Set - C

Q.P. SET CODE
C

MT - y

Seat No. ☐☐☐☐☐☐☐

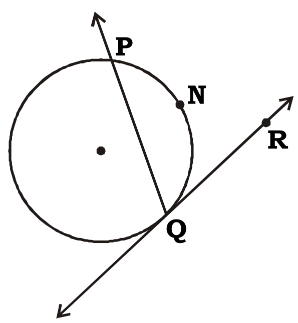
2013 ___ ___ 1100 - **MT - y** MATHEMATICS (71) GEOMETRY - PAPER C (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Q.1. Solve the following : (Any 5) **5**

(i) In ΔDEF, m $\angle D = 90°$, m $\angle E = 45°$, m $\angle F = 45°$. If EF = $8\sqrt{2}$ cm, find DE.

(ii) If m (arc PNQ) = 140°, find m $\angle PQR$.



(iii) The terminal arm is on negative Y-axis, what are the possible angles ? What can you say about this angle ?

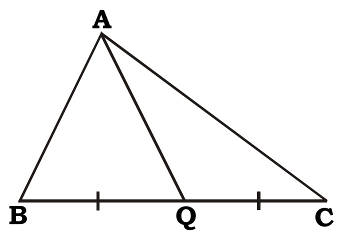
(iv) Write the equation of X-axis.

(v) $\Delta ABC \sim \Delta APQ$, if $\dfrac{A(\Delta ABC)}{A(\Delta APQ)} = \dfrac{1}{4}$, find $\dfrac{BC}{PQ}$.

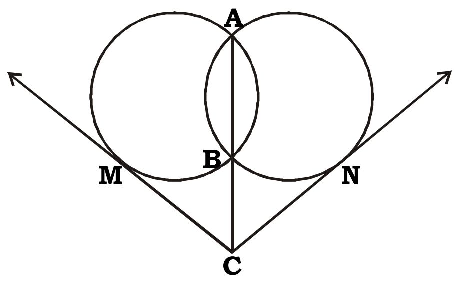
(vi) Perimeter of one face of a cube is 24 cm. Find the length of its side.

Q.2. Solve the following : (Any 4) **8**

(i) In the adjoining figure, $AB^2 + AC^2 = 122$, BC = 10. Find the length of the median on side BC.



(ii) In the adjoining figure, two circles intersect each other in two points A and B. Seg AB is the chord of both circles. Point C is the exterior point of both the circles on the line AB. From the point C tangents are drawn to the circles touches at M and N. Prove that CM = CN.



(iii) If $\tan A + \frac{1}{\tan A} = 2$, show that $\tan^2 A + \frac{1}{\tan^2 A} = 2$.

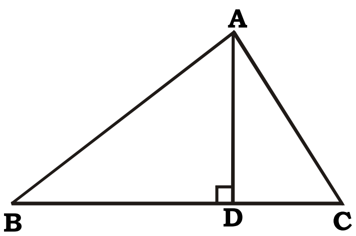
(iv) Write the equation of the line passing through P(-3,7) and slope $\frac{1}{2}$.

(v) If the angle $\theta = -60^\circ$, find the value of $\sin\theta$, $\cos\theta$, $\sec\theta$ and $\tan\theta$.

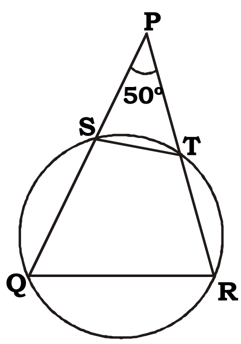
(vi) Write the equation 2x - 9 = 3 in double intercept form. Write x-intercept and y-intercept.

Q.3. Solve the following : (Any 3) **9**

(i) The perpendicular AD on the base BC of ΔABC intersects BC at D so that BD = 3 CD. Prove that $2AB^2 = 2AC^2 + BC^2$.



(ii) In the adjoining figure, in the isosceles triangle PQR, the vertical $\angle P = 50^\circ$. The circle passing through Q and R cuts PQ . in S and PR in T. ST is joined. Find $\angle PST$.



(iii) Construct tangents to the circle from point B with radius 3.5 cm and centre A. Point B is at a distance 7.3 cm from the centre.

(iv) The vertices of a triangle are A (3, – 4), B (5, 7) and C (– 4, 5). Find the slope of each side of the triangle ABC.

