MT

2014 ___ ___ 1100 **Seat No.**

MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 4 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY Five of the following : **5**

(i) Write the first five terms of the following Arithmetic Progressions where, the common difference 'd' and the first term 'a' are given :
$a = 5, d = 2$

(ii) Determine whether the given values of 'x' is a roots of given quadratic equation.
$x^2 + x - 1 = 0, x = 2$

(iii) Find the value of discriminant of following equations :
$x^2 + 4x + 1 = 0$

(iv) Write D_x for the given simultaneous equation. :
$3x + 4y = 8$; $x - 2y = 5$

(v) If $A = 22.5$, $\bar{d} = 2.92$ and $h = 5$ then find mean.

(vi) For a pie diagram, $\theta = 90°$, Total = 54000. Find the data.

Q.2. Solve ANY FOUR of the following : **8**

(i) Find t_{11} from the following A.P. 4, 9, 14, ...

(ii) From the quadratic equation if its roots are.
3 and 10

(iii) If $12x + 13y = 29$ and $13x + 12y = 21$, Find $x + y$.

(iv) In each of the following experiments, write the sample space S, number of sample point n (S), event A, B, C and n (A), n (B), n (C). Also find complementary events, mutually exclusive events :
Two coins are tossed :
(a) A is the event of getting at most one head
(b) B is the event getting both heads, C is the event of getting same face on both the coins.

(v) Which term of an A.P. is 55, if a = 3 and d = 1.3.

(vi) A die is thrown then find the probability of getting
(a) an odd number
(b) a perfect square

Q.3. Solve ANY THREE of the following : 9

(i) There is an auditorium with 35 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. Find the number of seats in the twenty fifth row.

(ii) Solve the given quadratic equations by completing square.
$x (x - 1) = 1$

(iii) One card is drawn from a well- shuffled deck of 52 cards. Find the probability of getting
(a) king of red colour. (b) a face card. (c) a red face card.

(iv) What is the probability that a leap year has 53 Sundays ?

(v) The following table gives information about the monetary investment by some residents in a city :

Mode of investment	Shares	Mutual funds	Real estate	Gold	Government bonds
Percentage of residents	10	20	35	30	5

Draw pie diagram to represent the data.

Q.4. Solve ANY TWO of the following : 8

(i) Solve the given simultaneous equations using graphical method :
$3x + 4y + 5 = 0; y = x + 4$

(ii) Represent the following data using, histogram and hence draw frequency polygon :

No. of words typed per minute	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79
No. of typists	2	8	15	12	3

(iii) Second and fourth term of on A.P. is 12 and 20 respectively. Find the sum of first 25 terms of that A.P.

Q.5. Solve ANY TWO of the following : 10

(i) Solve the given equations :

$$2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$$

(ii) Below is given frequency distribution of I.Q. (Intelligent Quotient) of 80 candidates.

I.Q,	70–80	80–90	90–100	100–110	110 – 120	120–130	130 – 140
No. of candidates	7	16	20	17	11	7	2

Find median I.Q. of a candidate.

(iii) Sharad bought a table and a fan together for Rs. 5000. After sometime he sold the table at the gain of 25% and the fan at a gain of 20%. Thus he gained 23% on the whole. Find the cost of the fan.

Best Of Luck

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MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 4 (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) $a = 5, d = 2$

Here, $t_1 = a = 5$

$t_2 = t_1 + d = 5 + 2 = 7$

$t_3 = t_2 + d = 7 + 2 = 9$

$t_4 = t_3 + d = 9 + 2 = 11$

$t_5 = t_4 + d = 11 + 2 = 13$

$\therefore$ The first five terms of the A.P. are 5, 7, 9, 11 and 13. **1**

(ii) $x^2 + x - 1 = 0, x = 2$

Putting x = 2 in L.H.S., we get,

L.H.S. $= (2)^2 + 2 - 1$

$= 4 + 1$

$= 5$

$\neq$ R.H.S.

$\therefore$ L.H.S. $\neq$ R.H.S.

Thus equation is not satisfied.

