Set - D

Q.P. SET CODE
D

MT - z

Seat No. | | | | | | | |

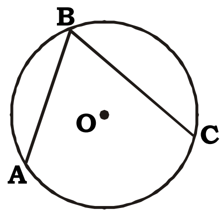
2013 ___ ___ 1100 - MT - z MATHEMATICS (71) GEOMETRY - PAPER D (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Q.1. Solve the following : (Any 5) **5**

(i) In ΔPQR, m $\angle Q = 90^\circ$, m $\angle P = 30^\circ$, m $\angle R = 60^\circ$. If PR = 8 cm, find QR.

(ii) O is the centre of the circle. If m $\angle ABC = 80^\circ$, the find m (arc AC) and m (arc ABC).



(iii) The terminal arm is in II quadrant, what are the possible angles ?

(iv) A line has the equation $y = 3x - 2$. State its y-intercept.

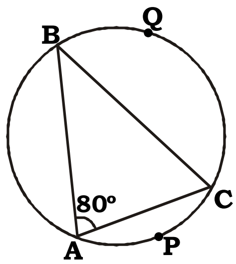
(v) $\Delta ABC \sim \Delta APQ$, if $\frac{A(\Delta ABC)}{A(\Delta APQ)} = \frac{1}{4}$, find $\frac{BC}{PQ}$.

(vi) Find the area of a circle with radius 7 cm.

Q.2. Solve the following : (Any 4) **8**

(i) The ratio of the areas of two triangles with the common base is 6 : 5. Height of the larger triangle is 9 cm. Then find the corresponding height of the smaller triangle.

(ii) In the adjoining figure, if m (arc APC) = 60° and m $\angle BAC = 80^\circ$ Find (a) $\angle ABC$ (b) m (arc BQC).



(iii) Prove : $\sec^2 \theta + \text{cosec}^2 \theta = \sec^2 \theta \, . \, \text{cosec}^2 \theta$

(iv) Check whether points (4,-5), (7,8) and (-2,-3) are collinear.

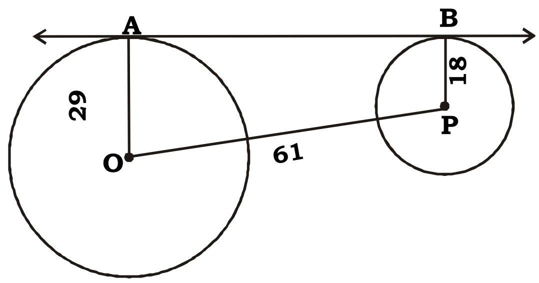
(v) $3 \sin \alpha - 4 \cos \alpha = 0$, then find the values of $\tan \alpha$ and $\sec \alpha$ where α is an acute angle.

(vi) Write the equation $2x+3y-7 = 0$ in double intercept form and write x-intercept and y-intercept.

Q.3. Solve the following : (Any 3) **9**

(i) Adjacent sides of a parallelogram are 11 cm and 17 cm. If the length of one of its diagonals is 26 cm. Find the length of the other.

(ii) In the adjoining figure, line AB is tangent to both the circles touching at A and B. OA = 29, BP = 18, OP = 61 then find AB.



(iii) Draw tangents to the circle with centre P and radius 2.9 cm. From a point Q which is at a distance 8.8 cm from the centre.

(iv) Show that the line joining (– 1, 1) and (– 9, 6) is parallel to the line joining (– 2, 14) and (6, 9).

(v) Construct a circumcircle of ΔABC such that AB = 5 cm, AC = 12 cm, $\angle BAC = 90^\circ$.

Q.4. Solve the following : (Any 2) **8**

(i) Prove : The lengths of the two tangent segments to a circle drawn from an external point are equal.

(ii) $\Delta LMN \sim \Delta XYZ$, In ΔLMN, LM = 6 cm, MN = 6.8 cm, LN = 7.6 cm and $\frac{LM}{XY} = \frac{4}{3}$; construct ΔXYZ.

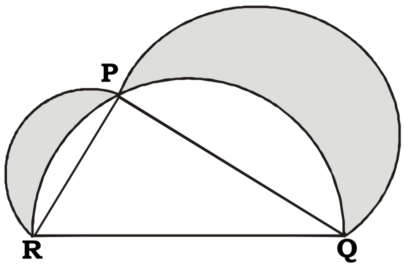
(iii) A roller of diameter 0.9 m and length 1.8 m is used to press the ground. Find the area of ground pressed by it in 500 revolutions. (Given $\pi = 3.14$)

Q.5. Solve the following : (Any 2) **10**

(i) Prove : In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides.