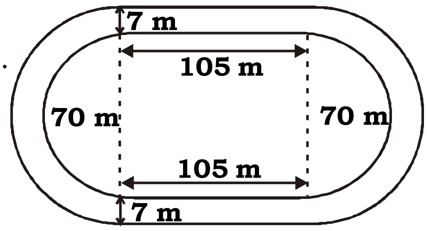
(v) Construct a right angled triangle ΔPQR where PQ = 6 cm, $\angle QPR = 40^\circ$, $\angle PRQ = 90^\circ$. Draw circumcircle of ΔPQR.

Q.4. Solve the following : (Any 2) **8**

(i) Prove : The opposite angles of a cyclic quadrilateral are supplementary.

(ii) $\Delta SHR \sim \Delta SVU$, In ΔSHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and $\frac{SH}{SV} = \frac{3}{5}$; construct ΔSVU.

(iii) Adjoining figure depicts a racing track whose left and right ends are semicircular. The distance between two inner parallel line segments is 70 m and they are each 105 m long. If the track is 7 m wide, find the difference in the lengths of the inner edge and outer edge of the track.

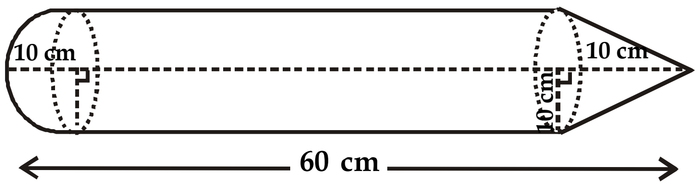


Q.5. Solve the following : (Any 2) **10**

(i) Prove : The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

(ii) A tree 12m high, is broken by the wind in such a way that its top touches the ground and makes an angle 60° with the ground. At what height from the bottom, the tree is broken by the wind ? ($\sqrt{3}$ = 1.73)

(iii) A toy is a combination of a cylinder, hemisphere and a cone, each with radius 10cm. Height of the conical part is 10 cm and total height is 60cm. Find the total surface area of the toy. (π = 3.14, $\sqrt{2}$ = 1.41)



Best Of Luck

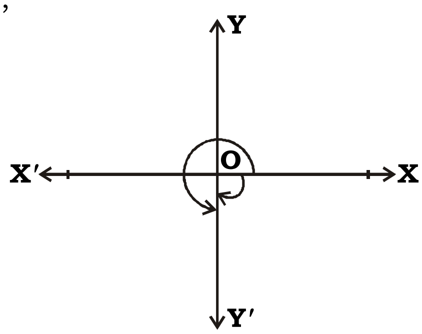
Set - C

A.P. SET CODE
C

MT - y

2013 ___ ___ 1100 - **MT - y** MATHEMATICS (71) GEOMETRY - PAPER C (E)

Time : 2 Hours **Prelim - I Model Answer Paper** **Max. Marks : 40**

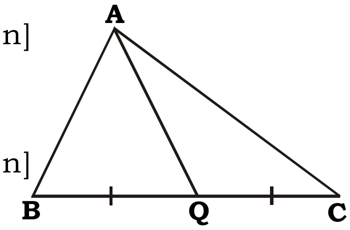
Q.1.	**Solve the following : (Any 5)**	
(i)	In ΔDEF, $\angle D = 90°, \angle E = 45°, \angle F = 45°$ [Given] $\therefore$ ΔDEF is 45° - 45° - 90° triangle	½
	$\therefore$ $DE = \frac{1}{\sqrt{2}} \times EF$ $\therefore$ $DE = \frac{1}{\sqrt{2}} \times 8\sqrt{2}$ $\therefore$ $\boxed{DE = 8 \text{ cm}}$	½
(ii)	$m\angle PQR = \frac{1}{2} m(\text{arc } PNQ)$ [Tangent-secant theorem]	½
	$\therefore$ $m\angle PQR = \frac{1}{2} \times 140°$ $\therefore$ $\boxed{m\angle PQR = 70°}$	½
(iii)	The terminal arm is on negative y-axis, the possible angles are 270° and – 90°. These angles are called quadrantal angles.  	1
(iv)	Equation of X-axis is y = 0	1
(v)	$\Delta ABC \sim \Delta APQ$ [Given] $\therefore$ $\frac{A(\Delta ABC)}{A(\Delta APQ)} = \frac{BC^2}{PQ^2}$ [Areas of similar triangles]	½
	$\therefore$ $\frac{1}{4} = \frac{BC^2}{PQ^2}$ [Given]	

