Set - A

Q.P. SET CODE
A

MT - w

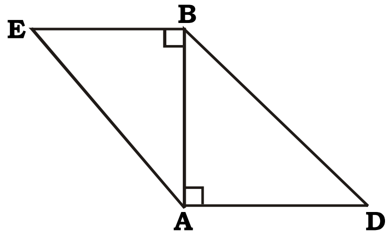
Seat No. ☐☐☐☐☐☐☐

2013 ___ ___ 1100 - MT - w MATHEMATICS (71) GEOMETRY - PAPER A (E)

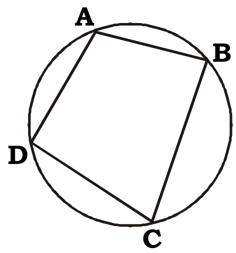
Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Q.1. Solve the following : (Any 5) **5**

(i) In the adjoining figure, seg BE $\perp$ seg AB and seg BA $\perp$ seg AD. If BE = 6 and AD = 9 find $\frac{A(\Delta ABE)}{A(\Delta ABD)}$



(ii) What is the relation between $\angle ABC$ and $\angle ADC$ of cyclic ☐ABCD ?



(iii) Find where the angle lies if the terminal arm passes through (5, – 7)

(iv) Find the slope of a line whose inclination is 45°.

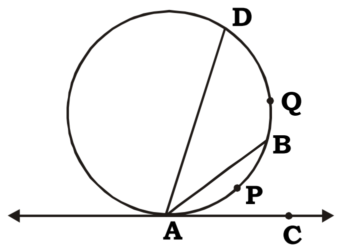
(v) A (ΔPQR) = 24 cm^2, the height QS is 8 cm. What is the length of side PR ?

(vi) Volume of a cube is 1000 cm^3, find the length of its side.

Q.2. Solve the following : (Any 4) **8**

(i) A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

(ii) In the adjoining figure, seg AB and seg AD are chords of the circle. C be a point on tangent to the circle at point A. If m (arc APB) = 80° and ∠BAD = 30°, then find (i) ∠BAC (ii) m (arc BQD)



(iii) Eliminate θ, if x = a sec θ, y = b tan θ

(iv) If (– 2, – 3) is a point on the line 2y = mx + 5, find m.

(v) Find the trigonometric ratios in standard position whose terminal arm passes through the points (4, 3)

(vi) Write the equation of a line passing through the origin and the point (– 3, 5)

Q.3. Solve the following : (Any 3) **9**

(i) □ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

(ii) If the chord AB of a circle is parallel to the tangent at C, then prove that AC = BC.

(iii) Construct tangents to the circle from point B with radius 3.5 cm and centre A. Point B is at a distance 7.3 cm from the centre.

(iv) If the points $\left(\frac{2}{5}, \frac{1}{3}\right)$, $\left(\frac{1}{2}, k\right)$ and $\left(\frac{4}{5}, 0\right)$ are collinear then find the value of k.

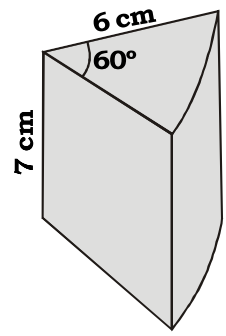
(v) Construct ΔLEM such that, LE = 6cm, LM = 7.5 cm, ∠LEM = 90° and draw its circumcircle.

Q.4. Solve the following : (Any 2) **8**

(i) Prove : The lengths of the two tangent segments to a circle drawn from an external point are equal.

(ii) ΔSHR ~ ΔSVU, In ΔSHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and $\frac{SH}{SV} = \frac{3}{5}$; construct ΔSVU.

(iii) A piece of cheese is cut in the shape of the sector of a circle of radius 6 cm. The thickness of the cheese is 7 cm. Find
(i) The curved surface area of the cheese.
(ii) The volume of the cheese piece.



Q.5. Solve the following : (Any 2) **10**

(i) Prove : In a triangle, the angle bisector divides the side opposite to the angle in the ratio of the remaining sides.

(ii) A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river. ($\sqrt{3}$ = 1.73)

(iii) A 10 m deep well of diameter 1.4 m is dug up in a field and the earth from digging is spread evenly on the adjoining cuboid field. The length and breadth of that filled are 55m and 14 m respectively. Find the thickness of the earth layer spread.

