MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 3 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY Five of the following : **5**

(i) Write the first five terms of the following Arithmetic Progressions where, the common difference 'd' and the first term 'a' is given :
$a = 4, d = 0$

(ii) Determine whether the given values of 'x' is a roots of given quadratic equation.
$x^2 - x = 0, x = 0$

(iii) Find the value of discriminant of the following equation.
$x^2 - 6x + 7 = 0$

(iv) If the value of the determinant $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix}$ is 31 , find m.

(v) If A = 57, $\bar{d}$ = 0.12, and h = 6 then, find mean.

(vi) For pie diagram, = θ = 100°, Total = 54000. Find the data.

Q.2. Solve ANY FOUR of the following : **8**

(i) Find the first three terms of the sequences for which S_n is given below :
$S_n = \dfrac{n^2(n+1)^2}{4}$

(ii) Form the quadratic equation if its roots are 0 and – 4

(iii) If the point (3, 2)lies on the graph of the equation 5x + ay = 19, then find a.

(iv) In each of the following experiments, write the sample space S, number of sample point n (S), event A, B, C and n (A), n (B), n (C). Also find complementary events, mutually exclusive events :
A die is thrown.
(a) A is the event that prime number comes up.
(b) B is the event that the number is divisible by three comes up,
(c) C is the event that a perfect square number comes up.

(v) Which term of an A.P. is 55, if a = 3 and d = 1.3.

(vi) A coin is tossed three times then find the probability of
(a) getting head on middle coin
(b) getting exactly one tail

Q.3. Solve ANY THREE of the following : 9

(i) Mangala started doing physical exercise 10 minutes for the first day. She will increase the time of exercise by 5 minutes per day, till she reaches 45 minutes. How many days are required to reach 45 minutes ?

(ii) Solve the given quadratic equation by completing square.
$y^2 = 3 + 4y$

(iii) One card is drawn from a well- shuffled pack of 52 cards. Find the probability of getting (a) the jack of hearts (b) a spade (c) the queen of diamonds.

(iv) Three horses A, B and C are in a race, A is twice as like to win as B and B is twice as like to win as C. What are their probabilities of winning ?

(v) Electricity used by farmers during different parts of a day for irrigation is as follows. Draw pie diagram :

Part of day	Morning	Afternoon	Evening	Night
Percentage of electricity used	30	40	20	10

Q.4. Solve ANY TWO of the following : **8**

(i) Solve the given simultaneous equations using graphical method :

$$2x + y = 6; \frac{4 - 3x}{4} = y$$

(ii) Following is the frequency distribution of customers in a certain year at the departmental store :

No. of customers	50 - 100	100 - 150	150 - 200	200 - 250	Total
No. of days	90	98	138	39	365

Draw histogram and hence draw frequency curve.

(iii) Find the sum of the first n odd natural numbers.
Hence find 1 + 3 + 5 + ... + 101.

Q.5. Solve ANY TWO of the following : **10**

(i) A man travels by boat 36 km down a river and back in 8 hours. If the speed of his boat in still water is 12 km per hour, find the speed of the river curved.

(ii) Following table shows frequency distribution of no. of rooms occupied in a hotel per day.

No. of rooms occupied	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of days	5	27	17	11	9	1

Find median number of rooms occupied per day in hotel.

(iii) Students of a school were made to stand in rows for drill. If 3 students less were standing in each row, 10 more rows were required and if 5 students more were standing in each row then the number of rows was reduced by 10. Find the number of students participating in the drill.

Best Of Luck

MT

2014 ___ ___ 1100 **Seat No.**

MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 3 (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) $a = 4, d = 0$

Here, $t_1 = a = 4$

$t_2 = t_1 + d = 4 + 0 = 4$

$t_3 = t_2 + d = 4 + 0 = 4$

$t_4 = t_3 + d = 4 + 0 = 4$

$t_5 = t_4 + d = 4 + 0 = 4$

∴ The first five terms of the A.P. are 4, 4, 4, 4 and 4. **1**

(ii) $x^2 - x = 0, x = 0$

Putting x = 0 in L.H.S., we get,

L.H.S. $= (0)^2 - 0$

$= 0 - 0$

$= 0$

$=$ R.H.S.