$\therefore \boxed{\dfrac{BC}{PQ} = \dfrac{1}{2}}$ [Taking square roots] ½

(vi) Perimeter of one face of a cube = 24 cm
Perimeter of one face of a cube = $4l$ ½
$\therefore$ $24 = 4l$
$\therefore$ $l = \dfrac{24}{4}$
$\therefore$ $l = 6$
$\therefore$ The length of the side of a cube is 6 cm. ½

Q.2. Solve the following : (Any 4)

(i) In ΔABC,
seg AQ is the median [Given]



$\therefore$ $BQ = QC = \dfrac{1}{2} \times BC$
$\therefore$ $BQ = QC = \dfrac{1}{2} \times 10$ [Given]
$\therefore$ $BQ = QC = 5$ units ......(i) ½
$AB^2 + AC^2 = 2AQ^2 + 2BQ^2$ [By Appollonius theorem] ½
$\therefore$ $122 = 2AQ^2 + 2(5)^2$ [From (i) and given]
$\therefore$ $122 = 2AQ^2 + 2(25)$
$\therefore$ $122 = 2AQ^2 + 50$ ½
$\therefore$ $2AQ^2 = 122 - 50$
$\therefore$ $2AQ^2 = 72$
$\therefore$ $AQ^2 = 36$
$\therefore$ $\boxed{AQ = 6 \text{ units}}$ [Taking square roots] ½

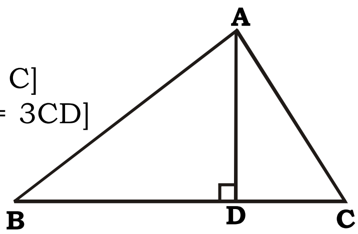
(ii) Line CBA is a secant intersecting the circle at points B and A and line CM is a tangent to the circle at point M.
$\therefore$ $CM^2 = CB \times CA$ ....(i) [Tangent secant property] ½
Line CBA is a secant intersecting the circle at points B and A and line CN is a tangent to the circle at point N.
$\therefore$ $CN^2 = CB \times CA$ ......(ii) [Tangent secant property] ½
$\therefore$ $CM^2 = CN^2$ [From (i) and (ii)] ½
$\therefore$ $\boxed{CM = CN}$ [Taking square roots] ½

(iii) $\tan A + \dfrac{1}{\tan A} = 2$

$\therefore$ $\left(\tan A + \dfrac{1}{\tan A}\right)^2 = 4$ [Squaring both sides] ½

	$\therefore \quad \tan^2 A + 2 \tan A \cdot \frac{1}{\tan A} + \frac{1}{\tan^2 A} = 4$	½
	$\therefore \quad \tan^2 A + 2 + \frac{1}{\tan^2 A} = 4$	
	$\therefore \quad \tan^2 A + \frac{1}{\tan^2 A} = 4 - 2$	½
	$\therefore \quad \tan^2 A + \frac{1}{\tan^2 A} = 2$	½
(iv)	$P \equiv (-3, 7) \equiv (x_1, y_1)$	
	$m = \frac{1}{2}$	
	The equation of the line passing through P (– 3, 7) and having slope $m = \frac{1}{2}$ is given by slope point form	½
	$(y - y_1) = m (x - x_1)$	½
	$\therefore \quad (y - 7) = \frac{1}{2} [x - (-3)]$	
	$\therefore \quad y - 7 = \frac{1}{2} (x + 3)$	½
	$\therefore \quad 2 (y - 7) = x + 3$	
	$\therefore \quad 2y - 14 = x + 3$	
	$\therefore \quad x - 2y + 3 + 14 = 0$	
	$\therefore \quad x - 2y + 17 = 0$	
	$\therefore$ The required equation of the line is $x - 2y + 17 = 0$	½
(v)	$\theta = -60°$	
	$\sin(-\theta) = -\sin\theta$	
	$\therefore \quad \sin(-60) = -\sin 60$	
	$\therefore \quad \sin(-60) = -\frac{\sqrt{3}}{2}$	½
	$\sec(-\theta) = \sec\theta$	
	$\therefore \quad \sec(-60) = \sec 60$	
	$\therefore \quad \sec(-60) = 2$	½
	$\cos(-\theta) = \cos\theta$	
	$\therefore \quad \cos(-60) = \cos 60$	
	$\therefore \quad \cos(-60) = \frac{1}{2}$	½
	$\tan(-\theta) = -\tan\theta$	