Best Of Luck 👍

Set - A

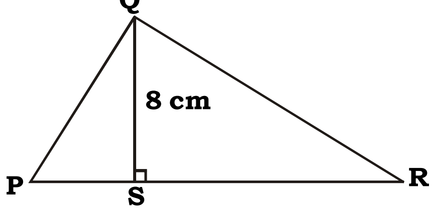
A.P. SET CODE
A

MT - w

2013 ___ ___ 1100 - MT - w MATHEMATICS (71) GEOMETRY - PAPER A (E)

Time : 2 Hours **Prelim - I Model Answer Paper** **Max. Marks : 40**

Q.1.	**Solve the following : (Any 5)**	
(i)	$\dfrac{A(\Delta ABE)}{A(\Delta ABD)} = \dfrac{BE}{AD}$ [Triangles with common base]	½
	$\therefore \dfrac{A(\Delta ABE)}{A(\Delta ABD)} = \dfrac{6}{9}$	
	$\therefore \boxed{\dfrac{A(\Delta ABE)}{A(\Delta ABD)} = \dfrac{2}{3}}$	½
(ii)	□ABCD is cyclic [Given]	
	$\therefore$ m $\angle$ABC + m $\angle$ADC = 180° [Opposite angles of quadrilateral are supplementary]	½
	$\therefore$ $\angle$ABC and $\angle$ADC are supplementary.	½
(iii)	(5, – 7)	
	$\because$ x is positive and y is negative	½
	$\therefore$ The terminal arm is in IV quadrant.	½
(iv)	Inclination of the line (θ) = 45°	½
	$\therefore$ Slope of the line = tan θ = tan 45° = 1	
	$\therefore$ $\boxed{\text{Slope of the line is 1.}}$	½
(v)	Area of a triangle = $\dfrac{1}{2}$ × base × height	½
	$\therefore$ A (ΔPQR) = $\dfrac{1}{2}$ × PR × QS	
	$\therefore$ 24 = $\dfrac{1}{2}$ × PR × 8	



∴ $24 = PR \times 4$

∴ $PR = \frac{24}{4}$ ½

∴ **$PR = 6$ m** ½

(vi) Volume of a cube $= 1000 \text{ cm}^3$

Volume of a cube $= l^3$

∴ $l^3 = 100$

∴ $l = 10$ [Taking cube roots] ½

∴ **Length of the side of cube is 10 cm.**

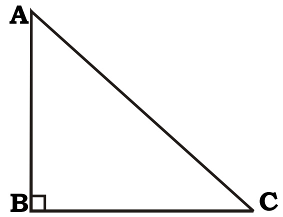
Q.2. Solve the following : (Any 4)

(i) In the adjoining figure,

seg AB represents the wall

seg AC represents the ladder ½

seg BC represents the distance of the foot of the ladder from the base of the wall



[½ mark for figure]

AC = 10 m

AB = 8 m

In ΔABC,

m ∠ABC = 90° [Given] ½

$AC^2 = AB^2 + BC^2$ [By Pythagoras Theorem] ½

∴ $(10)^2 = (8)^2 + BC^2$

∴ $100 = 64 + BC^2$

∴ $BC^2 = 100 - 64$

∴ $BC^2 = 36$ [Taking square roots]

∴ BC = 6 m

∴ **The distance of the foot of the ladder from the base of the wall is 6 m.** ½

(ii) m ∠BAC $= \frac{1}{2}$ m(arc APB) [Tangent secant theorem] ½

∴ m ∠BAC $= \frac{1}{2} \times 80$

∴ **m ∠BAC = 40°** ½

m ∠BAD $= \frac{1}{2}$ m (arc BQD) [Inscribed angle theorem] ½

∴ $30 = \frac{1}{2}$ m (arc BQD)

∴ m (arc BQD) $= 30 \times 2$

∴ **m (arc BQD) = 60°** ½

(iii)

$x = a \sec \theta$ — ½

$\therefore \sec \theta = \dfrac{x}{a}$ ......(i)

