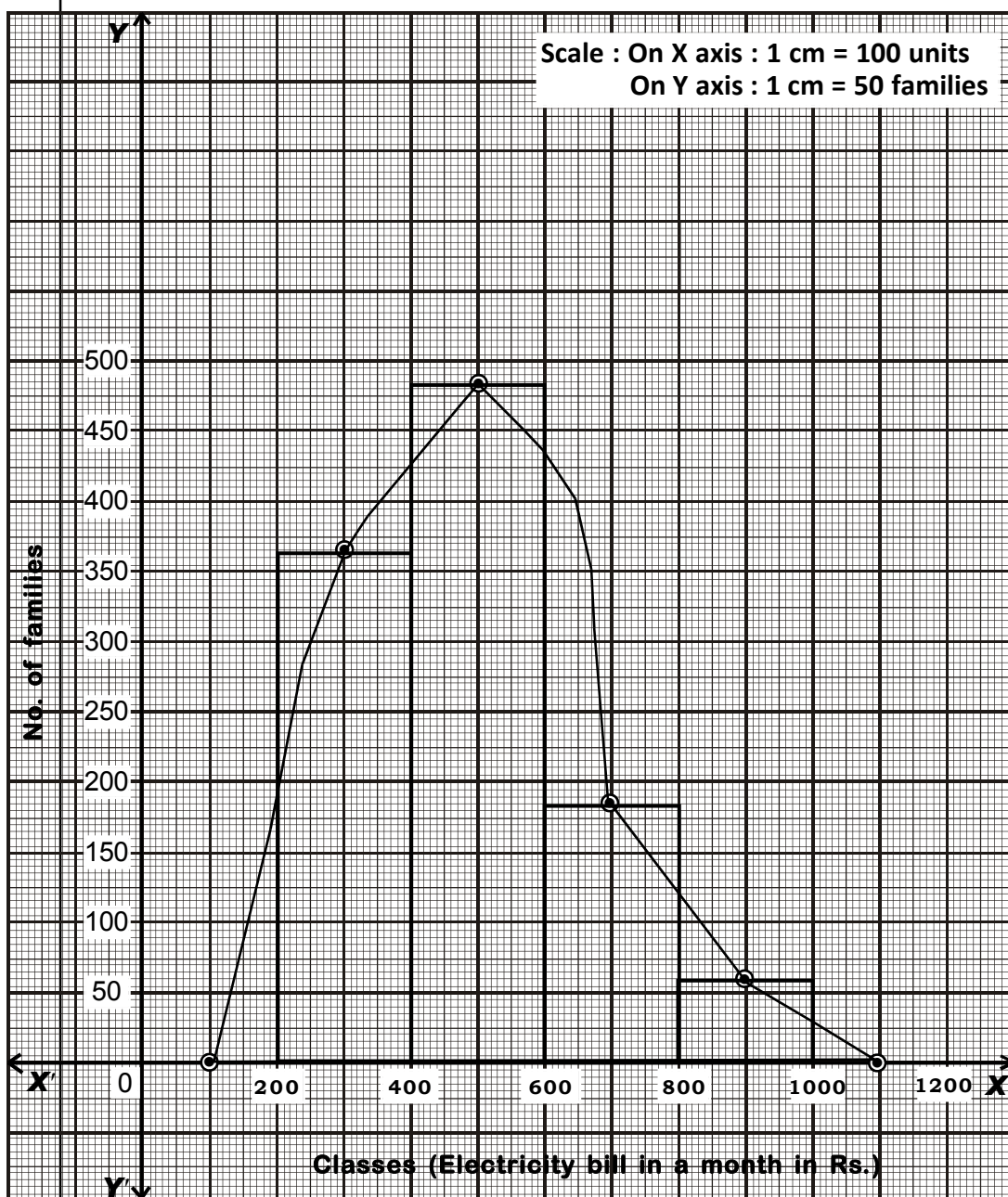
$\therefore x = -2$ and $y = -3$ is the solution of given simultaneous equations.

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(ii)

Electricity bill in a month in Rs.	Class mark	No. of families
200 - 400	300	362
400 - 600	500	490
600 - 800	700	185
800 - 1000	900	63