So 2 is not the root of the given quadratic equation. **1**

(iii) $x^2 + 4x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$

we have $a = 1, b = 4, c = 1$

$\Delta = b^2 - 4ac$

$= (4)^2 - 4 (1) (1)$

$= 16 - 4$

$= 12$

$\therefore$ $\Delta = 12$ **1**

(iv) $3x + 4y = 8$

$x - 2y = 5$

$D_x = \begin{vmatrix} 8 & 4 \\ 5 & -2 \end{vmatrix}$ **1**

(v)	Mean $(\bar{x}) = A + d$ $= 22.5 + 2.92$ $= 25.42$ $\therefore$ **Mean is 25.42 units.**	1
(vi)	$\theta = \frac{\text{Data}}{\text{Total}} \times 360$ $\therefore 90 = \frac{\text{Data}}{54000} \times 360$ $\therefore \text{Data} = \frac{90 \times 54000}{360}$ $\therefore$ **Data = 13500**	1
A.2.	**Solve ANY Four of the following :**	
(i)	For the A.P. 4, 9, 14, ... a = 4, d = 5 $t_n = a + (n - 1) d$ $\therefore t_{11} = 4 + (11 - 1) 5$ $\therefore t_{11} = 4 + 50$ $\therefore$ **$t_{11} = 54$**	1 1
(ii)	The roots of the quadratic equation are 3 and 10 Let $\alpha = 3$ and $\beta = 10$ $\therefore \alpha + \beta = 3 + 10 = 13$ $\alpha . \beta = 3 \times 10 = 30$ We know that, $x^2 - (\alpha + \beta) x + \alpha.\beta = 0$ $\therefore x^2 - 13x + 30 = 0$ $\therefore$ **The required quadratic equation is $x^2 - 13x + 30 = 0$**	 1 1
(iii)	$12x + 13y = 29$ ......(i) $13x + 12y = 21$ .....(ii) Adding (i) and (ii), $12x + 13y = 29$ $13x + 12y = 21$ $25x + 25y = 50$ Dividing throughout by 25 we get, $x + y = \frac{50}{25}$ $\therefore$ **$x + y = 2$**	 1 1

(iv)	Two coins are tossed	
	∴ S = { HH, HT, TH, TT }	
	∴ n (S) = 4	
	A is the event of getting at the most one head	1
	∴ A = { HT, TH, TT }	
	∴ n (A) = 3	
	B is the event of getting both heads.	
	∴ B = { HH }	
	∴ n (B) = 1	
	C is the event of getting same face on both the coins	
	∴ C = { HH, TT }	
	∴ n (C) = 2	
	$A \cap B = \phi$ and $A \cup B = S$	
	∴ A and B are complementary and mutually exclusive events.	1
	∴ $A \cup C = S$	
	∴ A and C are exhaustive events.	
(v)	For an A.P. $a = 3$, $d = 1.3$, $t_n = 55$	
	∴ $tn = a + (n - 1)d$	1
	∴ $55 = 3 + (n-1)(1.3)$	
	∴ $55 - 3 = 1.3n - 1.3$	
	∴ $52 + 1.3 = 1.3n$	
	∴ $53 - 3 = 1.3n$	
	∴ $n = 41$	
	∴ 41^{st} term of an A.P. is 55	1
(vi)	When a die is thrown	
	S = { 1, 2, 3, 4, 5, 6 }	
	n (S) = 6	
	(i) Let A be the event of getting an odd number	
	A = { 1, 3, 5 }	
	n (A) = 3	
	$P(A) = \frac{n(A)}{n(S)}$	
	∴ $P(A) = \frac{3}{6}$	
	∴ $P(A) = \frac{1}{2}$	1

(ii) Let B be the event of getting a perfect square

$B = \{1, 4\}$

$n(B) = 2$

$P(B) = \dfrac{n(B)}{n(S)}$

$\therefore P(B) = \dfrac{2}{6}$

$\therefore \boxed{P(B) = \dfrac{1}{3}}$ **1**

A.3. Solve ANY THREE of the following :

(i) Since the no. of seats in each row of the auditorium are 20, 22, 24, ... **1**

The no. of seats in each row form an A.P.