(ii) A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river. ($\sqrt{3}$ = 1.73)

(iii) In the adjoining figure, PR = 6 units and PQ = 8 units. Semicircles are draw taking sides PR, RQ and PQ as diameters as shown in the figure. Find out the area of the shaded portion. (π = 3.14)



Best Of Luck

Set - D

A.P. SET CODE

D

MT - z

2013 ___ ___ 1100 - **MT - z** MATHEMATICS (71) GEOMETRY - PAPER D (E)

Time : 2 Hours **Prelim - I Model Answer Paper** **Max. Marks : 40**

Q.1. Solve the following : (Any 5)

(i) In ΔPQR,
m $\angle P = 30^\circ$
m $\angle R = 60^\circ$ [Given]
m $\angle Q = 90^\circ$

$\therefore$ ΔPQR is a $30^\circ - 60^\circ - 90^\circ$ triangle ½

$\therefore$ By $30^\circ - 60^\circ - 90^\circ$ triangle theorem,

QR = $\frac{1}{2}$ PR [Side opposite to 30°]

$\therefore$ QR = $\frac{1}{2} \times 8$

$\therefore$ **QR = 4 cm** ½

(ii) m $\angle ABC = \frac{1}{2}$ m (arc AC) [By Inscribed angle theorem]

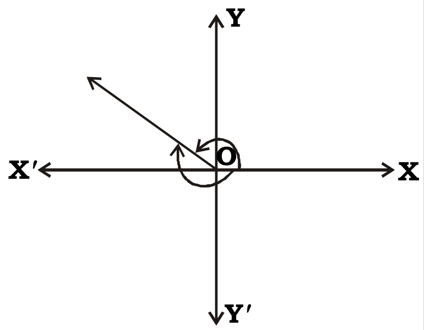
$\therefore$ $80^\circ = \frac{1}{2}$ m (arc AC)

$\therefore$ m (arc AC) = 160° ½

m (arc ABC) = 360° – m (arc AC)
= 360 – 160

$\therefore$ **m (arc ABC) = 200°** ½

(iii) The terminal arm is in II quadrant, the angle is in between 90° and 180° if the initial arm rotates anticlockwise direction or the angle is between –270° and – 180° if the initial arm rotates clockwise. 1



(iv) Equation of the line is y = 3x – 2

Comparing the given equation with slope-intercept form y = mx + c, ½

c = – 2

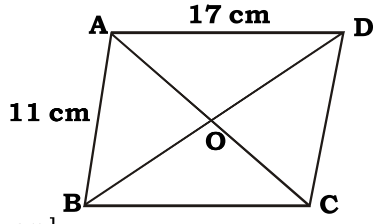
$\therefore$ **y intercept of the line is – 2.** ½

(v)	$\Delta ABC \sim \Delta APQ$ [Given]	
	$\therefore \dfrac{A(\Delta ABC)}{A(\Delta APQ)} = \dfrac{BC^2}{PQ^2}$ [Areas of similar triangles]	½
	$\therefore \dfrac{1}{4} = \dfrac{BC^2}{PQ^2}$ [Given]	
	$\therefore \boxed{\dfrac{BC}{PQ} = \dfrac{1}{2}}$ [Taking square roots]	½
(vi)	Radius of circle (r) = 7 cm	
	$\therefore$ Area of the circle = πr^2	½
	$= \dfrac{22}{7} \times 7 \times 7$	
	= 154 cm^2	
	$\therefore$ The area of a circle is 154 cm^2.	½
Q.2.	**Solve the following : (Any 4)**	
(i)	Let the areas of the larger and the smaller triangle be A_1 and A_2 respectively.	
	Let their heights be h_1 and h_2 respectively.	½
	$\dfrac{A_1}{A_2} = \dfrac{6}{5}$ and h_1 = 9 cm [Given]	
	The two triangles have a common base [Given]	
	$\therefore \dfrac{A_1}{A_2} = \dfrac{h_1}{h_2}$ [Triangles with common base]	½
	$\therefore \dfrac{6}{5} = \dfrac{9}{h_2}$	
	$\therefore h_2 = \dfrac{5 \times 9}{6}$	½
	$\therefore h_2 = \dfrac{15}{2}$	
	$\therefore h_2 = 7.5$	
	$\therefore$ The corresponding height of the smaller triangle is 7.5 cm.	½
(ii)	(a) m $\angle$ABC = $\dfrac{1}{2}$ m(arc APC) [Inscribed angle theorem]	½
	$\therefore$ m $\angle$ABC = $\dfrac{1}{2} \times 60$	
	$\therefore \boxed{m\angle ABC = 30^\circ}$	½
	(b) m $\angle$BAC = $\dfrac{1}{2}$ m(arc BQC) [Inscribed angle theorem]	½

