Set - B

Q.P. SET CODE
B

MT - x

Seat No. | | | | | | | |

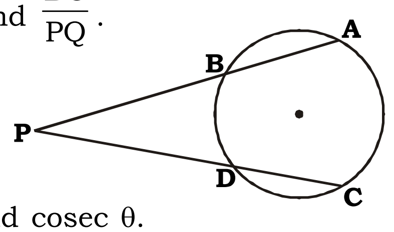
2013 ___ ___ 1100 - MT - x MATHEMATICS (71) GEOMETRY - PAPER B (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Q.1. Solve the following : (Any 5) **5**

(i) $\Delta ABC \sim \Delta APQ$, if $\dfrac{A(\Delta ABC)}{A(\Delta APQ)} = \dfrac{1}{4}$, find $\dfrac{BC}{PQ}$.

(ii) If PB = 3, PD = 4, PA = 6, find PC.



(iii) If the angle $\theta = -60°$, find $\cos\theta$ and $\operatorname{cosec}\theta$.

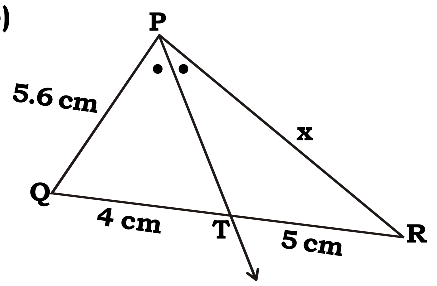
(iv) Find the slope of line with inclination 90°

(v) If $\Delta ABC \sim \Delta DEF$, $A(\Delta ABC) = 36\ cm^2$, $A(\Delta DEF) = 64\ cm^2$, what is the ratio of the length of sides BC and EF ?

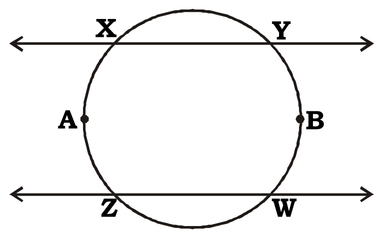
(vi) The radius of the base of a cone is 7 cm and its height is 24 cm. What is its slant height ?

Q.2. Solve the following : (Any 4) **8**

(i) Ray PT is the angle bisector of $\angle QPR$. Find the value of x



(ii) In the adjoining figure, m (arc XAZ) = m (arx YBW). Prove that XY || ZW



(iii) Find the trigonometric ratios in standard portion whose terminal arm passes through (5, – 12)

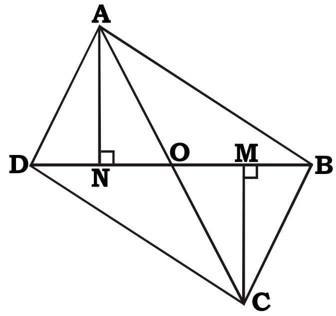
(iv) Write the equation of the line passing through A(-3,4) and B(4,5).

(v) If x = a sin θ, y = b tan θ, then prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

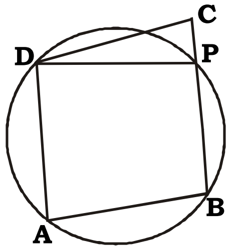
(vi) Write the equation of a line passing through point (3,4) and having slope 5.

Q.3. Solve the following : (Any 3) **9**

(i) In the adjoining figure, ΔADB and ΔCDB have the same base DB. If AC and BD intersect at O then prove that $\frac{A(\Delta ADB)}{A(\Delta CDB)} = \frac{AO}{CO}$.



(ii) □ABCD is a parallelogram. A circle passing through D, A, B cuts BC in P. Prove that DC = DP.



(iii) Construct the circumcircle of ΔKLM in which KM = 7 cm, ∠K = 60°, ∠M = 55°.

(iv) The points (k, 3), (2, – 4) and (– k + 1, – 2) are collinear, find k.