No. of seats in first row (a) = 20

Difference in no. of seats in two successive rows is (d) = 2

No. of seats in 25th row = t_{25} = ?

$t_n = a + (n + 1)\, d$ **1**

$\therefore t_{25} = a + (25 - 1)\, d$

$\therefore t_{25} = 20 + 24\,(2)$

$\therefore t_{25} = 20 + 48$

$\therefore t_{25} = 68$

$\therefore$ There are 68 seats in 25^{th} row. **1**

(ii) $x(x - 1) = 1$

$\therefore x^2 - x = 1$ .... (i)

Third term $= \left(\dfrac{1}{2} \times \text{coefficient of } x\right)^2$ **1**

$= \left(\dfrac{1}{2} \times -1\right)^2$

$= \left(\dfrac{-1}{2}\right)^2$

$= \dfrac{1}{4}$

Adding $\dfrac{1}{4}$ to both sides of equation (i) we get,

$x^2 - x + \dfrac{1}{4} = 1 + \dfrac{1}{4}$

	$\therefore \left(x - \frac{1}{2}\right)^2 = \frac{4+1}{4}$	
	$\therefore \left(x - \frac{1}{2}\right)^2 = \frac{5}{4}$	
	Taking square root on both the sides we get,	1
	$x - \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$	
	$x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$	
	$\therefore x = \frac{1 \pm \sqrt{5}}{2}$	
	$\therefore x = \frac{1}{2} + \frac{\sqrt{5}}{2}$ or $x = \frac{1}{2} - \frac{\sqrt{5}}{2}$	
	$\therefore x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$	
	$\therefore$ $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$ are the roots of the given quadratic equations.	1
(iii)	There are 52 cards in a pack	
	$\therefore$ n (S) = 52	
	(a) Let A be the event that card drawn is a king of red colour	
	$\because$ There are 2 kings of red colour	
	n (A) = 2	
	$P(A) = \frac{n(A)}{n(S)}$	
	$\therefore P(A) = \frac{2}{52}$	
	$\therefore P(A) = \frac{1}{26}$	1
	(b) Let B be the event that card drawn is a face card	
	$\because$ There are 3 face cards is each of the 4 types	
	$\therefore$ The total no. of face cards n (B) = 4 × 3 = 12	
	$\therefore P(B) = \frac{n(B)}{n(S)}$	

$\therefore$ P (B) = $\frac{12}{52}$

$\therefore$ $\boxed{P(B) = \frac{3}{13}}$ **1**

(c) Let C be the event that the card drawn is a red face card

$\because$ There are 3 face cards is each of the 2 red types

$\therefore$ The total no. of red face cards n (C) = 2 × 3 = 6

P (C) = $\frac{n(C)}{n(S)}$

$\therefore$ P (C) = $\frac{6}{52}$

$\therefore$ $\boxed{P(C) = \frac{3}{26}}$ **1**

(iv) A leap year has 366 days, which is equivalent to 52 weeks and 2 days.

52 weeks will have 52 Sundays.

The remaining 2 days can be as follows :

$\therefore$ S = { Sunday - Monday, Monday - Tuesday, Tuesday - Wednesday, Wednesday - Thusday, Thusday - Friday, Friday - Saturday, Saturday - Sunday } **1**