	$\therefore \quad 80 = \frac{1}{2}$ m(arc BQC) $\therefore$ m(arc BQC) = 80 × 2 $\therefore$ $\boxed{\text{m(arc BQC)} = 160°}$	½
(iii)	L.H.S. $= \sec^2\theta + \text{cosec}^2\theta$ $= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$ $\left[\because \sec\theta = \frac{1}{\cos\theta}, \text{cosec}\,\theta = \frac{1}{\sin\theta}\right]$ $= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta . \sin^2\theta}$ $= \frac{1}{\cos^2\theta . \sin^2\theta}$ $\quad [\because \sin^2\theta + \cos^2\theta = 1]$ $= \sec^2\theta . \text{cosec}^2\theta$ = R.H.S. $\therefore \sec^2\theta + \text{cosec}^2\theta = \sec^2\theta . \text{cosec}^2\theta$	½ ½ ½ ½
(iv)	Let, A ≡ (4, – 5) ≡ (x_1, y_1) B ≡ (7, 8) ≡ (x_2, y_2) C ≡ (– 2, – 3) ≡ (x_3, y_3) Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{8 - (-5)}{7 - 4}$ $= \frac{8 + 5}{3}$ $= \frac{13}{3}$ Slope of line BC $= \frac{y_3 - y_2}{x_3 - x_2}$ $= \frac{-3 - 8}{-2 - 7}$ $= \frac{-11}{-9}$ $= \frac{11}{9}$ $\therefore$ Slope of line AB and slope of line BC are not equal. $\therefore$ The points (4, – 5), (7, 8) and (– 2, – 3) are not collinear.	½ ½ ½ ½

(v)	$3\sin\alpha - 4\cos\alpha = 0$	½
	$\therefore\ 3\sin\alpha = 4\cos\alpha$	
	$\therefore\ \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{4}{3}$	
	$\therefore\ \boxed{\tan\alpha = \dfrac{4}{3}}$	½
	$1 + \tan^2\alpha = \sec^2\alpha$	½
	$\therefore\ 1 + \left(\dfrac{4}{3}\right)^2 = \sec^2\alpha$	
	$\therefore\ 1 + \dfrac{16}{9} = \sec^2\alpha$	
	$\therefore\ \dfrac{9+16}{9} = \sec^2\alpha$	
	$\therefore\ \dfrac{25}{9} = \sec^2\alpha$	
	$\therefore\ \boxed{\sec\alpha = \dfrac{5}{3}}$ [Taking square roots]	½
(vi)	$2x + 3y - 7 = 0$	
	$\therefore\ 2x + 3y = 7$	½
	Dividing throughout by 7,	
	$\therefore\ \dfrac{2x}{7} + \dfrac{3y}{7} = \dfrac{7}{7}$	½
	$\therefore\ \dfrac{x}{\frac{7}{2}} + \dfrac{y}{\frac{7}{3}} = 1$	
	$\therefore$ x intercept of line $2x + 3y - 7 = 0$ is $\dfrac{7}{2}$ y intercept of line $2x + 3y - 7 = 0$ is $\dfrac{7}{3}$	1

Q.3. (i)	**Solve the following : (Any 3)**	



$\square$ABCD is a parallelogram [Given]

$OB = OD = \frac{1}{2} \times BD$ .....(i) [$\because$ Diagonals of parallelogram bisect each other] ½

$\therefore$ $OB = OD = \frac{1}{2} \times 26$ [Given]

$\therefore$ $OB = OD = 13$ cm ½

In ΔADB,

seg AO is the median [From (i) and by definition]