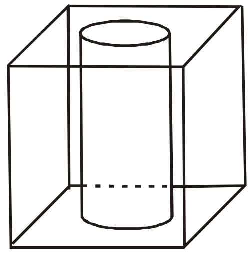
(v) Construct ΔDAT such that DA = 6.4 cm, ∠D = 120, ∠A = 25 and draw incircle of ΔDAT.

Q.4. Solve the following : (Any 2) **8**

(i) Prove : The opposite angles of a cyclic quadrilateral are supplementary.

(ii) ΔAMT ~ ΔAHE, In ΔAMT, MA = 6.3 cm, ∠MAT = 120°, AT = 4.9 cm and $\frac{MA}{HA} = \frac{7}{5}$, construct ΔAHE.

(iii) A cylindrical hole of diameter 30 cm is bored through a cubical wooden block with side 1 meter. Find the volume of the object so formed (π = 3.14)

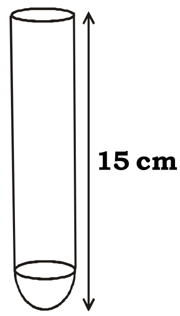


Q.5. Solve the following : (Any 2) **10**

(i) Prove : If a line parallel to a side of a triangle intersects other sides in two distinct points, then the line divides those sides in proportion.

(ii) A tree is broken by the wind. The top struck the ground at an angle of 30° and at a distance of 30 m from the root. Find the whole height of the tree. ($\sqrt{3}$ = 1.73)

(iii) A test tube has diameter 20 mm and height is 15 cm. The lower portion is a hemisphere in the adjoining figure. Find the capacity of the test tube. (π = 3.14)



Best Of Luck 👍

Set - B

A.P. SET CODE
B

MT - x

2013 ___ ___ 1100 - MT - x MATHEMATICS (71) GEOMETRY - PAPER B (E)

Time : 2 Hours **Prelim - I Model Answer Paper** **Max. Marks : 40**

Q.1. Solve the following : (Any 5)

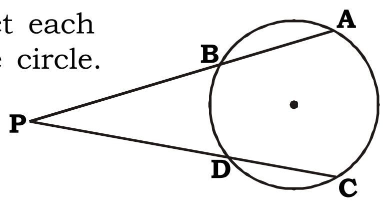
(i) $\Delta ABC \sim \Delta APQ$ [Given]

$\therefore \dfrac{A(\Delta ABC)}{A(\Delta APQ)} = \dfrac{BC^2}{PQ^2}$ [Areas of similar triangles] ½

$\therefore \dfrac{1}{4} = \dfrac{BC^2}{PQ^2}$ [Given]

$\therefore \boxed{\dfrac{BC}{PQ} = \dfrac{1}{2}}$ [Taking square roots] ½

(ii) Chords AB and CD intersect each other at point P outside the circle. ½



$\therefore$ PA × PB = PC × PD

$\therefore$ 6 × 3 = PC × 4

$\therefore$ PC = $\dfrac{6 \times 3}{4}$

$\therefore$ PC = $\dfrac{9}{2}$ ½

$\therefore \boxed{PC = 4.5 \text{ units}}$

(iii) $\theta = -60^\circ$

$\cos(-\theta) = \cos\theta$

$\therefore \cos(-60) = \cos 60$

$\therefore \boxed{\cos(-60) = \dfrac{1}{2}}$ ½

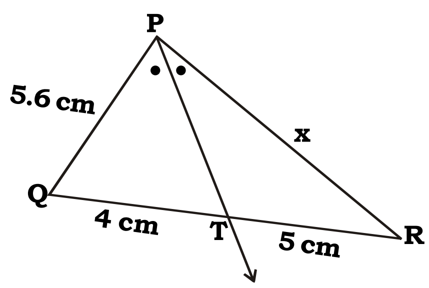
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