$\therefore$ n (S) = 7

Let A be the event of getting 53rd Sunday in remaining 2 days

$\therefore$ A = { Saturday - Sunday, Sunday - Monday }

$\therefore$ n (A) = 2

P (A) = $\frac{n(A)}{n(S)}$ **1**

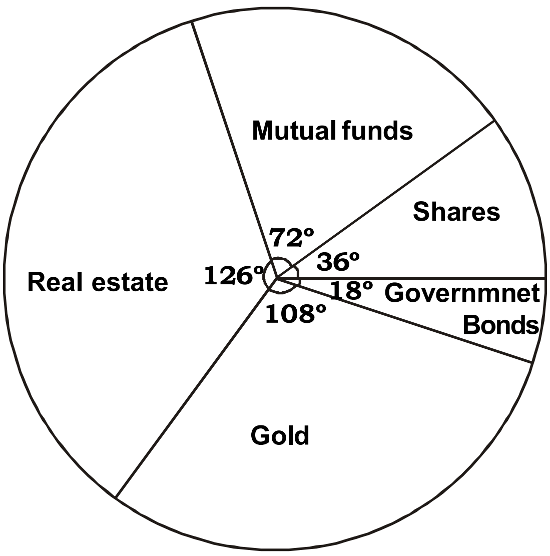
$\therefore$ P (A) = $\frac{2}{7}$

$\therefore$ The probability that a leap year selected at random will have 53 Sundays is $\frac{2}{7}$. **1**

(v)

Mode of investment	Percentage of residents	Measure of central angle
Shares	10	$\frac{10}{100} \times 360^\circ = 36^\circ$
Mutual funds	20	$\frac{20}{100} \times 360^\circ = 72^\circ$
Real estate	35	$\frac{35}{100} \times 360^\circ = 126^\circ$
Gold	30	$\frac{30}{100} \times 360^\circ = 108^\circ$
Government bonds	5	$\frac{5}{100} \times 360^\circ = 18^\circ$
Total	**100**	**360°**

1



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A.4. Solve ANY TWO of the following :