∴ L.H.S. = R.H.S.

Thus equation is satisfied.

So 0 is the root of the given quadratic equation. **1**

(iii) $x^2 - 6x + 7 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 1, b = -6, c = 7$

$\Delta = b^2 - 4ac$

$= (-6)^2 - 4(1)(7)$

$= 36 - 28$

$= 8$

∴ $\Delta = 8$ **1**

(iv) $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix} = 31$

∴ $(m \times 7) - (2 \times -5) = 31$

∴ $7m + 10 = 31$

∴ $7m = 31 - 10$

∴ $7m = 21$

	$\therefore \quad m = \frac{21}{7}$	
	$\therefore \quad \boxed{m = 3}$	1
(v)	$\text{Mean } (\overline{x}) = A + \overline{d}$	
	$= 57 + 0.12$	
	$= 57.12$	
	$\therefore$ mean is 57.12 units.	1
(vi)	$\therefore \quad \theta = \frac{\text{Data}}{\text{Total}} \times 360$	
	$\therefore \quad 100 = \frac{\text{Data}}{54000} \times 360$	
	$\therefore \quad \boxed{\text{Data} = 15000}$	1
A.2.	**Solve ANY Four of the following :**	
(i)	$S_n = \frac{n^2 (n+1)^2}{4}$	
	$\therefore \quad S_1 = \frac{1^2 (1+1)^2}{4} = \frac{1 (2)^2}{4} = \frac{1 \times 4}{4} = 1$	1
	$\therefore \quad S_2 = \frac{2^2 (2+1)^2}{4} = \frac{4 (3)^2}{4} = 9$	
	$\therefore \quad S_3 = \frac{3^2 (3+1)^2}{4} = \frac{9 \times 4^2}{4} = 9 \times 4 = 36$	
	We know that,	
	$t_1 = S_1 = 1$	
	$t_2 = S_2 - S_1 = 9 - 1 = 8$	
	$t_3 = S_3 - S_2 = 36 - 9 = 27$	
	$\therefore$ The first three terms of the sequence are 1, 8 and 27.	1
(ii)	The roots of the quadratic equation are 0 and – 4.	
	Let $\alpha = 0$ and $\beta = -4$	
	$\therefore \quad \alpha + \beta = 0 + (-4) = -4$	1
	and $\alpha.\beta = 0 \times -4 = 0$	
	We know that,	
	$x^2 - (\alpha + \beta)x + \alpha.\beta = 0$	
	$\therefore \quad x^2 - (-4)x + 0 = 0$	
	$\therefore \quad x^2 + 4x = 0$	
	$\therefore$ The required quadratic equation is $x^2 + 4x = 0$	1

(iii)	$\because$ (3, 2) lies on the graph of the equation 5x + y = 19. It satisfies the equation, $\therefore$ Substituting x = 3 and y = 2 in the equation we get, $5(3) + a(2) = 19$ $\therefore 15 + 2a = 19$ $\therefore 2a = 19 - 15$ $\therefore 2a = 4$ $\therefore a = \frac{4}{2}$ $\therefore \boxed{a = 2}$	1 1
(iv)	A die is thrown $\therefore$ S = { 1, 2, 3, 4, 5, 6 } $\therefore$ n (S) = 6 A is the event that a prime number comes up $\therefore$ A = { 2, 3, 5 } $\therefore$ n (A) = 3 B is the event that a number divisible by 3 comes up $\therefore$ B = { 3, 6 } $\therefore$ n (B) = 2 C is the event that a perfect square number comes up $\therefore$ C = { 1, 4 } $\therefore$ n (C) = 2 $B \cap C = \phi$ B and C are mutually exclusive events. $A \cap C = \phi$ $\therefore$ A and C are mutually exclusive events.	1 1
(v)	For an A.P. a = 1, d = 2, t_n = 149 $t_n = a + (n - 1)d$ $\therefore 149 = 1 + (n - 1)(2)$ $\therefore 149 - 1 = 2n - 2$ $\therefore 148 + 2 = 2n$ $\therefore 150 = 2n$ $\therefore n = 75$ $\therefore$ $\boxed{75^{th} \text{ term of an A.P. is } 149.}$	1 1