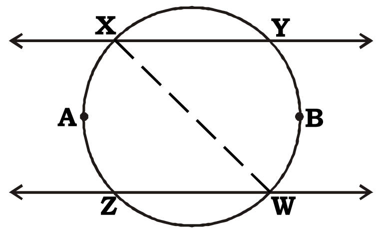
$\therefore \operatorname{cosec}(-60) = -\operatorname{cosec} 60$

$\therefore \boxed{\operatorname{cosec}(-60) = -\dfrac{2}{\sqrt{3}}}$ ½

(iv) Inclination of the line = 90°

$\therefore$ Slope of the line = $\tan\theta$ ½

	$= \tan 90^\circ$ $=$ Not defined $\therefore$ Slope of the line is not defined	½
(v)	$\Delta ABC \sim \Delta DEF$ [Given]	
	$\dfrac{A(\Delta ABC)}{A(\Delta DEF)} = \dfrac{BC^2}{EF^2}$ [Areas of similar triangles]	½
	$\therefore \dfrac{36}{64} = \dfrac{BC^2}{EF^2}$	
	$\therefore \dfrac{BC}{EF} = \dfrac{6}{8}$ [Taking square roots]	
	$\therefore \boxed{\dfrac{BC}{EF} = \dfrac{3}{4}}$	½
(vi)	Radius of base of cone (r) = 7 cm Its height (h) = 24 cm	½
	$l^2 = r^2 + h^2$ $\therefore l^2 = 7^2 + 24^2$ $\therefore l^2 = 49 + 576$ $\therefore l^2 = 625$ $\therefore l = 25$ [Taking square roots]	
	$\therefore$ Slant height of cone is 25 cm.	½
Q.2.	**Solve the following : (Any 4)**	
(i)	In ΔPQR, ray PT bisects $\angle QPR$ [Given]	
	$\therefore \dfrac{PQ}{PR} = \dfrac{QT}{TR}$ [Property of angle bisector of a triangle]	**1**
	$\therefore \dfrac{5.6}{x} = \dfrac{4}{5}$ 	
	$\therefore x = \dfrac{5 \times 5.6}{4}$	
	$\therefore x = 7$ $\therefore PR = 7$ cm	½
	$QR = QT + TR$ [$\because$ Q - T - R] $\therefore QR = 4 + 5$	
	$\therefore \boxed{QR = 9 \text{ cm}}$	½

(ii)	Construction : Draw seg XW 	½
	Proof : m $\angle XWZ = \frac{1}{2}$ m (arc XAZ) ....(i) m $\angle WXY = \frac{1}{2}$ m (arc YBW) ....(ii) [Inscribed angle theorem]	½
	But, m (arc XAZ) = m (arc YBW) ....(iii) [Given]	
	$\therefore$ m $\angle XWZ$ = m $\angle WXY$ [From (i), (ii) and (iii)]	½
	$\therefore$ line XY ‖ line ZW [Alternate angles test]	½
(iii)	The terminal arm passes through P (5, – 12) $\therefore$ x = 5 and y = – 12 $r = \sqrt{x^2 + y^2}$ $= \sqrt{(5)^2 + (-12)^2}$ $= \sqrt{25 + 144}$ $= \sqrt{169}$	
	$\therefore$ r = 13 units	½
	Let the angle be θ $\therefore \sin\theta = \frac{y}{r} = \frac{-12}{13}$ $\text{cosec}\,\theta = \frac{r}{y} = \frac{-13}{12}$ $\cos\theta = \frac{x}{r} = \frac{5}{13}$ $\sec\theta = \frac{r}{x} = \frac{13}{5}$ $\tan\theta = \frac{y}{x} = \frac{-12}{5}$ $\cot\theta = \frac{x}{y} = \frac{-5}{12}$	1½
(iv)	A ≡ (– 3, 4) ≡ (x_1, y_1) B ≡ (4, 5) ≡ (x_2, y_2)	½
	The equation of line AB by two point form is, $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$	½
	$\therefore \frac{x - (-3)}{(-3) - 4} = \frac{y - 4}{4 - 5}$ $\therefore \frac{x + 3}{-7} = \frac{y - 4}{-1}$ $\therefore$ x + 3 = 7 (y – 4)	½
	$\therefore$ x + 3 = 7y – 28	