(i) $3x + 4y + 5 = 0 \qquad y = x + 4$

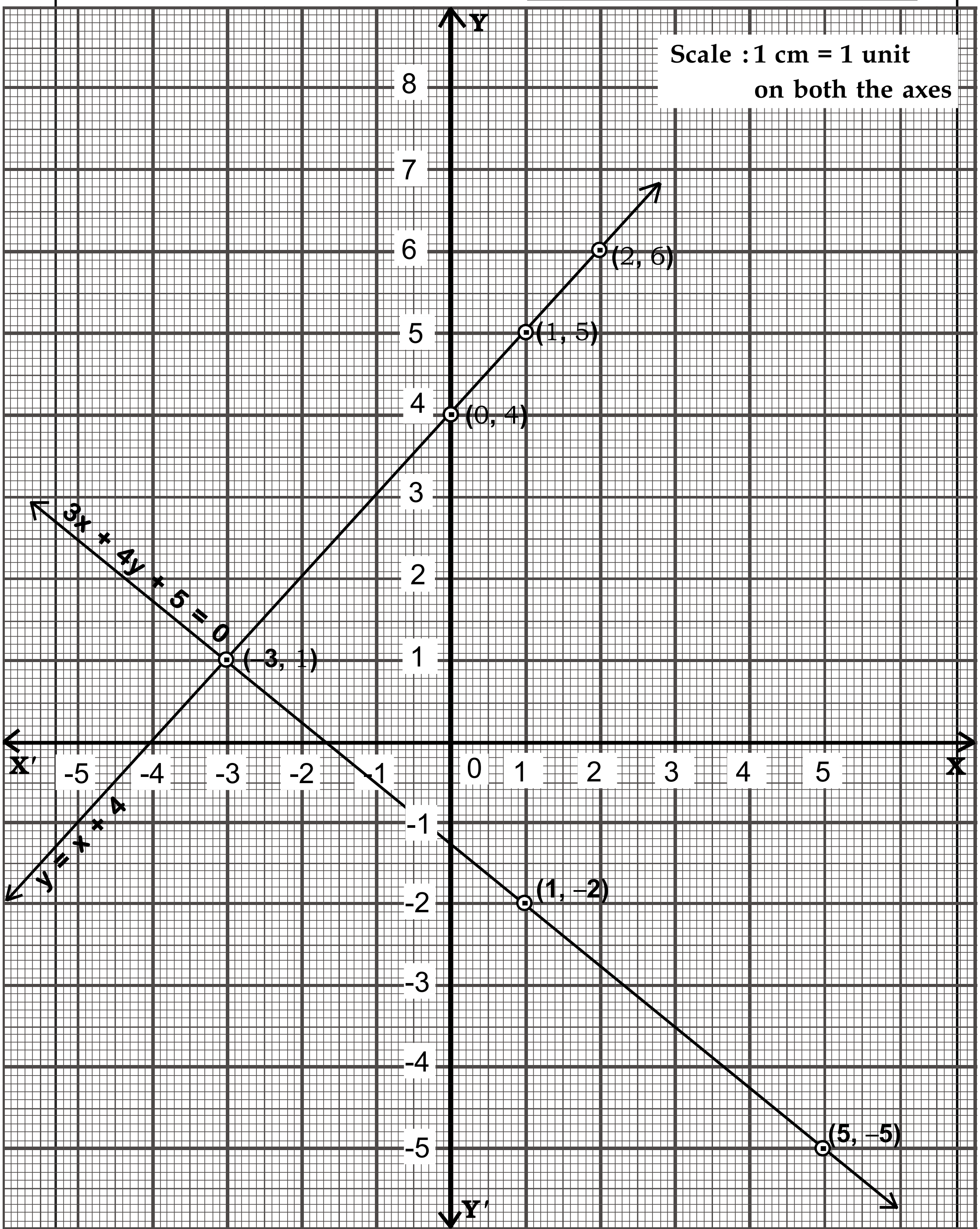
$\therefore \; 3x = -5 - 4y$

$\therefore \; x = \frac{-5 - 4y}{3}$

x	–3	1	5
y	1	–2	–5
(x, y)	(–3, 1)	(1, –2)	(5, –5)

x	0	1	2
y	4	5	6
(x, y)	(0, 4)	(1, 5)	(2, 6)

1



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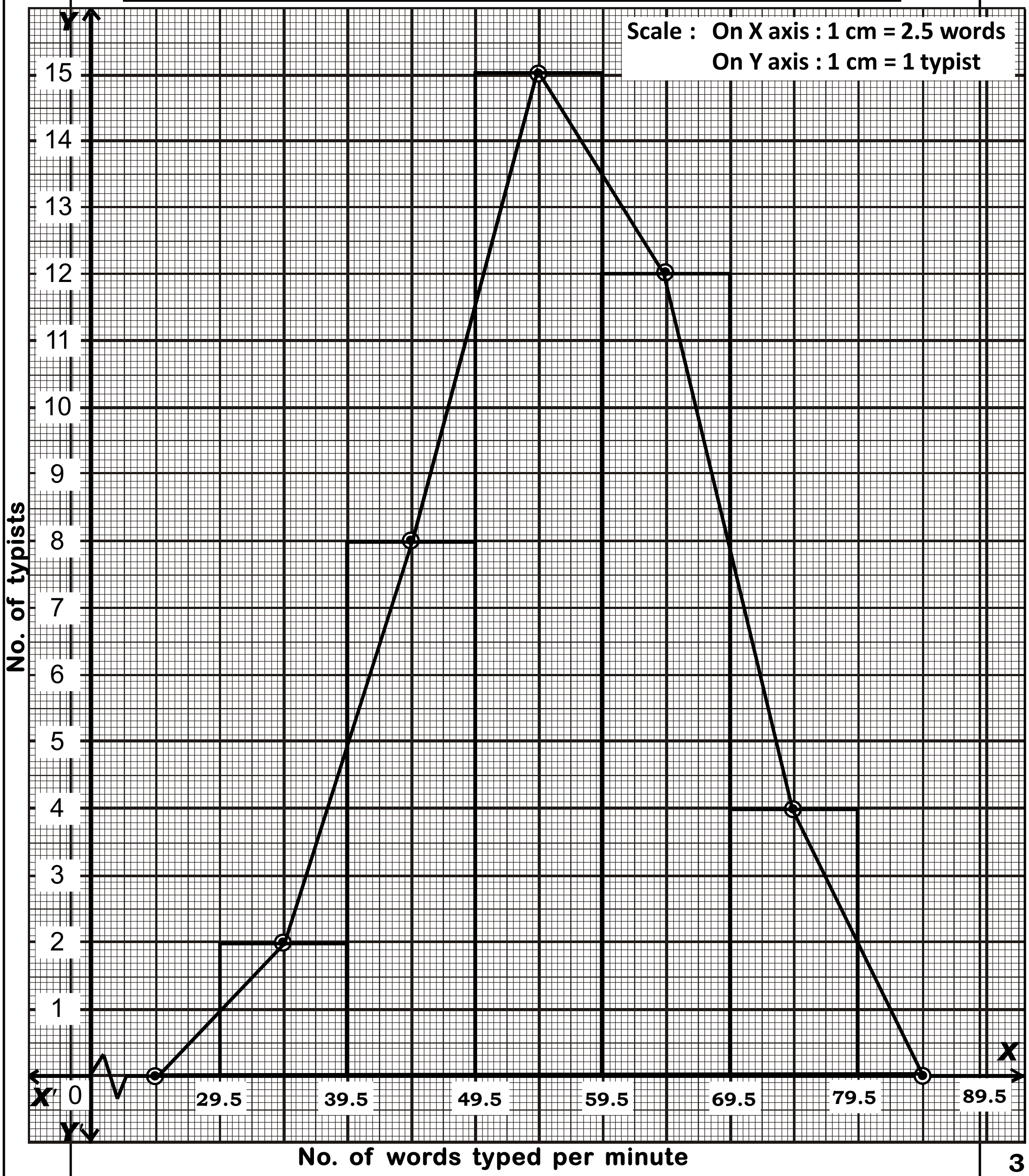
∴ x = – 3 and y = 1 is the solution of given simultaneous equations. 1

