MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 5 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY Five of the following : **5**

(i) Write the first five terms of the following Arithmetic Progressions where, the common difference 'd' and the first term 'a' are given :
$a = 3, d = 4$

(ii) Determine whether the given values of 'x' is a roots of given quadratic equation.
$x^2 - 4x + 4 = 0, x = 0$

(iii) Find the value of discriminant of the following equations :
$3x^2 + 2x - 1 = 0$

(iv) Write D_x for the given simultaneous equation. :
$5x = 10 - 2y$; $y = 3x - 11$

(v) If $A = 45$, $\bar{d} = 3.15$ and $h = 10$ then find mean.

(vi) For a pie diagram, $\theta = 45°$, Total = 54000. Find the data.

Q.2. Solve ANY FOUR of the following : **8**

(i) Find the first three terms of the sequences for which S_n is given below :
$$\frac{n(n+1)(2n+1)}{6}$$

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(ii) From the quadratic equation whose roots are.
3 and 10

(iii) What is the equation of X - axis? Hence, find the point of intersection of the graph of the equation x + y = 3 with the X - axis.

(iv) In each of the following experiments write the sample space S, number of sample points n (S), events P, Q, R using set and n (P), n (Q) and n (R).
Find the events among the events defined above which are : complementary events, mutually exclusive events and exhaustive events.
Form two digit numbers using the digits 0, 1, 2, 3, 4, 5 without repeating the digits.
(a) P is the event that the number so formed is even.
(b) Q is the event that the number so formed is divisible by 3.
(c) R is the event that the number so formed is greater than 50.

(v) Which term of an A.P. is 55, if a = 3 and d = 1.3.

(vi) Two coins are tossed. Find the probability of the events.
(a) head appears on both the coins.
(b) head does not appear.

Q.3. Solve ANY THREE of the following : 9

(i) Neela saves in a 'Mahila Bachat gat' Rs. 2 on the first day, Rs.4 on the second day, Rs. 6 on the third day and so on. What will be her saving in the month of February 2010 ?

(ii) Solve the given quadratic equation by completing square.
$x^2 + 3x + 1 = 0$

(iii) If a card is drawn from a pack of 52 cards, find the probability of getting:
(a) a black card
(b) not a black card
(c) a card bearing number between 2 to 5 including 2 and 5

(iv) Savita and Hamida are friends. What is the probability that both will have (a) different birthdays (b) the same birthday (ignoring a leap year).

(v) The number of hours, spent by a school boy in different activities in a day is given below.

Activity	Sleep	School	Play	Home work	Other	Total
No. of hours	8	7	2	4	3	24

Represent the above information using pie diagram.

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Q.4. Solve ANY TWO of the following : **8**

(i) Solve the given simultaneous equations using graphical method : $x + y = 8, x - y = 2$

(ii) Draw histogram and frequency polygon for the following frequency distribution :

Class	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30
Frequency	20	30	50	40	10

(iii) In the A.P. 7, 14, 21, ... How many terms are there to consider for getting sum 5740.

Q.5. Solve ANY TWO of the following : **10**

(i) Solve the following equations :

$$9\left[x^2 + \frac{1}{x^2}\right] - 3\left[x - \frac{1}{x}\right] - 20 = 0$$

(ii) The following table shows ages of 300 patients getting medical treatment in a hospital on a particular day.

Age (in years)	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of patients	60	42	55	70	53	20

(iii) The weight of a bucket is 15 kg, when it is filled with water $\frac{3}{5}$ of its capacity while it weights 19 kg, if it is filled with water $\frac{4}{5}$ of its capacity. Find the weight of bucket, if it is completely filled with water.

Best Of Luck 👍

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MT

2014 ___ ___ 1100 Seat No.

MT - MATHEMATICS (71) ALGEBRA - PRELIM II - PAPER - 5 (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) $a = 3, d = 4$

Here, $t_1 = a = 3$

$t_2 = t_1 + d = 3 + 4 = 7$

$t_3 = t_2 + d = 7 + 4 = 11$

$t_4 = t_3 + d = 11 + 4 = 15$

$t_5 = t_4 + d = 15 + 4 = 19$

$\therefore$ The first five terms of the A.P. are 3, 7, 11, 15 and 19. **1**

(ii) $x^2 - 4x + 4 = 0, x = 0$

Putting $x = 0$ in L.H.S., we get,

L.H.S. $= (0)^2 + 4(0) + 4$

$= 0 - 0 + 4$

$= 4$

$\neq$ R.H.S.