$\therefore$ $x - 7y + 3 + 28 = 0$

$\therefore$ $x - 7y + 31 = 0$

$\therefore$ **The equation of line AB is $x - 7y + 31 = 0$** ½

(v)

$$x = a \sin \theta$$

$\therefore$ $\dfrac{1}{\sin \theta} = \dfrac{a}{x}$

$\therefore$ $\text{cosec } \theta = \dfrac{a}{x}$ ......(i) $\left[\because \text{cosec } \theta = \dfrac{1}{\sin \theta}\right]$ ½

$$y = b \tan \theta$$

$\therefore$ $\dfrac{1}{\tan \theta} = \dfrac{b}{y}$

$\therefore$ $\cot \theta = \dfrac{b}{y}$ .....(ii) $\left[\because \cot \theta = \dfrac{1}{\tan \theta}\right]$ ½

We know,

$$1 + \cot^2 \theta = \text{cosec}^2 \theta$$ ½

$\therefore$ $\text{cosec}^2 \theta - \cot^2 \theta = 1$

$\therefore$ $\left(\dfrac{a}{x}\right)^2 - \left(\dfrac{b}{y}\right)^2 = 1$ [From (i) and (ii)]

$\therefore$ $\dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = 1$ ½

(vi) Let $A \equiv (3, 4) \equiv (x_1, y_1)$ and $m = 5$ ½

The equation of the line passing through A and having slope 5 by slope point form is, ½

$$y - y_1 = m(x - x_1)$$ ½

$\therefore$ $y - 4 = 5(x - 3)$

$\therefore$ $y - 4 = 5x - 15$

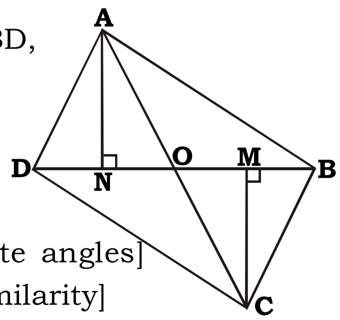
$\therefore$ $5x - y - 15 + 4 = 0$

$\therefore$ $5x - y - 11 = 0$

$\therefore$ **The equation of the line passing through the points (3, 4) and having slope 5 is $5x - y - 11 = 0$.** ½

Q.3. Solve the following : (Any 3)

(i) ΔADB and ΔCDB have a common base BD,



$\therefore$ $\dfrac{A(\Delta ADB)}{A(\Delta CDB)} = \dfrac{AN}{CM}$ ....(i) [Triangles with common base] 1