(ii)

No. of words typed per minute	Continuous Classes	Frequency No. of typists
30 - 39	29.5 - 39.5	2
40 - 49	39.5 - 49.5	8
50 - 59	49.5 - 59.5	15
60 - 69	59.5 - 69.5	12
70 - 79	69.5 - 79.5	3

1



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(iii)	$t_2 = 12, \quad t_4 = 20$ $t_n = a + (n - 1)d$ $t_2 = a + (2 - 1)d$ $12 = a + d$ $\therefore a + d = 12$ ....(i)	1
	$t_4 = a + (4 - 1)d$ $20 = a + 3d$ $\therefore a + 3d = 20$ ....(ii) Subtracting (ii) from (i), $a + d = 12$ $a + 3d = 20$ $(-) \quad (-) \quad (-)$ $-2d = -8$ $\therefore d = 4$	1
	Substituting d = 4 in (i), $a + 4 = 12$ $\therefore a = 12 - 4$ $\therefore a = 8$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $\therefore S_{25} = \frac{25}{2}[2a + (25 - 1)d]$	1
	$= \frac{25}{2}[2(8) + 24(4)]$ $= \frac{25}{2}[16 + 96]$ $= \frac{25}{2}[112]$ $S_{25} = 1400$ $\therefore$ Sum of 25 terms of the A.P is 1400.	1
A.5.	**Solve ANY TWO of the following :**	
(i)	$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$ ....(i) Substituting $x + \frac{1}{x} = m$ Squaring both the sides we get, $\left(x + \frac{1}{x}\right)^2 = m^2$	1

$$\therefore \quad (x)^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = m^2$$

$$\therefore \quad x^2 + 2 + \frac{1}{x^2} = m^2$$ 1

$$\therefore \quad x^2 + \frac{1}{x^2} = m^2 - 2$$

Equation (i) becomes

$$\therefore \quad 2(m^2 - 2) - 9m + 14 = 0$$

$$\therefore \quad 2m^2 - 4 - 9m + 14 = 0$$

$$\therefore \quad 2m^2 - 9m + 10 = 0$$

$$\therefore \quad 2m^2 - 4m - 5m + 10 = 0$$

$$\therefore \quad 2m(m - 2) - 5(m - 2) = 0$$

$$\therefore \quad (m - 2)(2m - 5) = 0$$

$$\therefore \quad m - 2 = 0 \quad \text{or} \quad 2m - 5 = 0$$

$$\therefore \quad m = 2 \quad \text{or} \quad 2m = 5$$ 1

$$\therefore \quad m = 2 \quad \text{or} \quad m = \frac{5}{2}$$

Resubstituting $m = x + \frac{1}{x}$ we get,

$$x + \frac{1}{x} = 2 \ \ldots\ldots \text{(ii)} \quad \text{or} \quad x + \frac{1}{x} = \frac{5}{2} \ \ldots\ldots \text{(iii)}$$