$\therefore$ $AB^2 + AD^2 = 2AO^2 + 2OB^2$ [By Appollonius theorem] ½

$\therefore$ $(11)^2 + (17)^2 = 2AO^2 + 2\,(13)^2$

$\therefore$ $121 + 289 = 2AO^2 + 2\,(169)$

$\therefore$ $410 = 2AO^2 + 338$ ½

$\therefore$ $410 - 338 = 2AO^2$

$\therefore$ $72 = 2AO^2$

$\therefore$ $AO^2 = 36$

$\therefore$ $AO = 6$ cm [Taking square roots]

$AO = \frac{1}{2} \times AC$ [$\because$ Diagonals of parallelogram bisect each other] ½

$\therefore$ $6 = \frac{1}{2} \times AC$

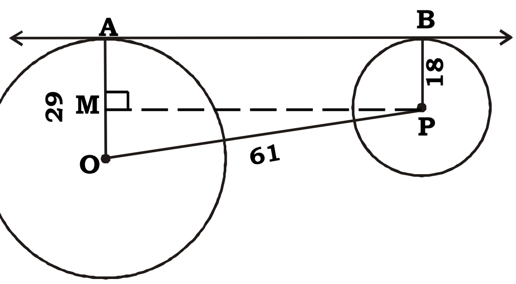
$\therefore$ $AC = 12$ cm

$\therefore$ Length of other diagonal is 12 cm. ½

(ii) Construction :

Draw seg PM $\perp$ seg AO, ½

A - M - O.



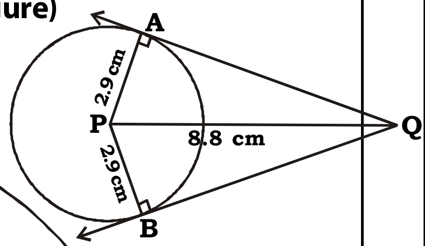
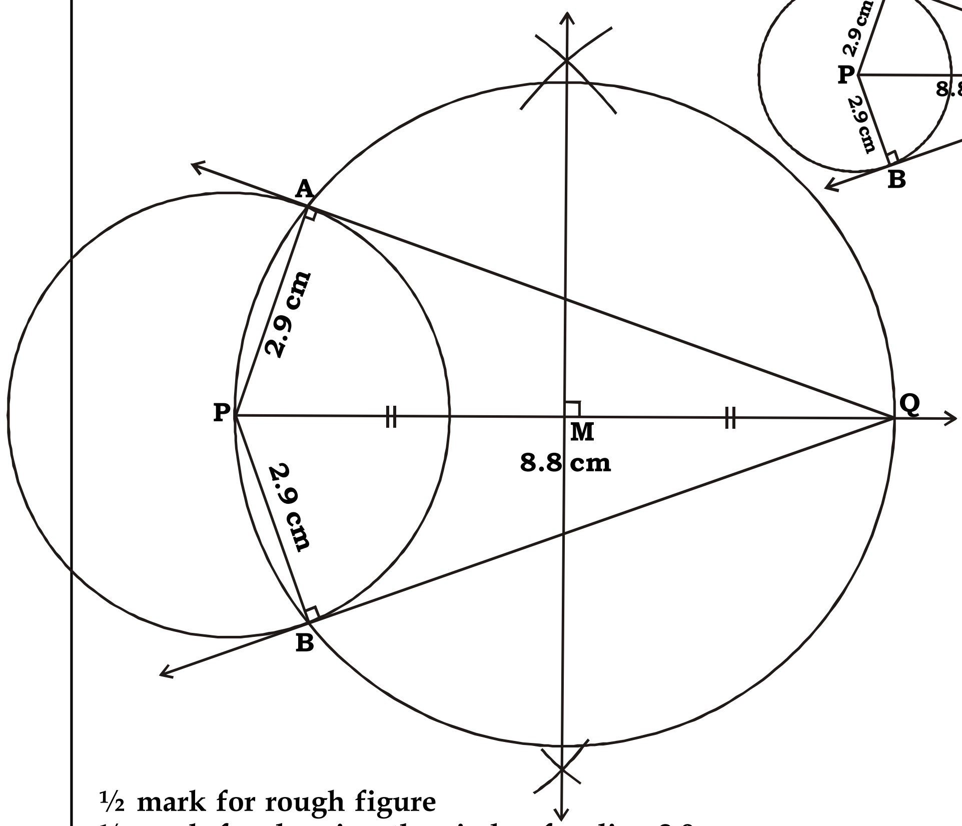
Proof :