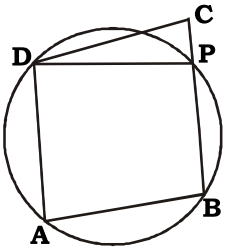
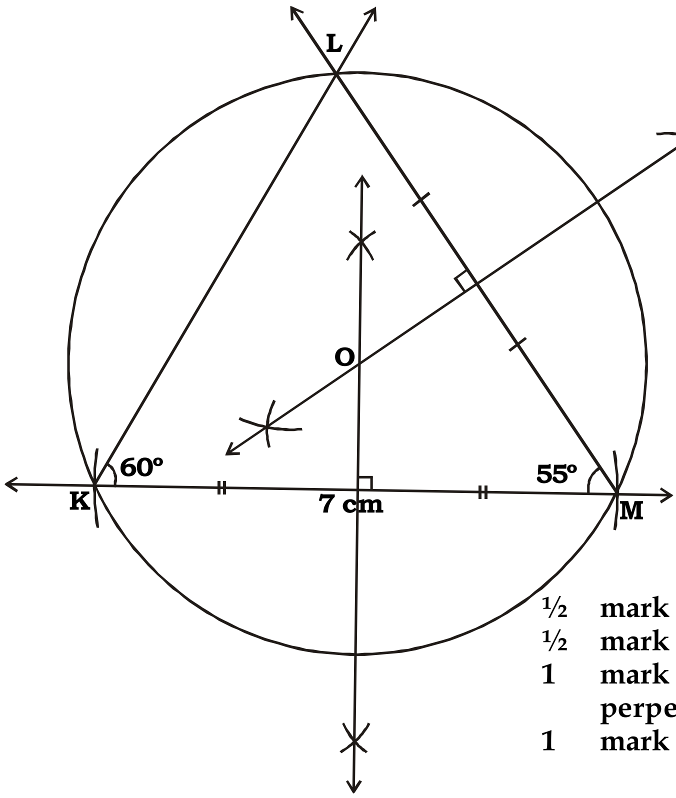
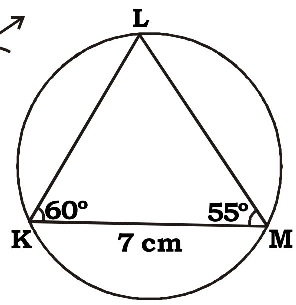
In ΔANO and ΔCMO,

$\angle ANO \cong \angle CMO$ [$\because$ Each is 90°]

$\angle AON \cong \angle COM$ [Vertically opposite angles]

$\therefore$ $\Delta ANO \sim \Delta CMO$ [By AA test of similarity] 1

$\therefore$ $\dfrac{AN}{CM} = \dfrac{AO}{CO}$ ......(ii) [c.s.s.t.] ½

	$\therefore \dfrac{A(\Delta ADB)}{A(\Delta CDB)} = \dfrac{AO}{CO}$ [From (i) and (ii)]	½
(ii)	Proof : □ABPD is cyclic [By definition]	½
	$\therefore$ ∠DPC ≅ ∠DAB .....(i) [An exterior angle of cyclic quadrilateral is congruent to the angle opposite to adjacent interior angle]	1
		
	□ABCD is parallelogram $\therefore$ ∠DCB ≅ ∠DAB ....(ii) [Opposite angles of a parallelogram are congruent]	½
	$\therefore$ ∠DPC ≅ ∠DCB ......(iii) [From (i) and (ii)] In ΔDPC, ∠DPC ≅ ∠DCP [From (ii) and C - P - B] $\therefore$ seg DP ≅ seg DC [Converse of isosceles triangle theorem] $\therefore$ DP = DC	1
(iii)	 **(Rough Figure)**  ½ mark for rough figure ½ mark for drawing ΔLKM 1 mark for drawing perpendicular bisectors 1 mark for drawing circumcircle	