$y = b \tan \theta$

$\therefore \tan \theta = \dfrac{y}{b}$ ......(ii) — ½

$1 + \tan^2 \theta = \sec^2 \theta$

$\therefore 1 + \left(\dfrac{y}{b}\right)^2 = \left(\dfrac{x}{a}\right)^2$ [From (i) and (ii)] — ½

$\therefore 1 + \dfrac{y^2}{b^2} = \dfrac{x^2}{a^2}$ — ½

$\therefore \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

(iv)

The equation of the line is 2y = mx + 5

Let P ≡ (– 2, – 3) — ½

Point P lies on the line 2y = mx + 5

∴ Co-ordinates of point P satisfies the equation of the line — ½

$\therefore 2(-3) = m(-2) + 5$

$\therefore -6 = -2m + 5$

$\therefore -2m = -6 - 5$ — ½

$\therefore -2m = -11$

$\therefore m = \dfrac{11}{2}$

∴ **The value of m is $\dfrac{11}{2}$** — ½

(v)

The terminal arm passes through P (4, 3)

∴ x = 4 and y = 3

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(4)^2 + (3)^2}$

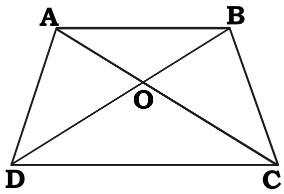
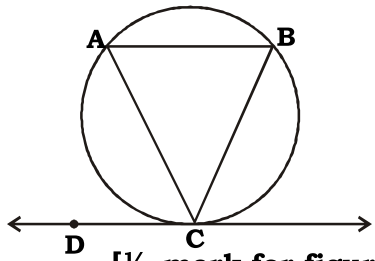
$= \sqrt{16 + 9}$

$= \sqrt{25}$

∴ r = 5 units

Let the angle be θ — ½

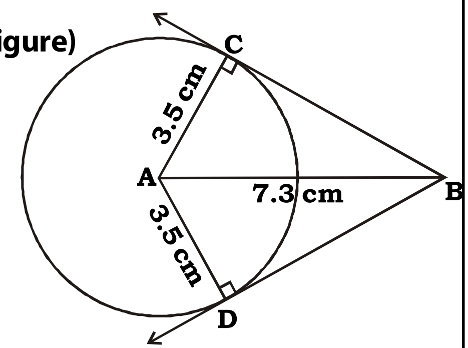
$\therefore \sin \theta = \dfrac{y}{r} = \dfrac{3}{5}$ | $\operatorname{cosec} \theta = \dfrac{r}{y} = \dfrac{5}{3}$