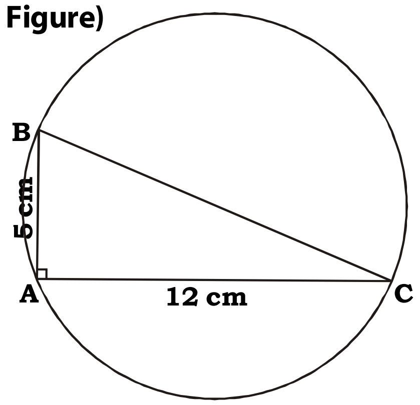
In $\square$PBAM,

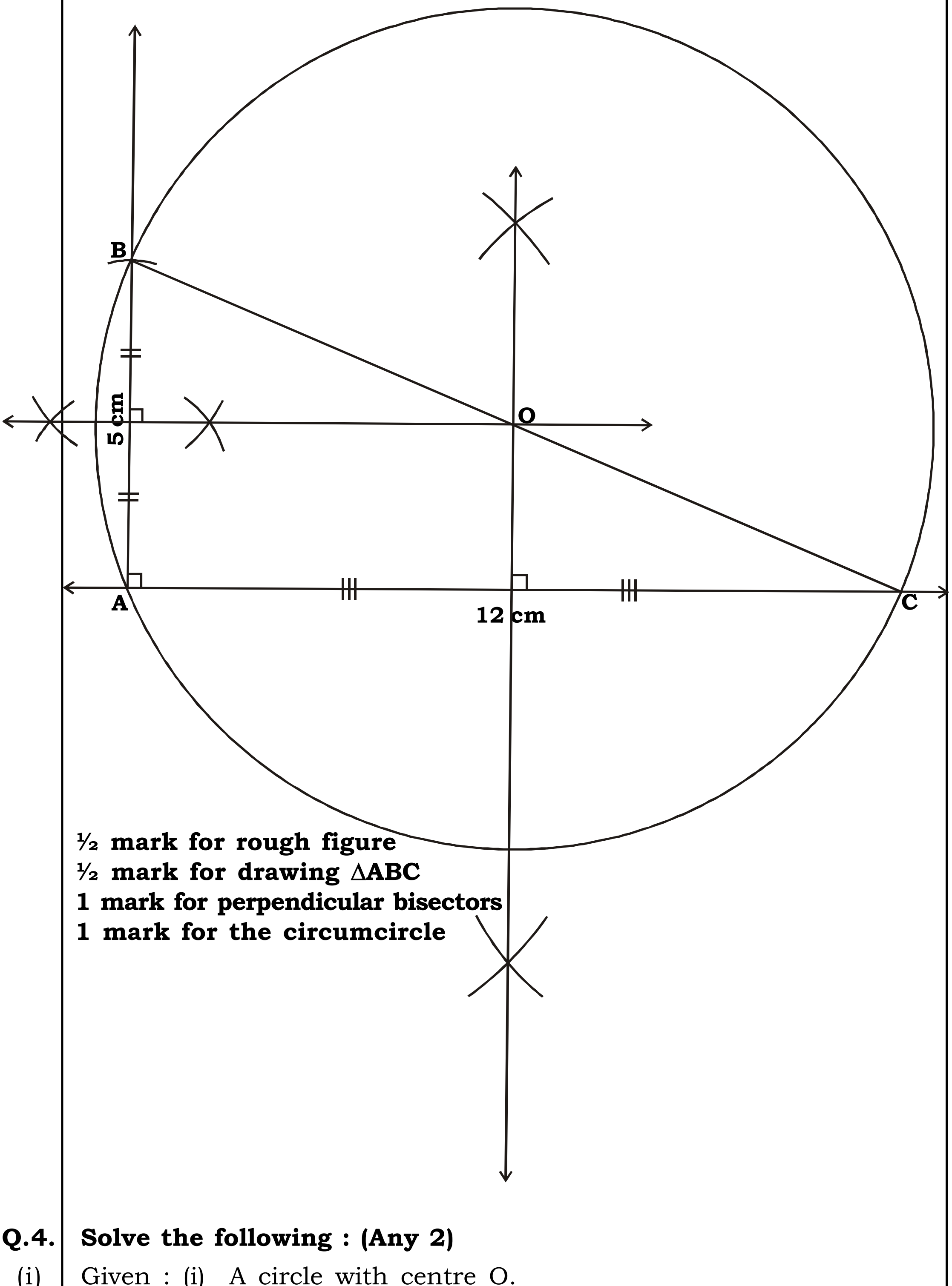
m $\angle$PBA = 90°
m $\angle$BAM = 90° } [Radius is perpendicular to the tangent] ½

m $\angle$PMA = 90° [Construction]