(iv) Let, A ≡ (k, 3), B ≡ (2, – 4), C ≡ (– k + 1, – 2) ½

∵ Points A, B and C are collinear

Slope of line AB = Slope of line BC ½

$\therefore \quad \frac{-4-3}{2-k} = \frac{-2-(-4)}{(-k+1)-2}$ ½

$\therefore \quad \frac{-7}{2-k} = \frac{-2+4}{-k+1-2}$

$\therefore \quad \frac{-7}{2-k} = \frac{2}{-k-1}$ ½

$\therefore \quad -7(-k-1) = 2(2-k)$

$\therefore \quad 7k + 7 = 4 - 2k$ ½

$\therefore \quad 7k + 2k = 4 - 7$

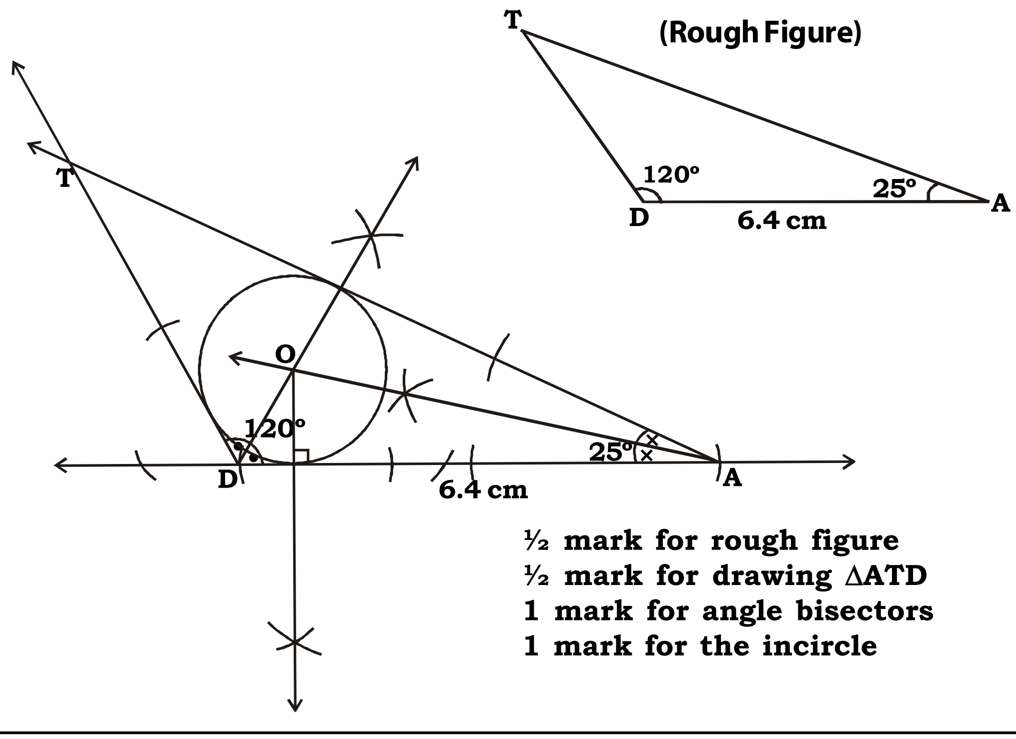
$\therefore \quad 9k = -3$

$\therefore \quad k = \frac{-3}{9}$

$\therefore \quad k = \frac{-1}{3}$

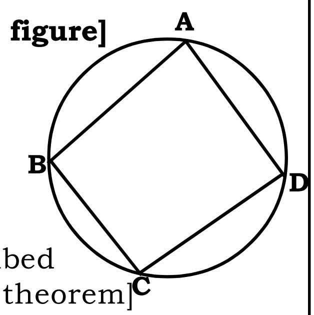
∴ **The value of k is $\frac{-1}{3}$.** ½

(v)



½ mark for rough figure
½ mark for drawing ΔATD
1 mark for angle bisectors
1 mark for the incircle

Q.4. Solve the following : (Any 2) **[½ mark for figure]** ½



(i) Given : □ABCD is a cyclic

To Prove : m ∠ABC + m ∠ADC = 180°

m ∠BAD + m ∠BCD = 180°

Proof: m ∠ABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ∠ADC = $\frac{1}{2}$ m (arc ABC) .....(ii) [Inscribed angle theorem] 1

Adding (i) and (ii), we get

m ∠ABC + m ∠ADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC) ½

∴ m ∠ABC + m ∠ADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)]

∴ m ∠ABC + m ∠ADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

∴ m ∠ABC + m ∠ADC = 180° ....(iii)