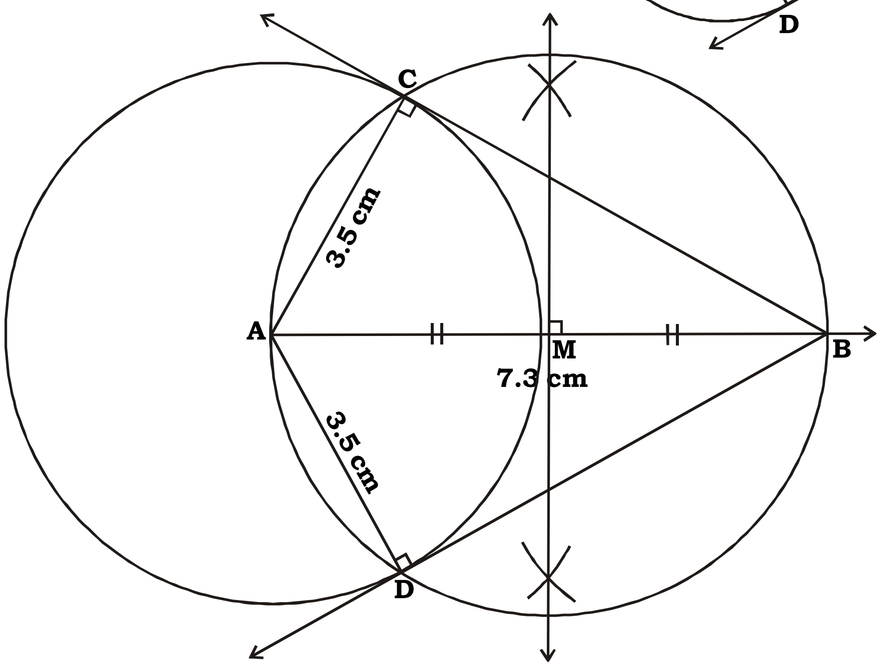
	$\cos\theta = \frac{x}{r} = \frac{4}{5}$ | $\sec\theta = \frac{r}{x} = \frac{5}{4}$ $\tan\theta = \frac{y}{x} = \frac{3}{4}$ | $\cot\theta = \frac{x}{y} = \frac{4}{3}$	1½
(vi)	Let O ≡ (0, 0) ≡ (x_1, y_1) A ≡ (– 3, 5) ≡ (x_2, y_2)	½
	∴ The equation of line OA by two point form is, $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$	½
	∴ $\frac{x - 0}{0 - (-3)} = \frac{y - 0}{0 - 5}$ ∴ $\frac{x}{3} = \frac{y}{-5}$	½
	∴ $-5x = 3y$ ∴ $5x + 3y = 0$ ∴ The equation of the line passing through the origin and the point (– 3, 5) is 5x + 3y = 0.	½
Q.3.	**Solve the following : (Any 3)**	
(i)	□ABCD is a trapezium side AB ‖ side DC [Given] ∴ On transversal AC, ∠BAC ≅ ∠DCA [Converse of alternate angles test] ∴ ∠BAO ≅ ∠DCO ......(i) [∵ A - O - C] 	1
	In ΔAOB and ΔCOD, ∠BAO ≅ ∠DCO [From (i)] ∠AOB ≅ ∠COD [Vertically opposite angles] ∴ ΔAOB ~ ΔCOD [By AA test of similarity]	1
	∴ $\frac{AO}{CO} = \frac{BO}{DO}$ [c.s.s.t.] ∴ $\frac{AO}{BO} = \frac{CO}{DO}$ [By Alternendo]	1
(ii)	Take a point D on the tangent at C as shown in the figure. seg AB ‖ line CD. [Given] ∴ On transversal AC,  **[½ mark for figure]**	½

	∠BAC ≅ ∠ACD ....(i) [Converse of alternate angles test]	½
	But, ∠ACD ≅ ∠ABC ....(ii) [Angles in alternate segment]	½
	In ΔABC,	
	∠BAC ≅ ∠ABC [From (i) and (ii)]	
	∴ seg AC ≅ seg BC [Converse of Isosceles triangle theorem]	
	∴ AC = BC	1

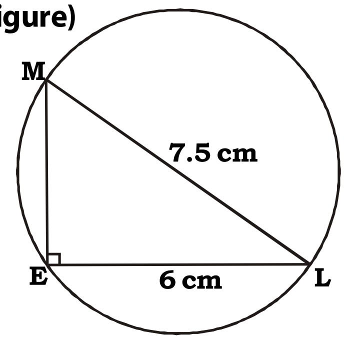
(iii)

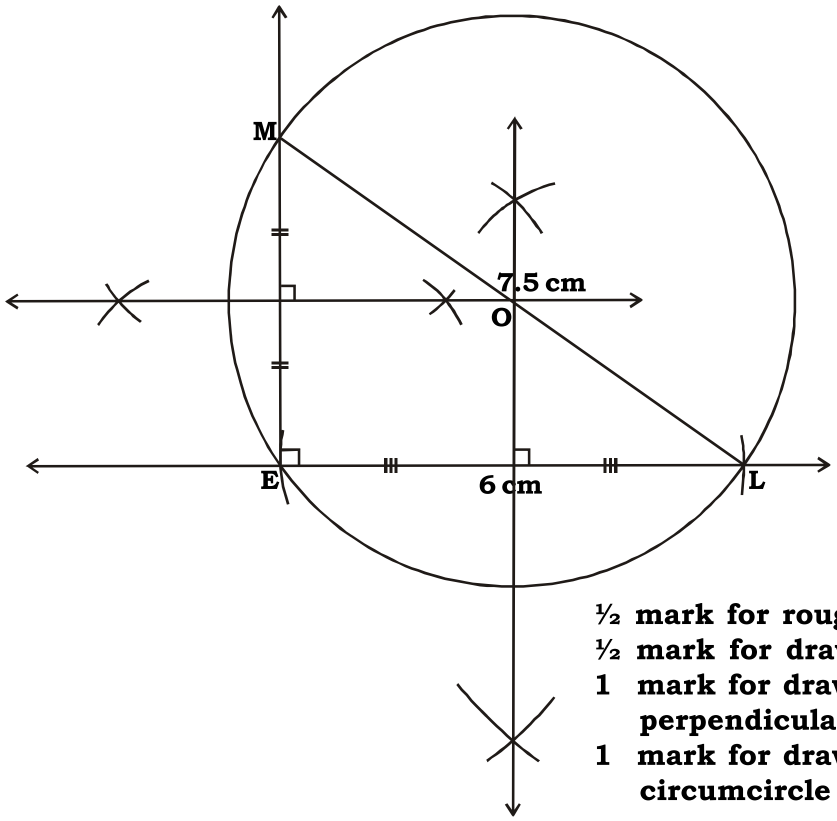
(Rough Figure)