$\therefore$ L.H.S. $\neq$ R.H.S.

Thus equation is not satisfied.

So 0 is not the root of the given quadratic equation. **1**

(iii) $3x^2 + 2x - 1 = 0$

Comparing with $ax^2 + bx + c = 0$

we have $a = 3, b = 2, c = -1$

$\Delta = b^2 - 4ac$

$= (2)^2 - 4(3)(-1)$

$= 4 + 12$

$= 16$

$\therefore$ $\Delta = 16$ **1**

(iv) $5x = 10 - 2y$

$\therefore$ $5x + 2y = 10$

$y = 3x - 11$

$\therefore$ $-3x + y = -11$

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$$D_x = \begin{vmatrix} 10 & 2 \\ -11 & 1 \end{vmatrix} = (10 \times 1) - (2 \times -11)$$
$$= 10 + 22 = 32$$
$$\therefore \boxed{Dx = 32}$$ **1**

(v) $$\text{Mean } (\bar{x}) = A + \bar{d}$$
$$= 45 + 3.15$$
$$= 48.15$$
$\therefore$ Mean is 48.15 units. **1**

(vi) $$\theta = \frac{\text{Data}}{\text{Total}} \times 360$$
$$\therefore 45 = \frac{\text{Data}}{54000} \times 360$$
$$\therefore \text{Data} = \frac{45 \times 54000}{360}$$
$$\therefore \boxed{\text{Data} = 6750}$$ **1**

A.2. Solve ANY Four of the following :

(i) $$S_n = \frac{n(n+1)(2n+1)}{6}$$
$$\therefore S_1 = \frac{1(1+1)[2(1)+1]}{6} = \frac{1 \times 2 \times 3}{6} = \frac{6}{6} = 1$$ **1**
$$\therefore S_2 = \frac{2(2+1)[2(2)+1]}{6} = \frac{2 \times 3 \times 5}{6} = \frac{30}{6} = 5$$
$$\therefore S_3 = \frac{3(3+1)[2(3)+1]}{6} = \frac{3 \times 4 \times 7}{6} = 14$$
We know that,
$t_1 = S_1 = 1$
$t_2 = S_2 - S_1 = 5 - 1 = 4$
$t_3 = S_3 - S_2 = 14 - 5 = 9$
$\therefore$ The first three terms of the sequence are 1, 4 and 9. **1**

(ii) The roots of the quadratic equation are 3 and 10
Let $\alpha = 3$ and $\beta = 10$
$\therefore \alpha + \beta = 3 + 10 = 13$
$\alpha . \beta = 3 \times 10 = 30$ **1**
We know that,
$x^2 - (\alpha + \beta) x + \alpha.\beta = 0$
$\therefore x^2 - 13x + 30 = 0$
$\therefore$ The required quadratic equation is $x^2 - 13x + 30 = 0$ **1**

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(iii)	The equation of X-axis is $y = 0$ Let the point of intersection of graph $x + y = 3$ with X-axis be $(h, 0)$ $\because$ $(h, 0)$ lies on the graph, it satisfies the equation $\therefore$ Substituting $x = h$ and $y = 0$ in the equation we get, $h + 0 = 3$ $\therefore$ $h = 3$ $\therefore$ The line $x + y = 3$ intersects the X-axis at $(3, 0)$.	1 1
(iv)	Two digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 without repeating digits are as follows : S = { 10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43, 45, 50, 51, 52, 53, 54 } n (S) = 25 P is the event that the number so formed is even P = { 10, 12, 14, 20, 24, 30, 32, 34, 40, 42, 50, 52, 54 } $\therefore$ n (P) = 13 Q is the event that the number so formed is divisible by 3 Q = { 12, 15, 21, 24, 30, 42, 45, 51, 54 } $\therefore$ n (Q) = 9 R is the event that the number so formed is greater than 50 R = { 51, 52, 53, 54 } $\therefore$ n (R) = 4	 1 1
(v)	For an A.P. $a = 1$, $d = 2$, $t_n = 149$ $tn = a + (n - 1)d$ $\therefore$ $149 = 1 + (n - 1)(2)$ $\therefore$ $149 - 1 = 2n - 2$ $\therefore$ $148 + 2 = 2n$ $\therefore$ $150 = 2n$ $\therefore$ $n = 75$ $\therefore$ 75^{th} term of an A.P. is 149.	 1 1
(vi)	(a) head appears on both the coins. Two coins are tossed S = { HH, HT, TH, TT } n (S) = 4	