	$\therefore$ tan (– 60) = – tan 60	
	$\therefore$ $\boxed{\tan(-60) = -\sqrt{3}}$	½
(vi)	2x – y = 3	
	Dividing throughout by 3,	½
	$\therefore \frac{2x}{3} - \frac{y}{3} = 1$	
	$\therefore \frac{x}{\frac{3}{2}} + \frac{y}{-3} = 1$	½
	$\therefore$ x intercept of line 2x – y = 3 is $\frac{3}{2}$ y intercept of line 2x – y = 3 is – 3	1
Q.3.	**Solve the following : (Any 3)**	
(i)	BC = BD + CD [B - D - C]	½
	$\therefore$ BC = 3CD + CD [$\because$ BD = 3CD]	
	$\therefore$ BC = 4CD .....(i)	
	In ΔADB,	
	m ∠ADB = 90° [Given]	
	$\therefore$ $AB^2 = AD^2 + BD^2$ [By Pythagoras theorem]	½
	$\therefore$ $AB^2 = AD^2 + (3CD)^2$ [$\because$ BD = 3CD]	
	$\therefore$ $AB^2 = AD^2 + 9CD^2$	
	$\therefore$ $AB^2 = AD^2 + CD^2 + 8CD^2$ .....(ii)	½
	In ΔADC,	
	m ∠ADC = 90° [Given]	
	$\therefore$ $AC^2 = AD^2 + CD^2$ .....(iii) [By Pythagoras theorem]	½
	$AB^2 = AC^2 + 8CD^2$ [From (ii) and (iii)]	
	$\therefore$ $AB^2 = AC^2 + 8\left(\frac{BC}{4}\right)^2$ [From (i)]	
	$\therefore$ $AB^2 = AC^2 + 8 \times \frac{BC^2}{16}$	½
	$\therefore$ $AB^2 = AC^2 + \frac{BC^2}{2}$	
	$\therefore$ $2AB^2 = 2AC^2 + BC^2$ [Multiplying throughout by 2]	½
(ii)	In ΔPQR	
	seg PQ ≅ seg PR [Given]	
	$\therefore$ ∠PQR ≅ ∠PRQ ......(i) [Isosceles triangle theorem]	½
	m ∠PQR + m ∠PRQ + m ∠QPR = 180° [Sum of the measures of angles of a triangle is 180°]	½



∴ m ∠PRQ + m ∠PRQ + 50 = 180° [From (i) and Given]

2m ∠PRQ = 180° – 50°

∴ 2m ∠PRQ = 130°

∴ m ∠PRQ = 65° ……(ii) ½

□SQRT is cyclic

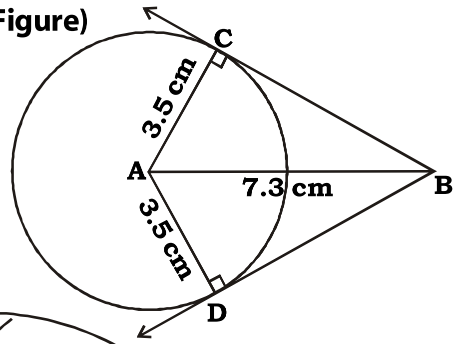
∴ ∠PST ≅ ∠TRQ [An exterior angle of cyclic quadrilateral is congruent to the angle opposite to adjacent interior angle] 1