½ mark for rough figure
½ mark for drawing the circle of radius 3.5 cm
½ mark for drawing the perpendicular bisector of seg AB
½ mark for drawing the circle with centre M
1 mark for drawing both the tangents from point B

(iv)	Let, $A \equiv \left(\frac{2}{5}, \frac{1}{3}\right) \equiv (x_1, y_1)$ $B \equiv \left(\frac{1}{2}, k\right) \equiv (x_2, y_2)$ $C \equiv \left(\frac{4}{5}, 0\right) \equiv (x_3, y_3)$	½
	$\because$ Points A, B and C are collinear Slope of line AB = Slope of line BC	½
	$\therefore \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$	½
	$\therefore \frac{k - \frac{1}{3}}{\frac{1}{2} - \frac{2}{5}} = \frac{0 - k}{\frac{4}{5} - \frac{1}{2}}$	
	$\therefore \frac{\frac{3k-1}{3}}{\frac{1}{10}} = \frac{-k}{\frac{3}{10}}$	½
	$\therefore \frac{3k-1}{3} \times 10 = -k \times \frac{10}{3}$	½
	$\therefore 3k - 1 = -k$ $\therefore 3k + k = 1$ $\therefore 4k = 1$ $\therefore k = \frac{1}{4}$	
	$\therefore$ The value of k is $\frac{1}{4}$	½
(v)	**(Rough Figure)**  	



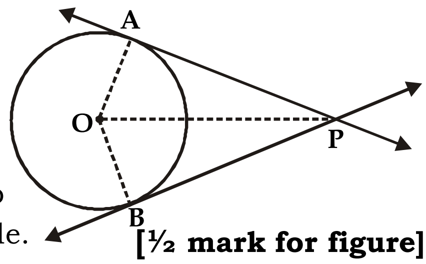
½ mark for rough figure
½ mark for drawing ΔLEM
1 mark for drawing the perpendicular bisectors
1 mark for drawing the circumcircle

Q.4. Solve the following : (Any 2)

(i) Given : (i) A circle with centre O.

(ii) P is a point in the exterior of the circle.

(iii) Points A and B are the points of contact of the two tangents from P to the circle.