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Let A be the event that head appears on both the coins

$A = \{HH\}$

$n(A) = 1$

$P(A) = \frac{n(A)}{n(S)}$

$\therefore$ $\boxed{P(A) = \frac{1}{4}}$ **1**

(b) head does not appear.

Let B be the event that head does not appear

$B = \{TT\}$

$n(B) = 1$

$P(B) = \frac{n(B)}{n(S)}$

$\therefore$ $\boxed{P(B) = \frac{1}{4}}$ **1**

A.3. Solve ANY THREE of the following :

(i) The savings done by Neela on each day is as follows 2, 4, 6, ...

These every day savings form an A.P. with

First day saving (a) = 2 **1**

Difference in savings made in two successive days (d) = 2

Total no. of days in the month of February 2010 (n) = 28

$\therefore$ Total savings for the month of February (S_{28}) = ?

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{28} = \frac{28}{2}[2(2) + (28 - 1)(2)]$

$= 14[4 + 27(2)]$ **1**

$= 14[4 + 54]$

$\therefore S_{28} = 14[58]$

$\therefore S_{28} = 812$

$\therefore$ Neela saved Rs. 812 in the month of February. **1**

(ii) $x^2 + 3x + 1 = 0$

$\therefore x^2 + 3x = -1$ ..... (i)

Third term $= \left(\frac{1}{2} \times \text{coefficient of } x\right)^2$ **1**

$= \left(\frac{1}{2} \times 3\right)^2$

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$$= \left(\frac{3}{2}\right)^2$$

$$= \frac{9}{4}$$

Adding $\frac{9}{4}$ to both the sides of (i) we get,

$$x^2 + 3x + \frac{9}{4} = -1 + \frac{9}{4}$$ **1**

$$\therefore \left(x+\frac{3}{2}\right)^2 = \frac{-4+9}{4}$$

$$\therefore \left(x+\frac{3}{2}\right)^2 = \frac{5}{4}$$

Taking square root on both the sides we get,

$$x + \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$$

$$\therefore x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$$

$$\therefore x = \frac{-3 \pm \sqrt{5}}{2}$$

$$\therefore x = \frac{-3 + \sqrt{5}}{2} \quad \text{or} \quad x = \frac{-3 - \sqrt{5}}{2}$$

$\therefore$ $\frac{-3 + \sqrt{5}}{2}$ and $\frac{-3 - \sqrt{5}}{2}$ are the roots of the given quadratic equation. **1**

(iii) There are 52 cards in a pack

$\therefore$ n (S) = 52

(a) Let A be event that the card drawn is a black card

Total no. of black cards = 26

$\therefore$ n (A) = 26

$$P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{26}{52}$$

$$\therefore P(A) = \frac{1}{2}$$ **1**

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(b) Let B be the event that the card drawn is not a black card

Total no. of red cards = 26

$\therefore$ n (B) = 26

$$P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{26}{52}$$

$$\therefore \boxed{P(B) = \frac{1}{2}}$$ **1**

(c) Let C be the event that card drawn bears number between 2 to 5 including 2 and 5

No. of cards between 2 to 5 including 2 and 5 is 4.

$\because$ There are 4 types of cards

$\therefore$ The total no. of cards between 2 to 5 including 2 and 5 is 4 × 4 = 16

n (C) = 16

$$P(C) = \frac{n(C)}{n(S)}$$

$$\therefore P(C) = \frac{16}{52}$$

$$\therefore \boxed{P(C) = \frac{4}{13}}$$ **1**

(iv) Since there are 365 days in a year

n (S) = 365 × 365

Let A be the event that both Savita and Hamida have the same day as a their birthday. **1**

$\therefore$ n (A) = 1 × 365

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1 \times 365}{365 \times 365}$$

$$\therefore \boxed{P(A) = \frac{1}{365}}$$ **1**

Let B be the event that both Savita and Hamida have their birth days on different days