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(iii)	<p>The natural numbers from 1 to 140 that are divisible by 4 are as follows : 4, 8, 12, 16,, 140</p> <p>These numbers form an A.P. with $a = 4$, $d = t_2 - t_1 = 8 - 4 = 4$</p> <p>Let, 140 be the n^{th} term of A.P.</p> $t_n = 140$ $t_n = a + (n - 1) d$ $\therefore 140 = 4 + (n - 1) 4$ $\therefore 140 = 4 + 4n - 4$ $\therefore 140 = 4n$ $\therefore n = \frac{140}{4}$ $\therefore n = 35$ <p>\therefore 140 is 35 term of A.P.</p> <p>\therefore We have to find sum of 35 terms i.e. S_{35},</p> $S_n = \frac{n}{2} [2a + (n - 1)d]$ $\therefore S_{35} = \frac{35}{2} [2(4) + (35 - 1) 4]$ $\therefore S_{35} = \frac{35}{2} [8 + 34(4)]$ $\therefore S_{35} = \frac{35}{2} [8 + 136]$ $\therefore S_{35} = \frac{35}{2} [144]$ $\therefore S_{35} = 2520$ <p>\therefore Sum of natural numbers from 1 to 140 that are divisible by 4 is 2520.</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
A.5.	Solve ANY TWO of the following :	
(i)	<p>Let the five consecutive natural numbers be x, $x + 1$, $x + 2$, $x + 3$ and $x + 4$ respectively.</p> <p>As per the given condition,</p> $x^2 + (x + 1)^2 + (x + 2)^2 + (x + 3)^2 + (x + 4)^2 = 1455$ $\therefore x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 + x^2 + 6x + 9 + x^2 + 8x + 16 = 1455$ $= 0$ $\therefore 5x^2 + 20x + 30 - 1455 = 0$ $\therefore 5x^2 + 20x - 1425 = 0$ <p>Dividing throughout by 5 we get,</p> $x^2 + 4x - 285 = 0$ $\therefore x^2 - 15x + 19x - 285 = 0$ $\therefore x(x - 15) + 19(x - 15) = 0$ $\therefore (x - 15)(x + 19) = 0$ $\therefore x - 15 = 0 \quad \text{or} \quad x + 19 = 0$ $\therefore x = 15 \quad \text{or} \quad x = -19$	<p>1</p> <p>1</p> <p>1</p>

\therefore x is a natural number $x \neq -19$

Hence $x = 15$

$\therefore x + 1 = 15 + 1 = 16$ $\therefore x + 2 = 15 + 2 = 17$

$\therefore x + 3 = 15 + 3 = 18$ $\therefore x + 4 = 15 + 4 = 19$

\therefore The required five consecutive natural numbers are 15, 16, 17, 18 and 19 respectively.

(ii)

Calories	Frequency (f_i) (No. of boys)	Cumulative frequency less than type
1000 - 1500	5	5
1500 - 2000	13	18
2000 - 2500	16	34 \rightarrow <i>c.f.</i>
2500 - 3000	18 \rightarrow f	52
3000 - 3500	27	79
3500 - 4000	10	89
4000 - 4500	4	93
Total	93 \rightarrow N	

Here total frequency = $\Sigma f_i = N = 93$

$$\therefore \frac{N}{2} = \frac{93}{2} = 46.5$$

Cumulative frequency (less than type) which is just greater than 46.5 is 52. Therefore corresponding class 2500 - 3000 is median class.

$L = 2500$, $N = 93$, $c.f. = 34$, $f = 18$, $h = 500$

$$\begin{aligned} \text{Median} &= L + \left(\frac{N}{2} - c.f. \right) \frac{h}{f} \\ &= 2500 + \left(\frac{93}{2} - 34 \right) \frac{500}{18} \\ &= 2500 + (46.5 - 34) \frac{500}{18} \\ &= 2500 + (12.5) \frac{500}{18} \\ &= 2500 + \frac{6250}{18} \\ &= 2500 + 347.22 \\ &= 2847.22 \end{aligned}$$

\therefore Median of calories consumed by boys is 2847.22 calories.

(iii)	<p>Let the speed of bus be x km/hr. and time taken be y hrs.</p> $\text{Distance} = \text{Speed} \times \text{Time}$ $\therefore \text{Distance} = xy \text{ km}$ <p>According to the first condition,</p> $(x + 15)(y - 2) = xy$ $\therefore x(y - 2) + 15(y - 2) = xy$ $\therefore xy - 2x + 15y - 30 = xy$ $\therefore -2x + 15y = 30 \quad \dots\dots(i)$ <p>According to the second condition,</p> $(x - 5)(y + 1) = xy$ $\therefore x(y + 1) - 5(y + 1) = xy$ $\therefore xy + x - 5y - 5 = xy$ $\therefore x - 5y = 5 \quad \dots\dots(ii)$ <p>Multiplying (ii) by 3 we get,</p> $3x - 15y = 15 \quad \dots\dots(iii)$ <p>Adding (i) and (iii) we get,</p> $\begin{array}{r} -2x + 15y = 30 \\ 3x - 15y = 15 \\ \hline x = 45 \end{array}$ <p>Substituting $x = 45$ in (ii),</p> $\therefore 45 - 5y = 5$ $\therefore -5y = 5 - 45$ $\therefore -5y = -40$ $\therefore y = \frac{-40}{-5}$ $\therefore y = 8$ $\therefore \text{Distance} = xy$ $= 45 \times 8$ $= 360$ <p>\therefore Distance covered by bus is 360 km.</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
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