[½ mark for figure] ½

To Prove : PA = PB

Construction : Draw seg OA, seg OB and seg OP. ½

Proof : In ΔPAO and ΔPBO,

m ∠PAO = m ∠PBO = 90°[Radius is perpendicular to the tangent] ½

Hypotenuse OP ≅ Hypotenuse OP [Common side] ½

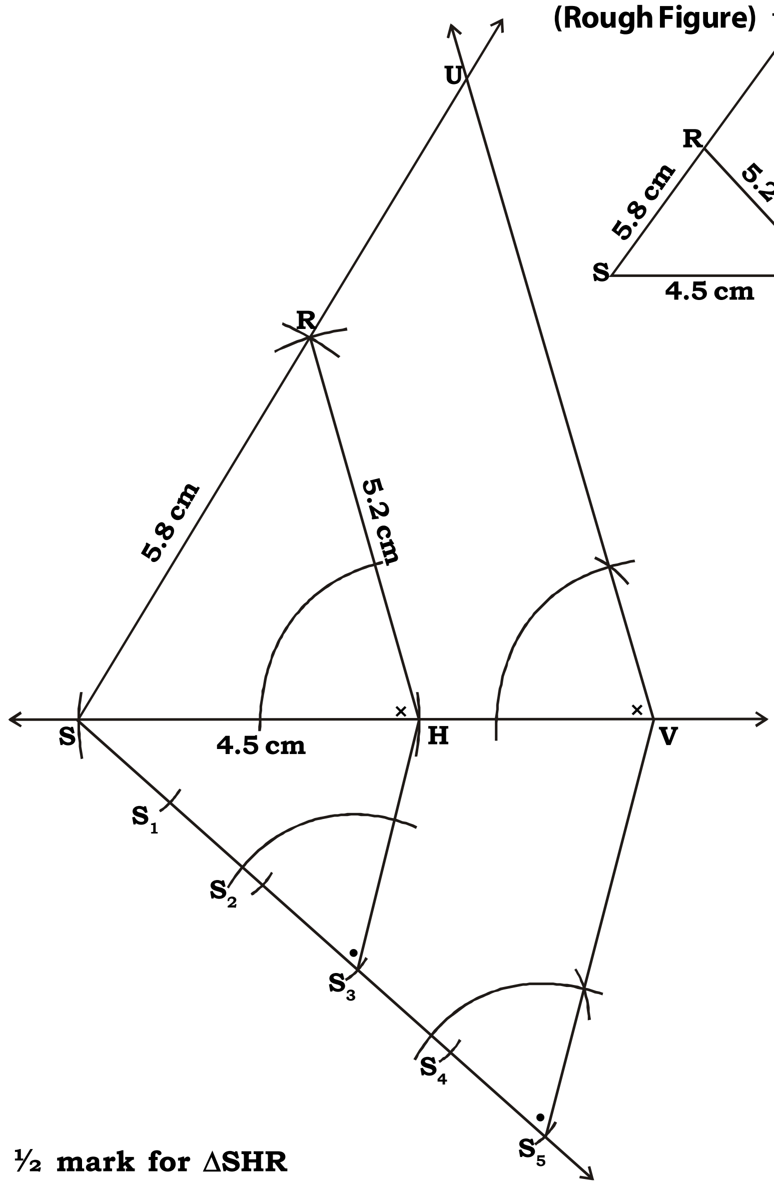
seg OA ≅ seg OB [Radii of same circle] ½

∴ ΔPAO ≅ ΔPBO [By hypotenuse - side theorem] ½

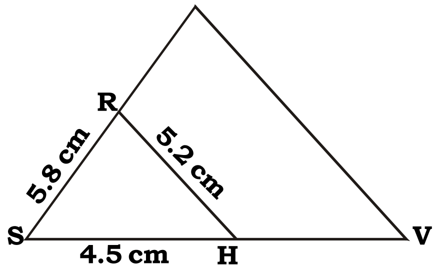
seg PA ≅ seg PB [c.s.c.t]

∴ PA = PB ½

(ii)



(Rough Figure)



½ mark for ΔSHR
1 mark for constructing 5 congruent parts
1 mark for constructing $\angle VS_5S \cong \angle HS_3S$
1 mark for constructing $\angle UVS \cong \angle RHS$
½ mark for required ΔSVU

(iii) For a sector,
Measure of arc (θ) = 60°
Radius (r) = 6 cm

(i) Curved surface area of the cheese = Length of arc × height ½

$$= \frac{\theta}{360} \times 2\pi r \times h$$ ½

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6 \times 7$ ½

$= 44 \text{ cm}^2$ ½

$\therefore$ The curved surface area of the cheese is 44 cm^2. ½

(ii) Volume of the cheese piece $= A \text{ (sector)} \times \text{height}$

$= \frac{\theta}{360} \times \pi r^2 \times h$ ½

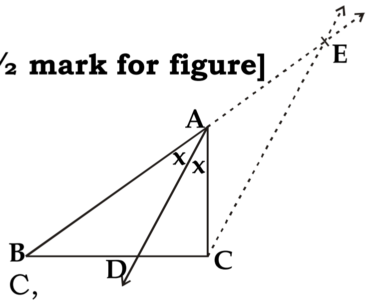
$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6 \times 7$ ½

$= 132 \text{ cm}^3$

$\therefore$ The volume of the cheese piece 132 cm^3. ½

Q.5. Solve the following : (Any 2)

[½ mark for figure]



(i) Given : In ΔABC,

ray AD is the bisector of $\angle BAC$ ½

such that B - D - C.