$\therefore$ A and B are two complementary events

$\therefore$ P (A) + P (B) = 1

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$\therefore \quad \frac{1}{365} + P(B) = 1$

$\therefore \quad P(B) = 1 - \frac{1}{365}$

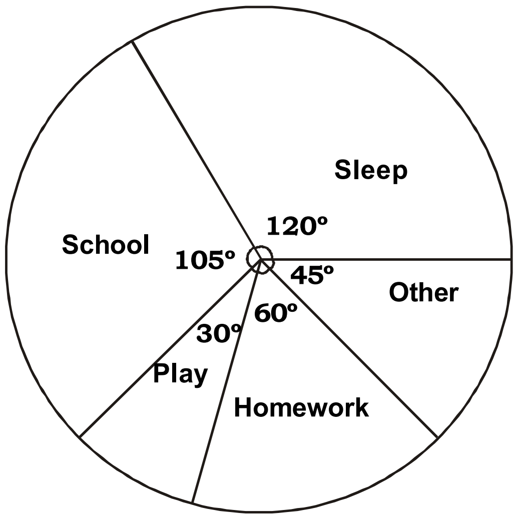
$\therefore \quad P(B) = \frac{365 - 1}{365}$

$\therefore \quad \boxed{P(B) = \frac{364}{365}}$ **1**

(v)

Activity	No. of hrs.	Measure of central angle (θ)
Sleep	8	$\frac{8}{24} \times 360° = 120°$
School	7	$\frac{7}{24} \times 360° = 105°$
Play	2	$\frac{2}{24} \times 360° = 30°$
Home work	4	$\frac{4}{24} \times 360° = 60°$
Other	3	$\frac{3}{24} \times 360° = 45°$
Total	**24**	**360°**

1



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A.4. **Solve ANY TWO of the following :**