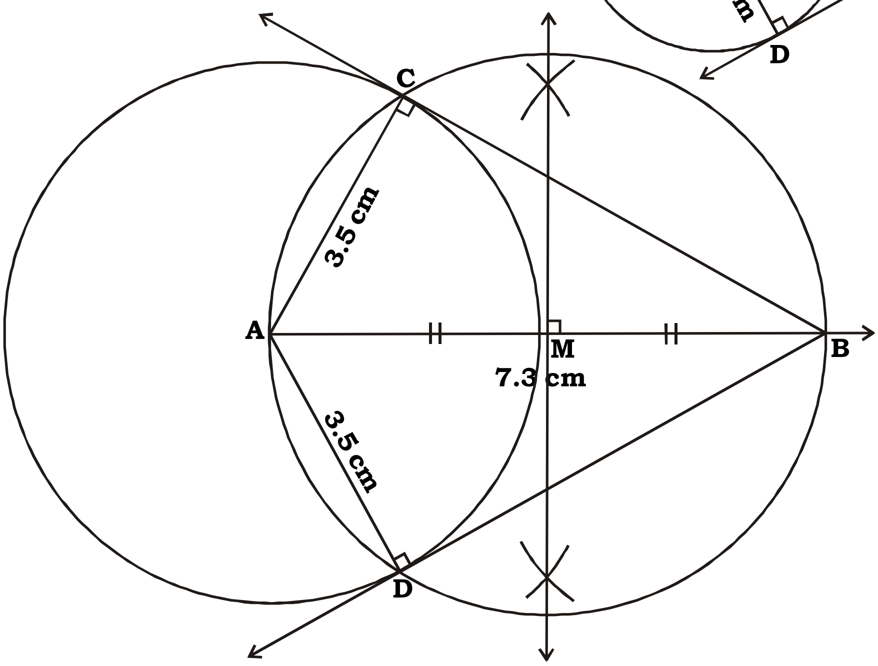
∴ ∠PST ≅ ∠PRQ [P - T - R]

∴ **∠PST = 65°** [From (ii)] ½

(iii)

(Rough Figure)





½ mark for rough figure
½ mark for drawing the circle of radius 3.5 cm
½ mark for drawing the perpendicular bisector of seg AB
½ mark for drawing the circle with centre M
1 mark for drawing both the tangents from point B

(iv) For ΔABC,

$A \equiv (3, -4)$, $B \equiv (5, 7)$, $C \equiv (-4, 5)$ ½

Slope of a line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope of side AB $= \dfrac{7-(-4)}{5-3}$

$= \dfrac{7+4}{2}$

$= \dfrac{11}{2}$

$\therefore$ **Slope of side AB $= \dfrac{11}{2}$** ½

Slope of side BC $= \dfrac{5-7}{-4-5}$ ½

$= \dfrac{-2}{-9}$

$= \dfrac{2}{9}$

$\therefore$ **Slope of side BC is $\dfrac{2}{9}$** ½

Slope of side AC $= \dfrac{5-(-4)}{-4-3}$ ½

$= \dfrac{5+4}{-7}$

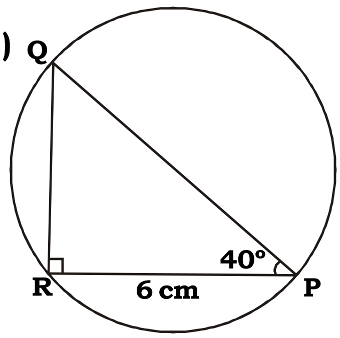
$= \dfrac{9}{-7}$

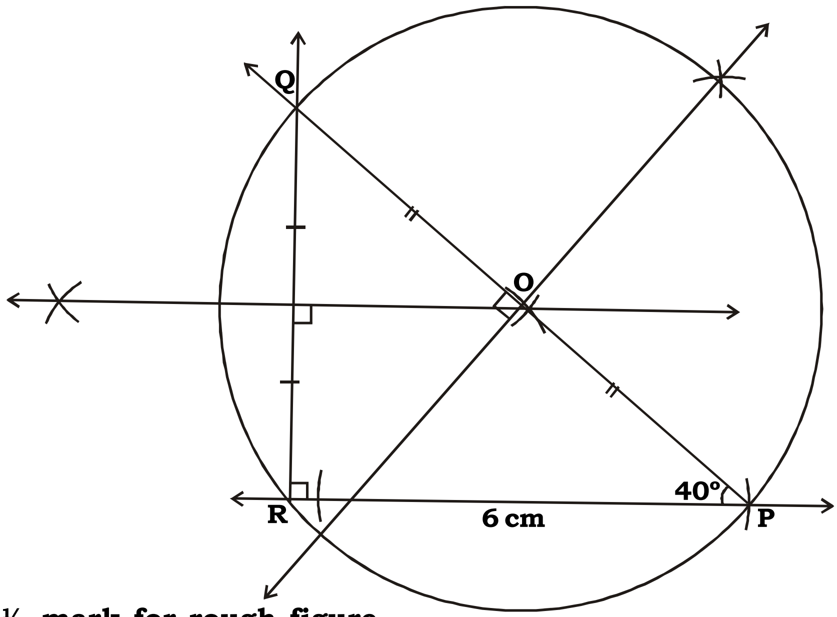
$= \dfrac{-9}{7}$

$\therefore$ **Slope of side AC is $\dfrac{-9}{7}$** ½

(v)