(vi) When a coin tossed three times

S = { HHH, HTH, THH, TTH, HHT, HTT, THT, TTT }

n (S) = 8

(a) Let A be the event of getting head on middle coin

A = { HHH, THH, HHT, THT }

n (A) = 4

$P(A) = \frac{n(A)}{n(S)}$

$\therefore P(A) = \frac{4}{8}$

$\therefore \boxed{P(A) = \frac{1}{2}}$ **1**

(b) Let B be the event of getting exactly one tail

B = { HTH, THH, HHT }

n (B) = 3

$P(B) = \frac{n(B)}{n(S)}$

$\therefore \boxed{P(B) = \frac{3}{8}}$ **1**

A.3. Solve ANY THREE of the following :

(i) Since the workout time Mangala increases by 5 minutes everyday after the first day, the successive workout times are in A.P. **1**

Workout time for first day (a) = 10 minutes.

Increases in workout time (d) = 5 minutes

Let No. of days required to reach workout time of 45 minutes be 'n' days.

$t_n = 45$

$\therefore t_n = a + (n - 1) d$ **1**

$\therefore 45 = 10 + (n - 1) 5$

$\therefore 45 = 10 + 5n - 5$

$\therefore 45 = 5 + 5n$

$\therefore 45 - 5 = 5n$

$\therefore 5n = 40$

$\therefore n = 8$

$\therefore$ 8 days required to reach work out time of 45 minutes. **1**

(ii) $y^2 = 3 + 4y$

$\therefore y^2 - 4y = 3$ .... (i)

Third term = $\left(\frac{1}{2} \times \text{coefficient of } y\right)^2$ **1**

	$= \left(\frac{1}{2} \times -4\right)^2$	
	$= (-2)^2$	
	$= 4$	
	Adding 4 to both sides of (i) we get,	
	$y^2 - 4y + 4 = 3 + 4$	**1**
	$\therefore (y - 2)^2 = 7$	
	Taking square root on both the sides we get,	
	$y - 2 = \pm\sqrt{7}$	
	$\therefore y = 2 \pm \sqrt{7}$	
	$\therefore y = 2 + \sqrt{7}$ or $y = 2 - \sqrt{7}$	
	$\therefore$ $2 + \sqrt{7}$ and $2 - \sqrt{7}$ are the roots of the given quadratic equations.	**1**
(iii)	There are 52 cards in a pack	
	$\therefore n(S) = 52$	
	(a) Let A be the event of getting the jack of hearts	
	$\because$ There is one jack card in hearts	
	$n(A) = 1$	
	$P(A) = \frac{n(A)}{n(S)}$	
	$\therefore P(A) = \frac{1}{52}$	**1**
	(b) Let B be the event of getting a spade card	
	$\because$ There are 13 spade cards	
	$n(B) = 13$	
	$\therefore P(B) = \frac{n(B)}{n(S)}$	
	$\therefore P(B) = \frac{13}{52}$	
	$\therefore P(B) = \frac{1}{4}$	**1**
	(c) Let C be the event that the card drawn is queen of diamonds	
	$\because$ There is one queen card in diamond	
	$n(C) = 1$	
	$P(C) = \frac{n(C)}{n(S)}$	
	$\therefore P(C) = \frac{1}{52}$	**1**