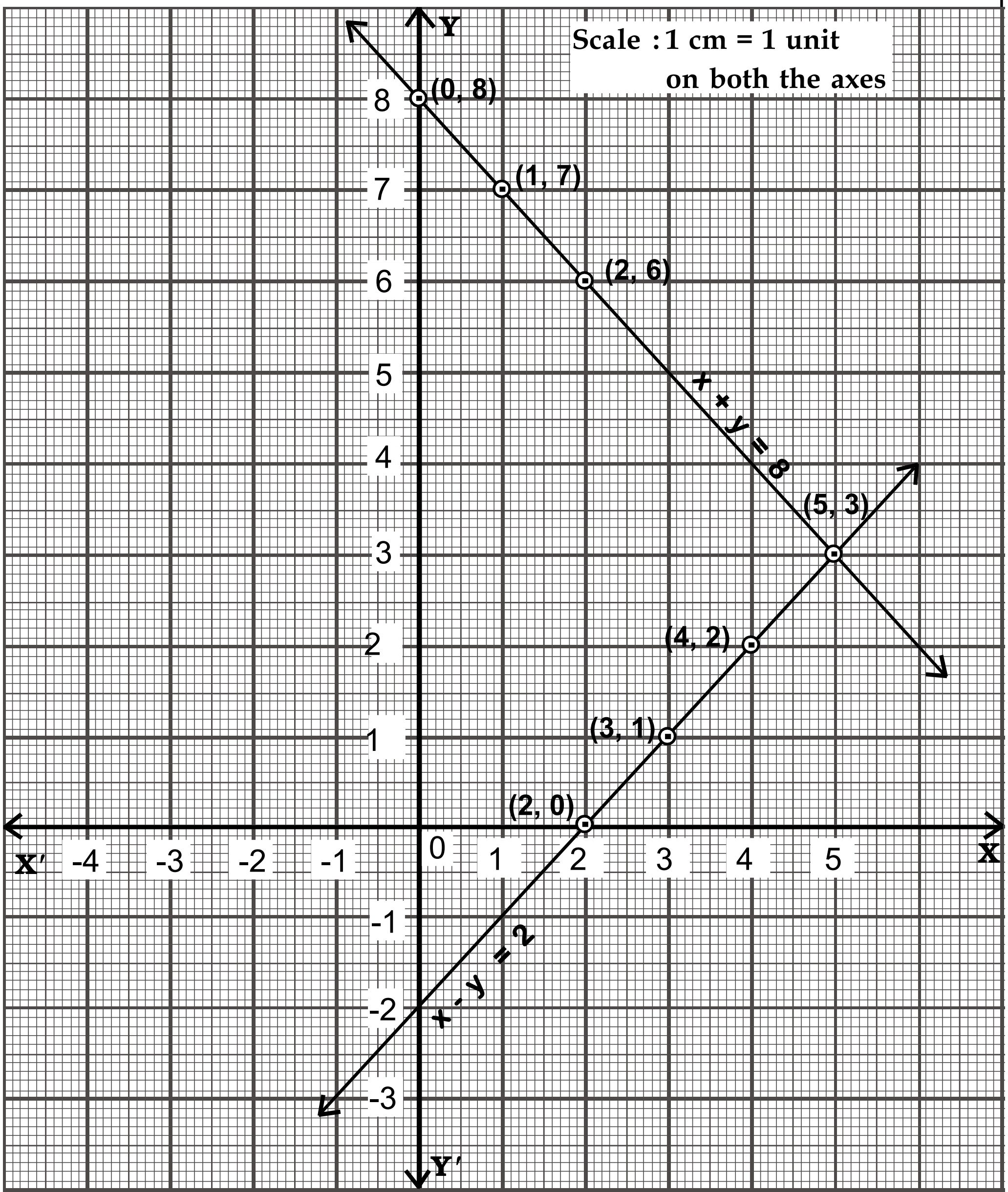
(i) $x + y = 8$ $\quad$ $x - y = 2$

$\therefore$ $y = 8 - x$ $\quad$ $\therefore$ $x = 2 + y$

x	0	1	2
y	8	7	6
(x, y)	(0, 8)	(1, 7)	(2, 6)

x	2	3	4
y	0	1	2
(x, y)	(2, 0)	(3, 1)	(4, 2)

1



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$\therefore$ x = 5 and y = 3 is the solution of given simultaneous equations. 1

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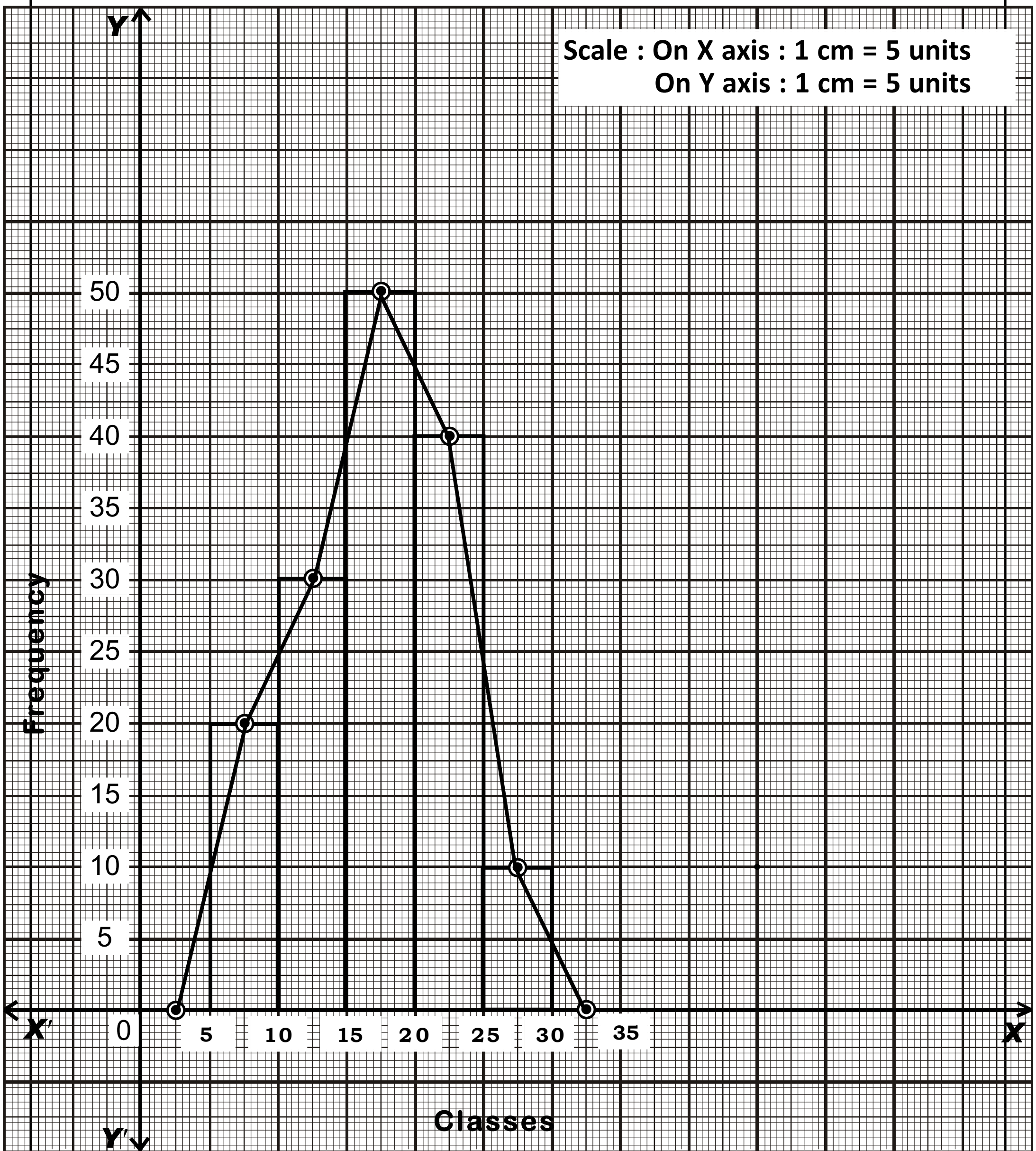
(ii)