(Rough Figure)

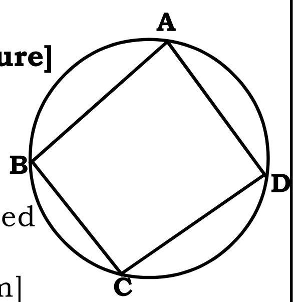




½ mark for rough figure
1 mark for drawing triangle
1 mark for drawing perpendicular bisectors
½ mark for drawing circumcircle

Q.4. Solve the following : (Any 2)

(i) Given : □ABCD is a cyclic **[½ mark for figure]** ½



To Prove : m ∠ABC + m ∠ADC = 180°
m ∠BAD + m ∠BCD = 180°

Proof: m ∠ABC = $\frac{1}{2}$ m (arc ADC) .....(i)
m ∠ADC = $\frac{1}{2}$ m (arc ABC) .....(ii) [Inscribed angle theorem] 1

Adding (i) and (ii), we get

m ∠ABC + m ∠ADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC) ½

∴ m ∠ABC + m ∠ADC = $\frac{1}{2}$ [m (arc ADC) + m (arc ABC)]

∴ m ∠ABC + m ∠ADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

∴ m ∠ABC + m ∠ADC = 180° ....(iii)

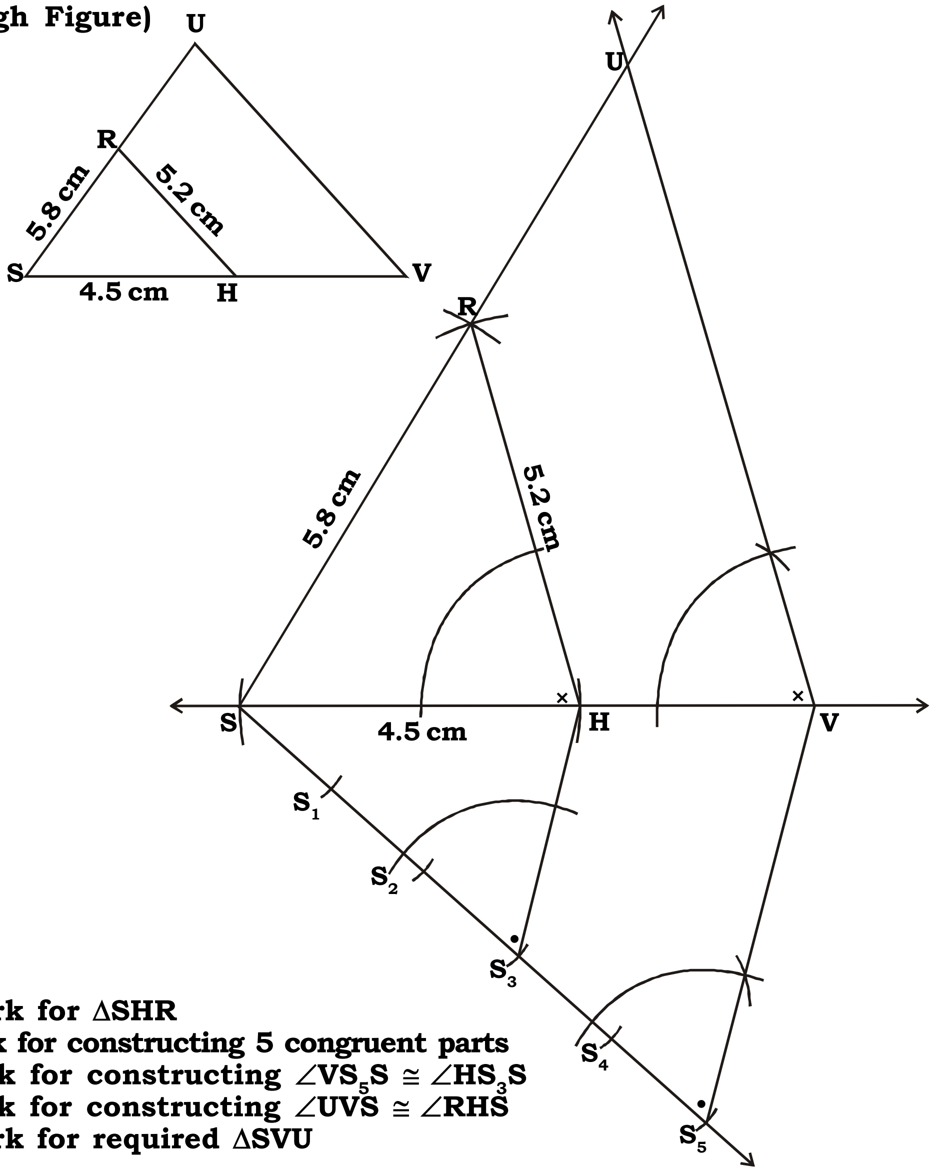
In □ABCD,

m ∠BAD + m ∠BCD + m ∠ABC + m ∠ADC = 360°
[∵ Sum of measure of angles of a quadrilateral is 360°] ½

∴ m ∠BAD + m ∠BCD + 180° = 360° [From (iii)]

∴ m ∠BAD + m ∠BCD = 180° ½

(ii) **(Rough Figure)**



½ mark for ΔSHR
1 mark for constructing 5 congruent parts
1 mark for constructing $\angle VS_5S \cong \angle HS_3S$
1 mark for constructing $\angle UVS \cong \angle RHS$
½ mark for required ΔSVU

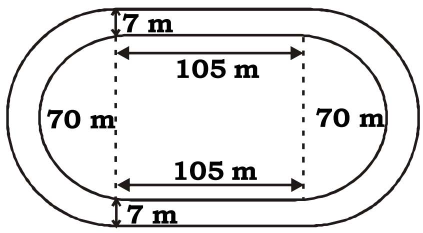
(iii) Diameter of inner circular edge (d_1) = 70 m ½

Width of the track = 7 m

Diameter of outer circular edge (d_2) = 70 + 7 + 7

= 84 m ½