(iv) Let probabilities of horses A, B and C be P (A), P (B), and P (C).

$$P(A) = 2P(B) \quad \text{.......(i)}$$
$$P(B) = 2P(C) \quad \text{......(ii)}$$

Substituting (ii) in (i),

$$P(A) = 2 \times 2\,P(C)$$
$$\therefore \quad P(A) = 4P(C) \quad \text{.....(iii)}$$

We know that,

$$P(A) + P(B) + P(C) = 1$$
$$\therefore \quad 4P(C) + 2\,P(C) + P(C) = 1$$
$$\therefore \quad 7P(C) = 1$$
$$\therefore \quad \boxed{P(C) = \frac{1}{7}} \quad \textbf{1}$$

Substituting the value of P (C) in (iii),

$$P(A) = 4\left(\frac{1}{7}\right)$$
$$\therefore \quad \boxed{P(A) = \frac{4}{7}} \quad \textbf{1}$$

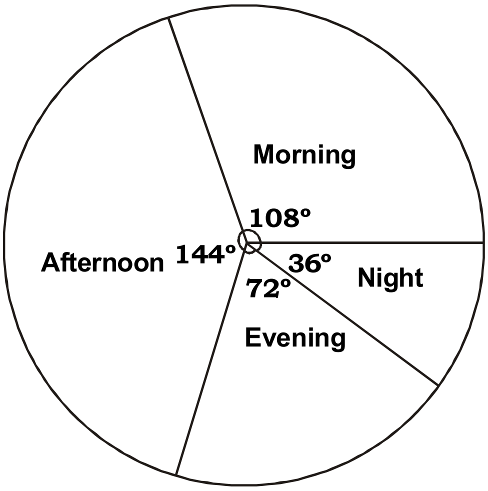
Substituting the value of P (C) in (ii),

$$P(B) = 2\left(\frac{1}{7}\right)$$
$$\therefore \quad \boxed{P(B) = \frac{2}{7}} \quad \textbf{1}$$

(v)

Part of day	Percentage of electricity used	Measure of central angle
Morning	30	$\frac{30}{100} \times 360^\circ = 108^\circ$
Afternoon	40	$\frac{40}{100} \times 360^\circ = 144^\circ$
Evening	20	$\frac{20}{100} \times 360^\circ = 72^\circ$
Night	10	$\frac{10}{100} \times 360^\circ = 36^\circ$
Total	100	360°

1



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A.4. Solve ANY TWO of the following :