Class	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30
Frequency	20	30	50	40	10

1



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(iii) For the A.P. 7, 14, 21

a = 7, d = 7

$S_n = 5740$ 1

$S_n = \frac{n}{2}[2a + (n - 1)d]$

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$$S_n = \frac{n}{2}[2(7) + (n-1)7]$$ **1**

$$5740 = \frac{n}{2}[14 + 7n - 7]$$

$$5740 = \frac{n}{2}[7n + 7]$$

$$11480 = 7n^2 + 7n$$

$\therefore\ 7n^2 + 7n - 11480 = 0$

Dividing through at by 7 we get,

$$n^2 + n - 1640 = 0$$

$\therefore\ n^2 + 41n - 40n - 1640 = 0$ **1**

$\therefore\ n(n + 41) - 40(n + 41) = 0$

$\therefore\ (n + 41)(n - 40) = 0$

$\therefore\ n + 41 = 0$ or $n - 40 = 0$

$\therefore\ n = -41$ or $n = 40$

$n = -41$ is not acceptable because no of terms cannot be negative

$\therefore\ n = 40$

$\therefore$ For the given sequence 40 terms have to be considered for getting sum of 5740. **1**

A.5. Solve ANY TWO of the following :

(i) $9\left[x^2 + \frac{1}{x^2}\right] - 3\left[x - \frac{1}{x}\right] - 20 = 0$ ....(i)

Substituting $x - \frac{1}{x} = m$ **1**

Squaring both the sides we get,

$$\left(x - \frac{1}{x}\right)^2 = m^2$$

$\therefore\ x^2 - 2 + \frac{1}{x^2} = m^2$

$\therefore\ x^2 + \frac{1}{x^2} = m^2 + 2$

$\therefore$ Equation (i) becomes,

$9(m^2 + 2) - 3m - 20 = 0$ **1**

$\therefore\ 9m^2 + 18 - 3m - 20 = 0$

$\therefore\ 9m^2 - 3m - 2 = 0$

$\therefore\ 9m^2 + 3m - 6m - 2 = 0$

$\therefore\ 3m(3m + 1) - 2(3m + 1) = 0$

$\therefore\ (3m + 1)(3m - 2) = 0$

$\therefore\ 3m + 1 = 0$ or $3m - 2 = 0$

$\therefore\ 3m = -1$ or $3m = 2$

$\therefore\ m = -\frac{1}{3}$ or $m = \frac{2}{3}$

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Resubstituting $m = x - \frac{1}{x}$ we get,