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$

Construction : Draw a line passing through C, ½

parallel to line AD and intersecting line BA at point E, B - A - E.

Proof : In ΔBEC,

line AD || side CE [Construction]

$\therefore \frac{BD}{DC} = \frac{AB}{AE}$ ....(i) [By B.P.T.] ½

line CE || line AD [Construction]

$\therefore$ On transversal BE,

$\angle BAD \cong \angle AEC$ ....(ii) [Converse of corresponding angles test] ½

Also, On transversal AC,

$\angle DAC \cong \angle ACE$ ....(iii) [Converse of alternate angles test] ½

But, $\angle BAD \cong \angle DAC$ ....(iv) [$\because$ ray AD bisects $\angle BAC$] ½

In ΔAEC,

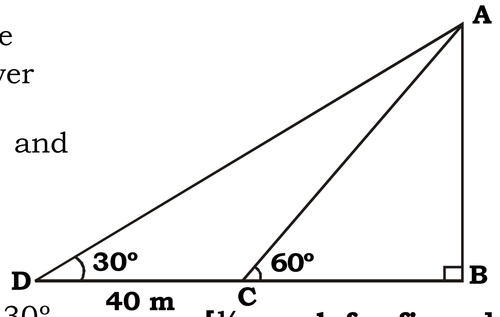
$\angle AEC \cong \angle ACE$ [From (ii), (iii) and (iv)]

$\therefore$ seg AC $\cong$ seg AE [Converse of Isosceles triangle theorem]

$\therefore$ AC = AE ....(v) 1

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$ [From (i) and (v)] ½

(ii) Let seg AB represents the tree
seg BC represents width of river
Let BC = x m
C and D represents the initial and final positions of the observer
DC = 40 m
∠ACB and ∠ADB are the angles of elevation
m ∠ACB = 60° and m ∠ADB = 30° — ½
In right angled ΔACB,



[½ mark for figure]

$\tan 60° = \frac{AB}{BC}$ [By definition] — ½

$\therefore \sqrt{3} = \frac{AB}{x}$

$\therefore AB = \sqrt{3}\,x$ m .....(i) — ½

In right angled ΔADB,

$\tan 30° = \frac{AB}{DB}$ [By definition] — ½

$\therefore \frac{1}{\sqrt{3}} = \frac{AB}{40 + x}$

$\therefore AB = \frac{40 + x}{\sqrt{3}}$ m .....(ii) — ½

From (i) and (ii) we get,

$\sqrt{3}\,x = \frac{40 + x}{\sqrt{3}}$ — ½

$\therefore 3x = 40 + x$

$\therefore 3x - x = 40$

$\therefore 2x = 40$

$\therefore x = 20$ — ½

$\therefore BC = 20$ m

$\therefore AB = 20\sqrt{3}$ m [From (i)] — ½

$\therefore AB = 20 \times 1.73$

$\therefore AB = 34.6$ m

∴ **Height of tree is 34.6 m and width of river is 20 m.** — ½

(iii) Diameter of well = 1.4 m

∴ Its radius (r) $= \frac{1.4}{2}$

$= 0.7$ m — ½

Its depth (h) = 10 m — ½

Volume of cylindrical well $= \pi r^2 h$ — ½

$= \frac{22}{7} \times 0.7 \times 0.7 \times 10$ ½

$= \frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} \times 10$

$= \frac{154}{10}$

$= 15.4 \text{ m}^3$

$\therefore$ Volume of earth dug is 15.4 m^3 ½

Now, Earth dug from the well is spread evenly on the adjoining cuboid field

Volume of cuboid = Volume of earth dug ½

= 15.4 m^3

Length of a cuboid (l) = 55 m ½

Its breadth (b) = 14 m

Volume of cuboid = $l \times b \times h$

$\therefore$ 15.4 = 55 × 14 × h ½

$\therefore \frac{154}{10 \times 55 \times 14} = h$ ½

$\therefore h = \frac{1}{50}$ m

$\therefore$ h = 0.02 m

$\therefore$ The thickness of the earth layer spread is 0.02 cm. ½

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