(i) $2x + y = 6$ $\quad \frac{4 - 3x}{4} = y$

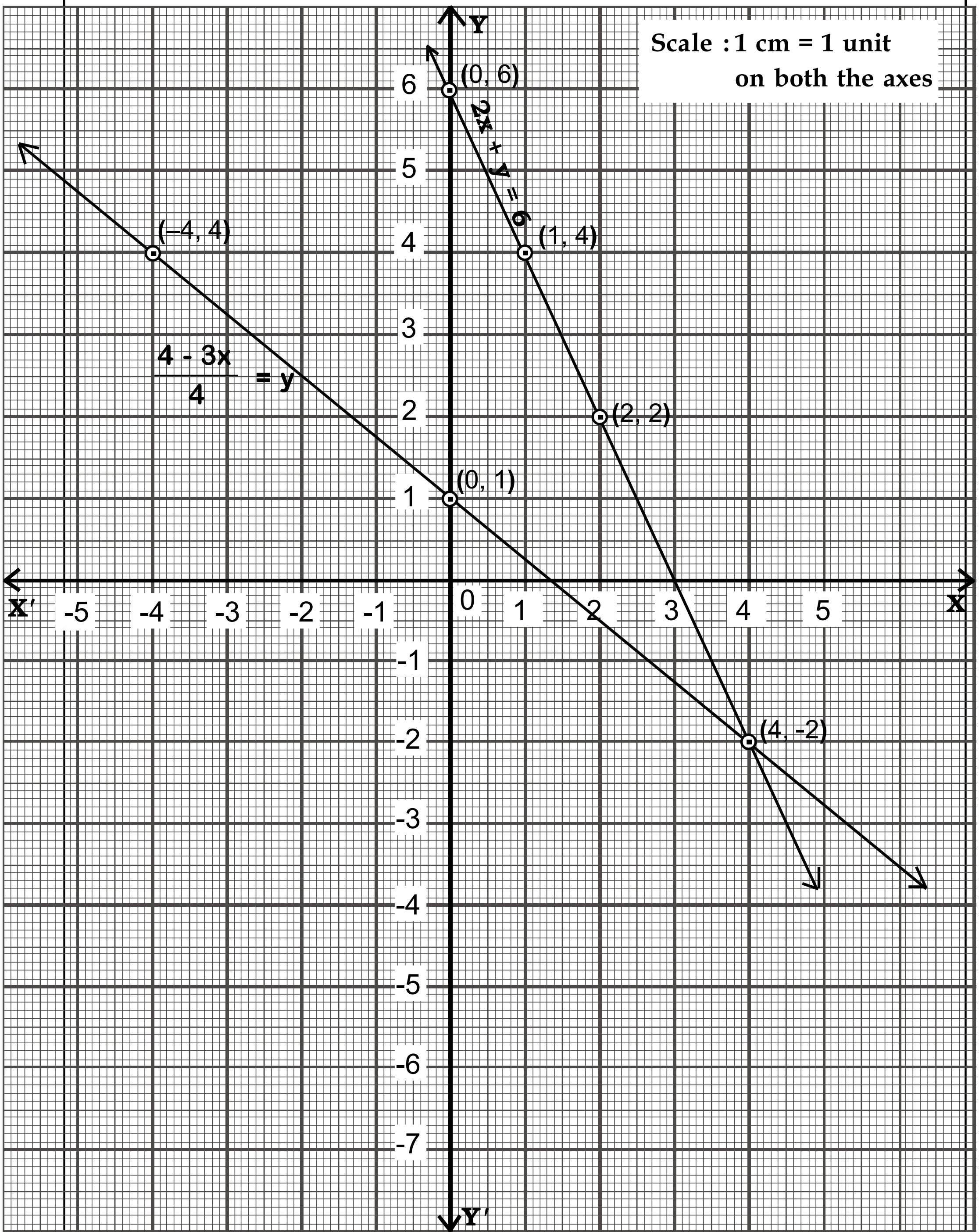
$\therefore \quad y = 6 - 2x \therefore \quad y = \frac{4 - 3x}{4}$

x	0	1	2
y	6	4	2
(x, y)	(0, 6)	(1, 4)	(2, 2)

x	0	4	– 4
y	1	–2	4
(x, y)	(0, 1)	(4, –2)	(– 4, 4)