In □ABCD,

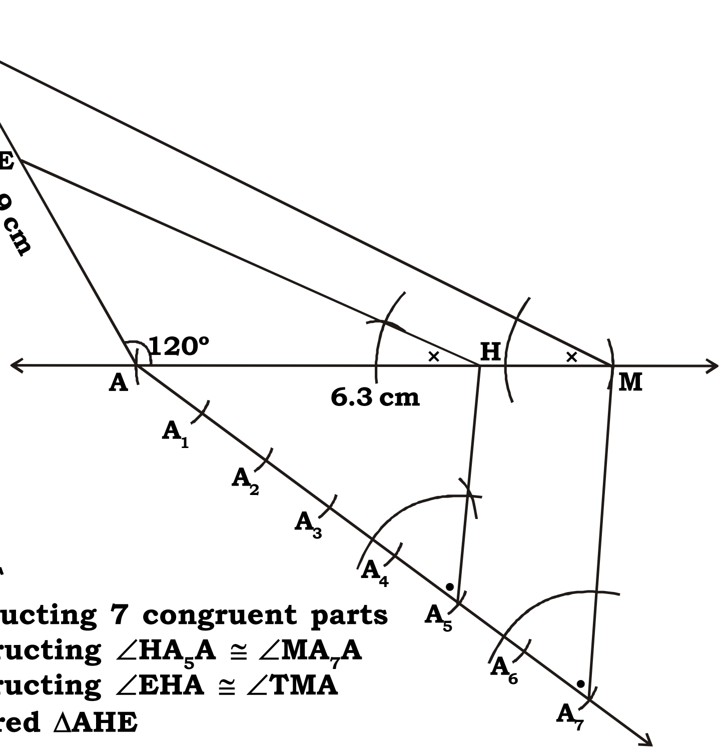
m ∠BAD + m ∠BCD + m ∠ABC + m ∠ADC = 360° ½

[∵ Sum of measure of angles of a quadrilateral is 360°]

∴ m ∠BAD + m ∠BCD + 180° = 360° [From (iii)]

∴ m ∠BAD + m ∠BCD = 180° ½

(ii)



½ mark for ΔAMT
1 mark for constructing 7 congruent parts
1 mark for constructing ∠HA_5A ≅ ∠MA_7A
1 mark for constructing ∠EHA ≅ ∠TMA
½ mark for required ΔAHE

(iii)

side of cubical wooden block $= 1$ m ½

$= 100$ cm

Volume of cubical wooden block $= l^3$

$= (100)^3$

$= 1000000 \text{ cm}^3$ ½

A cylindrical hole is bored through the cubical wooden block

$\therefore$ Height of cylindrical hole (h) $= 1$m

$= 100$ cm ½

Diameter of cylindrical hole $= 30$ cm

$\therefore$ Its radius (r) $= \frac{30}{2}$

$= 15$ cm ½

Volume of cylindrical hole $= \pi r^2 h$ ½

$= 3.14 \times 15 \times 15 \times 100$

$= 70650 \text{ cm}^3$ ½

Volume of the object so formed = Volume of cubical wooden block – Volume of cylindrical hole ½

$= 1000000 - 70650$

$= 929350 \text{ cm}^3$

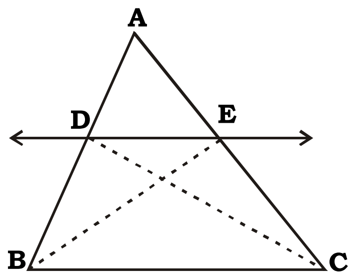
$\therefore$ **Volume of the object so formed is 929350 cm³.** ½

Q.5. Solve the following : (Any 2)

(i) Given : In ΔABC,

(i) line DE || side BC

(ii) Line DE intersects sides AB and AC at points D and E respectively. ½



[½ mark for figure]

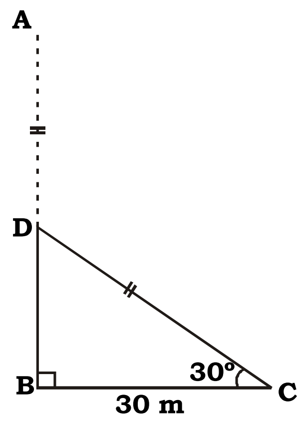
To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

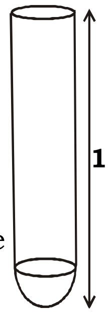
Construction : Draw seg BE and seg CD. ½

Proof : ΔADE and ΔBDE have a common vertex E and their bases AD and BD lie on the same line AB.