$x - \frac{1}{x} = -\frac{1}{3}$ ....(ii) or $x - \frac{1}{x} = \frac{2}{3}$ ....(iii)

From (ii), $x - \frac{1}{x} = -\frac{1}{3}$

Multiplying throughout by 3x we get,

$\therefore \quad 3x^2 - 3 = -x$

$\therefore \quad 3x^2 + x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 3$, $b = 1$, $c = -3$

$b^2 - 4ac = (1)^2 - 4(3)(-3)$

$= 1 + 36$

$= 37$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **1**

$= \frac{-1 \pm \sqrt{37}}{2(3)}$

$= \frac{-1 \pm \sqrt{37}}{6}$

$\therefore \quad x = \frac{1 + \sqrt{37}}{6}$ or $x = \frac{-1 - \sqrt{37}}{6}$

From (iii), $x - \frac{1}{x} = \frac{2}{3}$

Multiplying throughout by 3x, we get;

$3x^2 - 3 = 2x$

$\therefore \quad 3x^2 - 2x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 3$, $b = -2$, $c = -3$ **1**

$b^2 - 4ac = (-2)^2 - 4(3)(-3)$

$= 4 + 36$

$= 40$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{40}}{2(3)}$

$= \frac{-2 \pm \sqrt{4 \times 10}}{6}$

$= \frac{-2 \pm 2\sqrt{10}}{6}$

$= \frac{-1 \pm \sqrt{10}}{3}$

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$\therefore \quad x = \frac{-1+\sqrt{10}}{3}$ or $\frac{-1-\sqrt{10}}{3}$

$\therefore \quad x = \frac{1+\sqrt{37}}{6}$ or $x = \frac{-1-\sqrt{37}}{6}$ or $x = \frac{-1+\sqrt{10}}{3}$ or $x = \frac{-1-\sqrt{10}}{3}$ **1**

(ii)

Classes (Age in years)	**Frequency (f_i) (No. of patients)**	**Cumulative frequency less than type**
10 - 20	60	60
20 - 30	42	102 $\rightarrow$ *c.f.*
30 - 40	55 $\rightarrow f$	157
40 - 50	70	227
50 - 60	53	280
60 -70	20	300
Total	**300 $\rightarrow$ N**	

1

Here total frequency $= \Sigma f_i = N = 300$

$\therefore \quad \frac{N}{2} = \frac{300}{2} = 150$ **1**

Cumulative frequency (less than type) which is just greater than 150 is 157. Therefore corresponding class 30 - 40 is median class. **1**

L = 30, N = 300, *c.f.* = 102, f = 55, h = 10

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$ **1**

$$= 30 + \left(\frac{300}{2} - 102\right)\frac{10}{55}$$

$$= 30 + (150 - 102)\frac{10}{55}$$

$$= 30 + (48)\frac{10}{55}$$

$$= 30 + (48)\frac{2}{11}$$

$$= 30 + \frac{96}{11}$$

$$= 30 + 8.73$$

$$= 38.73$$

$\therefore$ Median of age is 38.73 years. **1**

(iii) Let weight of empty bucket be x kg. and let the weight of only water when the bucket is filled to its full capacity be y kg. **1**

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As per the first given condition,

$$x + \frac{3}{5}y = 15$$

Multiplying throughout by 5,

$$5x + 3y = 75 \quad \ldots\ldots.(i)$$

As per the second given condition, **1**

$$x + \frac{4}{5}y = 19$$

Multiplying throughout by 5,

$$5x + 4y = 95 \quad \ldots\ldots(ii)$$

Subtracting (ii) from (i),

$$\begin{array}{rcl} 5x + 3y & = & 75 \\ 5x + 4y & = & 95 \\ (-) \quad (-) & & (-) \\ \hline -y & = & -20 \end{array}$$

1

$\therefore$ $y = 20$

Substituting y = 20 in (i),

$5x + 3\ (20) = 75$ **1**

$\therefore$ $5x + 60 = 75$

$\therefore$ $5x = 75 - 60$

$\therefore$ $5x = 15$

$\therefore$ $x = 3$

$\therefore$ Weight of bucket when it is filled with water to its full capacity

$= x + y$

$\therefore$ $3 + 20 = 23$

$\therefore$ Weight of bucket when it is filled with water to its full capacity is 23 kg. **1**

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