1

2

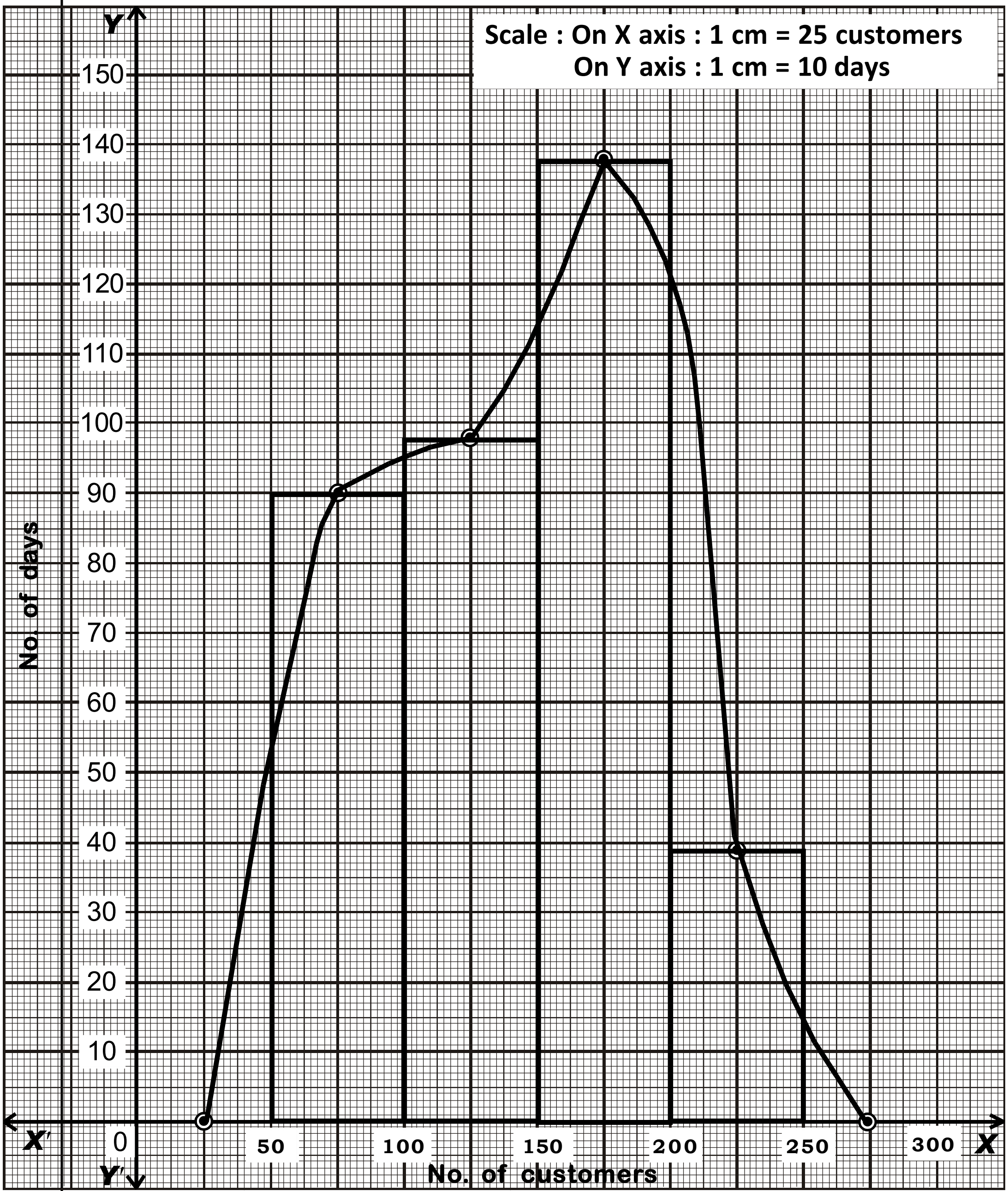


∴ x = 4 and y = – 2 is the solution of given simultaneous equations. 1

(ii)

No. of customers	50 - 100	100 - 150	150 - 200	200 - 250	Total
No. of days	90	98	138	39	365

1



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(iii) The first n odd natural numbers are as follows :

1, 3, 5, 7, ..., n

a = 1, d = $t_2 - t_1$ = 3 – 1 = 2

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_n = \frac{n}{2}[2(1) + (n-1)2]$

$\therefore S_n = \frac{n}{2}[2 + 2n - 2]$ **1**

$= \frac{n}{2}[2n]$

$\therefore S_n = n^2$ ......(i)

$1 + 3 + 5 + \ldots\ldots + 101$

Let, 101 be the n^{th} term of A.P.