$\therefore$ Their heights are equal

$\therefore \frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{AD}{DB}$ ....(i) [Triangles having equal heights] ½

	ΔADE and ΔCDE have a common vertex D and their bases AE and EC lie on the same line AC. $\therefore$ Their heights are equal.	
	$\therefore \frac{A(\Delta ADE)}{A(\Delta CDE)} = \frac{AE}{CE}$ ....(ii) [Triangles having equal heights]	½
	line DE || side BC [Given] ΔBDE and ΔCDE are between the same two parallel lines DE and BC. $\therefore$ Their heights are equal. Also, they have same base DE.	1
	$\therefore$ $A(\Delta BDE) = A(\Delta CDE)$ ......(iii) [Areas of two triangles having equal bases and equal heights are equal]	½
	$\therefore \frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{A(\Delta ADE)}{A(\Delta CDE)}$ ......(iv) [From (i), (ii) and (iii)]	½
	$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [From (i), (ii) and (iv)]	½
(ii)	seg AB represents the height of the tree The tree breaks at point D seg AD is the broken part of tree which then takes the position of DC $\therefore$ AD = DC $m\angle DCB = 30°$ BC = 30 m In right angled ΔDBC,  **[½ mark for figure]**	½
	$\tan 30° = \frac{DB}{BC}$ [By definition] $\therefore \frac{1}{\sqrt{3}} = \frac{DB}{30}$ $\therefore DB = \frac{30}{\sqrt{3}}$ $\therefore DB = \frac{30\sqrt{3}}{3}$	½
	$\therefore DB = 10\sqrt{3}$ m	½
	$\cos 30° = \frac{BC}{DC}$ [By definition] $\therefore \frac{\sqrt{3}}{2} = \frac{30}{DC}$	½

	$\therefore$ DC = $\frac{30 \times 2}{\sqrt{3}}$	½
	$\therefore$ DC = $\frac{30\sqrt{3} \times 2}{3}$	
	$\therefore$ DC = $20\sqrt{3}$ m	½
	$\therefore$ AD = DC = $20\sqrt{3}$ m	
	AB = AD + DB [$\because$ A - D - B]	½
	$\therefore$ AB = $20\sqrt{3} + 10\sqrt{3}$	
	$\therefore$ AB = $30\sqrt{3}$ m	
	$\therefore$ AB = 30 × 1.73	
	$\therefore$ AB = 51.9 m	½
	$\therefore$ The height of tree is 51.9 m.	½
(iii)	Diameter of a test tube = 20 mm 	
	$\therefore$ its radius (r) = $\frac{20}{2}$	
	= 10 mm	
	= 1 cm	½
	Its height (h) = 15 cm	
	Height of hemispherical part (h_1) = radius of hemisphere	½
	= 1 cm	
	$\therefore$ Height of cylindrical part (h_2) = $h - h_1$	½
	= 15 – 1	
	= 14 cm	½
	Volume of test tube = Volume of cylindrical part + Volume of hemispherical part	½
	$= \pi r^2 h_2 + \frac{2}{3}\pi r^3$	½
	$= \pi r^2 \left[h_2 + \frac{2}{3} r\right]$	½
	$= 3.14\ (1) \left[14 + \frac{2}{3}\right]$	½
	$= 3.14 \times \frac{44}{3}$	½
	$= \frac{138.16}{3}$	
	Volume of test tube = 46.05 cm³	
	$\therefore$ Capacity of a test tube is 46.05 cm³	½
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