The inner and outer edges of the racing tracks comprises of two semicircles and parallel segments of length 105 m each



$\therefore$ Length of outer edge $= \frac{1}{2}\pi d_2 + 105 + \frac{1}{2}\pi d_2 + 105$ ½

$= \pi d_2 + 210$ ½

$= (84\pi + 210)$ m

Length of inner edge $= \frac{1}{2}\pi d_1 + 105 + \frac{1}{2}\pi d_1 + 105$ ½

$= \pi d_1 + 210$ ½

$= (70\pi + 210)$ m

Difference in the lengths of inner and outer edge $= (84\pi + 210) - (70\pi + 210)$ ½

$= 84\pi + 210 - 70\pi - 210$

$= 14\pi$

$= 14 \times \frac{22}{7}$

$= 44$ m

$\therefore$ **The difference in the lengths of inner edge and outer edge of the track is 44 m.** ½

Q.5. Solve the following : (Any 2)

(i) Given : $\Delta ABC \sim \Delta PQR$.

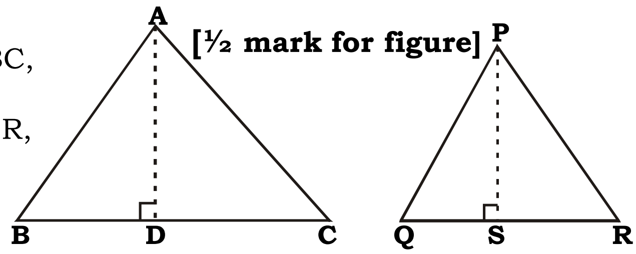
To Prove : $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ ½

Construction :

(i) Draw seg AD $\perp$ side BC, B - D - C

(ii) Draw seg PS $\perp$ side QR, Q - S - R

[½ mark for figure] ½



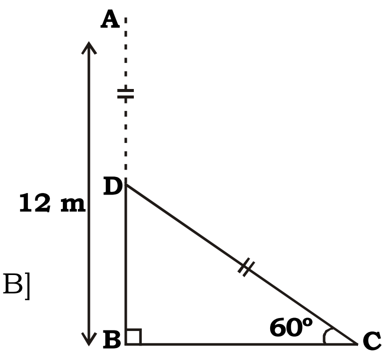
Proof : $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ [The ratio of the areas of two triangles is equal to ratio of the products of a base and its corresponding height] ½

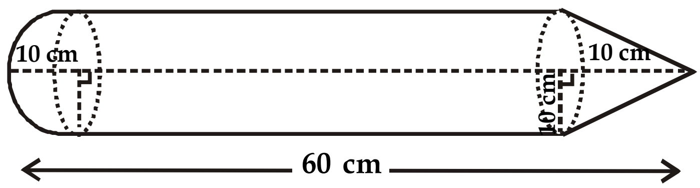
$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ ......(i) ½

$\Delta ABC \sim \Delta PQR$ [Given]

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$ ......(ii) [c.s.s.t.] ½

Also, $\angle B \cong \angle Q$ .....(iii) [c.a.s.t.]