$t_n = 101$ **1**

$t_n = a + (n-1)d$

$\therefore 101 = a + (n-1)d$

$\therefore 101 = 1 + (n-1)2$

$\therefore 101 = 1 + 2n - 2$

$\therefore 101 = 2n - 1$

$\therefore 101 + 1 = 2n$

$\therefore 2n = 102$ **1**

$\therefore n = 51$

$\therefore$ 101 is the 51^{st} term of A.P.,

$\therefore$ We have to find sum of 51 terms i.e. S_{51},

$S_n = n^2$ [From (i)]

$\therefore S_{51} = (51)^2$

$\therefore \boxed{S_{51} = 2601}$ **1**

A.5. Solve ANY TWO of the following :

(i) Speed of boat in still water = 12 km/hr.

Let speed of river current = x km/hr. **1**

$\therefore$ Speed of boat up the river = $(12 - x)$ km/hr.

Speed of boat down the river = $(12 + x)$ km/hr.

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\therefore$ Time taken by boat to travel 36 km down the river = $\left(\frac{36}{12 + x}\right)$ hrs. **1**

As per the given condition,

$\frac{36}{12 + x} + \frac{36}{12 - x} = 8$

$\therefore \quad 36\left[\frac{1}{12+x}+\frac{1}{12-x}\right] = 8$ **1**

$\therefore \quad 36\left[\frac{12-x+12+x}{(12+x)(12-x)}\right] = 8$

$\therefore \quad 36\left[\frac{24}{144-x^2}\right] = 8$ **1**

$\therefore \quad \frac{36 \times 24}{8} = 144 - x^2$

$\therefore \quad 108 = 144 - x^2$

$\therefore \quad x^2 = 144 - 108$

$\therefore \quad x^2 = 36$

$\therefore \quad x = \pm 6$ [Taking square roots]

$\therefore$ x = – 6 is not acceptable because speed cannot be negative. **1**

(ii)

No. of rooms occupied	Frequency (f_i) (No. of days)	Cumulative frequency less than type
0 - 10	5	5
10 - 20	27	32 → *c.f.*
20 - 30	17 → *f*	49
30 - 40	11	60
40 - 50	9	69
50 - 60	1	70
Total	**70 → N**	

1

Here total frequency = Σf_i = N = 70 **1**

$\therefore \quad \frac{N}{2} = \frac{70}{2} = 35$

Cumulative frequency (less than type) which is just greater than 35 is 49. Therefore corresponding class 20 - 30 is median class. **1**

L = 20, N = 70, *c.f.* = 32, f = 17, h = 10

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$

$$= 20 + \left(\frac{70}{2} - 32\right)\frac{10}{17}$$ **1**

$$= 20 + (35 - 32)\frac{10}{17}$$

$= 20 + \frac{30}{17}$
$= 20 + 1.76$
$= 21.76$

$\therefore$ **Median of rooms occupied is 21.76 rooms.** 1

(iii) Let the no. of students standing in each row be x and let no. of rows be y. 1

$\therefore$ Total no. of students participating in the drill = xy

As per the first given condition,

$(x - 3)(y + 10) = xy$ 1

$\therefore x(y + 10) - 3(y + 10) = xy$

$\therefore xy + 10x - 3y - 30 = xy$

$\therefore 10x - 3y = 30$ ....(i)

As per the second given condition,

$(x + 5)(y - 10) = xy$ 1

$\therefore x(y - 10) + 5(y - 10) = xy$

$\therefore xy - 10x + 5y - 50 = xy$

$\therefore -10x + 5y = 50$ ......(ii)

Adding (i) and (ii),

$10x - 3y = 30$
$-10x + 5y = 50$
$2y = 80$

$\therefore y = \frac{80}{2}$

$\therefore y = 40$

Substituting y = 40 in (i),

$10x - 3(40) = 30$ 1

$\therefore 10x - 120 = 30$

$\therefore 10x = 30 + 120$

$\therefore 10x = 150$

$\therefore x = \frac{150}{10}$

$\therefore x = 15$

$\therefore xy = 15 \times 40 = 600$

$\therefore$ **600 students were participating in the drill.** 1

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