From (ii), $x + \frac{1}{x} = 2$

Multiplying throughout by x we get,

$$x^2 + 1 = 2x$$

$$\therefore \quad x^2 - 2x + 1 = 0$$

$$\therefore \quad (x - 1)^2 = 0$$

Taking square root on both the sides we get,

$$x - 1 = 0$$

$$\therefore \quad x = 1$$ 1

From (iii), $x + \frac{1}{x} = \frac{5}{2}$

Multiplying throughout by 2x, we get;

$$2x^2 + 2 = 5x$$

$$\therefore \quad 2x^2 - 5x + 2 = 0$$

$$\therefore \quad 2x^2 - 4x - x + 2 = 0$$

$$\therefore \quad 2x(x - 2) - 1(x - 2) = 0$$

$$\therefore \quad (x - 2)(2x - 1) = 0$$

$$\therefore \quad x - 2 = 0 \quad \text{or} \quad 2x - 1 = 0$$

$$\therefore \quad x = 2 \quad \text{or} \quad 2x = 1$$

$$\therefore \quad x = 2 \quad \text{or} \quad x = \frac{1}{2}$$

$$\therefore \quad \boxed{x = 1 \quad \text{or} \quad x = 2 \quad \text{or} \quad x = \frac{1}{2}}$$ 1

(ii)

Classes (I.Q.)	Frequency (f_i) (No. of candidates)	Cumulative frequency less than type	
70 - 80	7	7	
80 - 90	16	23 $\rightarrow$ *c.f.*	
90 - 100	20 $\rightarrow f$	43	1
100 - 110	17	60	
110 - 120	11	71	
120 - 130	7	78	
130 - 140	2	80	
Total	**80** $\rightarrow$ **N**		

Here total frequency = Σf_i = N = 80 **1**

$\therefore \frac{N}{2} = \frac{80}{2} = 40$

Cumulative frequency (less than type) which is just greater than 40 is 43. Therefore corresponding class 90 - 100 is median class. **1**

L = 90, N = 80, *c.f.* = 23, f = 20, h = 10

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$

$$= 90 + \left(\frac{80}{2} - 23\right)\frac{10}{20}$$ **1**

$$= 90 + (40 - 23)\frac{1}{2}$$

$$= 90 + (17)\frac{1}{2}$$

$$= 90 + 8.5$$

$$= 98.5$$

$\therefore$ Median of I.Q.is 98.5. **1**

(iii) Let the cost of table be Rs. x **1**

Let the cost of fan be Rs. y

As per the first given condition,

$$x + y = 5000 \quad \ldots\ldots(i)$$

Profit earnt on table $= \frac{25}{100}x$

Profit earnt on fan $= \frac{20}{100}y$ **1**

Total Profit earnt $= \frac{23}{100} \times 5000$ **1**

As per the second condition,

$$\frac{25x}{100} + \frac{20y}{100} = \frac{115000}{100}$$

Multiplying throughout by 100,

$$25x + 20y = 115000$$

Dividing throughout by 5,

$$5x + 4y = 23000 \quad \text{......(ii)}$$

Multiplying (i) by 4,

$$4x + 4y = 20000 \quad \text{....(iii)}$$ **1**

Subtracting (iii) from (ii),

$\therefore$ $5x + 4y = 23000$

$\therefore$ $4x + 4y = 20000$

$x = 3000$

Substituting x = 3000 in (i),

$3000 + y = 5000$

$\therefore$ $y = 2000$ **1**

$\therefore$ **Cost of fan is Rs. 2000.**

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