	In ΔADB and ΔPSQ, $\angle ADB \cong \angle PSQ$ [Each is a right angle] $\angle B \cong \angle Q$ [From (ii)] $\therefore$ $\Delta ADB \sim \Delta PSQ$ [By A-A test of similarity]	½
	$\therefore \frac{AD}{PS} = \frac{AB}{PQ}$ .....(iv) [c.s.s.t.]	
	$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (i), (ii) and (iv)]	½
	$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ....(vi)	½
	$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ [From (ii) and (vi)]	½
(ii)	seg AB represents the tree AB = 12 m The tree breaks at point D seg AD is the broken part of tree which then takes the position of DC $\therefore$ AD = DC m $\angle DCB = 60^\circ$ Let DB = x m $\therefore$ AD + DB = AB [$\because$ A - D - B] $\therefore$ AD + x = 12 $\therefore$ AD = (12 – x) m $\therefore$ DC = (12 – x) m In right angled ΔDBC,  **[½ mark for figure]**	½ ½
	$\sin 60^\circ = \frac{DB}{DC}$ [By definition]	½
	$\therefore \frac{\sqrt{3}}{2} = \frac{x}{12 - x}$	
	$\therefore \sqrt{3}(12 - x) = 2x$	
	$\therefore 12\sqrt{3} - \sqrt{3}x = 2x$	½
	$\therefore 12\sqrt{3} = 2x + \sqrt{3}x$	
	$\therefore x(2 + \sqrt{3}) = 12\sqrt{3}$	
	$\therefore x = \frac{12\sqrt{3}}{2 + \sqrt{3}}$	½

	$\therefore$ DB = $\frac{12\sqrt{3}}{2+\sqrt{3}}$ m	
	$\therefore$ DB = $\frac{12\sqrt{3}\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$	½
	$\therefore$ DB = $\frac{24\sqrt{3}-12(3)}{(2)^2-\left(\sqrt{3}\right)^2}$	
	$\therefore$ DB = $\frac{24\sqrt{3}-36}{4-3}$	½
	$\therefore$ DB = $\frac{24(1.73)-36}{1}$	
	$\therefore$ DB = 41.52 – 36	½
	$\therefore$ DB = 5.52 m	
	$\therefore$ The height at which the tree is broken from the bottom by the wind is 5.52 m.	½
(iii)	 	
	A toy is a combination of cylinder, hemisphere and cone, each with radius 10 cm	
	$\therefore$ r = 10 cm	½
	$\therefore$ Height of the conical part (h) = 10 cm	
	Height of the hemispherical part = its radius = 10cm	½
	Total height of the toy = 60cm	
	$\therefore$ Height of the cylindrical part (h_1) = 60 – 10 – 10 = 60 – 20 = 40 cm	½
	$l^2 = r^2 + h^2$	
	$\therefore$ $l^2 = 10^2 + 10^2$	½
	$\therefore$ $l^2 = 100 + 100$	
	$l^2 = 200$	
	$\therefore$ $l = \sqrt{200}$ [Taking square roots]	
	$l = 10\sqrt{2}$ cm	½
	Slant height of the conical part (l) = $10\sqrt{2}$	

	$= 10 \times 1.41$	½
	$= 14.1$ cm	
	Total surface area of the toy = Curved surface area of the conical part + Curved surface area of the cylindrical part + Curved surface area of the hemispherical part	1
	$= \pi r l + 2\pi r h_1 + 2\pi r^2$	½
	$= \pi r (l + 2h_1 + 2r)$	
	$= 3.14 \times 10 (14.1 + 2 \times 40 + 2 \times 10)$	
	$= 31.4 (14.1 + 80 + 20)$	
	$= 31.4 \times 114.1$	
	$= 3582.74 \text{ cm}^2$	
	$\therefore$ Total surface area of the toy is 3582.74 cm^2.	½

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