$\therefore$ m $\angle$MPB = 90° [Remaining angle]

	∴ □PBAM is a rectangle [By definition]	½
	∴ PB = AM = 18 units [Opposite sides of a rectangle]	
	OA = OM + AM [∵ A - M - O]	
	∴ 29 = OM + 18	
	∴ OM = 29 – 18	
	∴ OM = 11 units	½
	In ΔPMO,	
	m ∠PMO = 90° [Construction]	
	∴ $OP^2 = OM^2 + PM^2$ [By Pythagoras theorem]	½
	∴ $(61)^2 = (11)^2 + PM^2$	
	∴ $3721 = 121 + PM^2$	
	∴ $PM^2 = 3721 - 121$	
	∴ $PM^2 = 3600$	
	∴ PM = 60 units	
	But, AB = PM [Opposite sides of a rectangle]	
	∴ **AB = 60 units**	½
(iii)	**(Rough Figure)**  	
	½ mark for rough figure **½ mark for drawing the circle of radius 2.9 cm** **½ mark for drawing the perpendicular bisector of seg PQ** **½ mark for drawing the circle with centre M** **1 mark for drawing both the tangents from point Q**	

(iv)	Let, A ≡ (– 1, 1), B ≡ (– 9, 6), C ≡ (– 2, 14), D ≡ (6, 9)	
	Slope of a line $= \dfrac{y_2 - y_1}{x_2 - x_1}$	½
	Slope of side AB $= \dfrac{6-1}{-9-(-1)}$	½
	$= \dfrac{5}{-9+1}$	
	$= \dfrac{5}{-8}$	
	∴ Slope of line AB $= \dfrac{-5}{8}$	½
	Slope of line CD $= \dfrac{9-14}{6-(-2)}$	½
	$= \dfrac{-5}{6+2}$	
	∴ Slope of line CD $= \dfrac{-5}{8}$	½
	∴ Slope of line AB and slope of line CD are equal.	
	∴ line AB || line CD	
	∴ The line joining (– 1, 1) and (– 9, 6) is parallel to the line joining (– 2, 14) and (6, 9).	½
(v)	(Rough Figure) 	

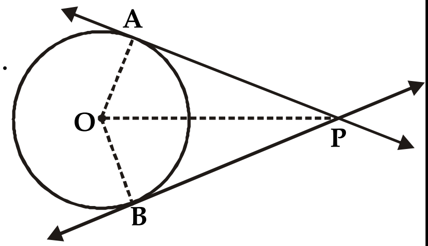


½ mark for rough figure
½ mark for drawing ΔABC
1 mark for perpendicular bisectors
1 mark for the circumcircle

Q.4. Solve the following : (Any 2)

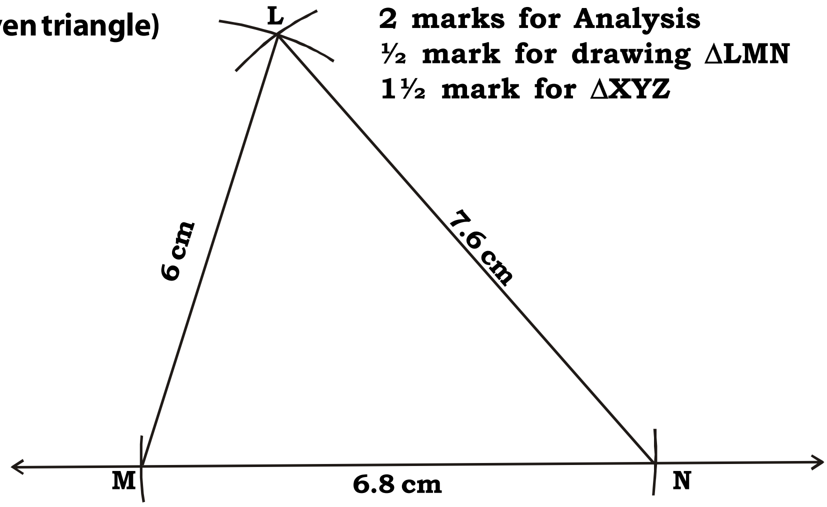
(i) Given : (i) A circle with centre O.

(ii) P is a point in the exterior of the circle.

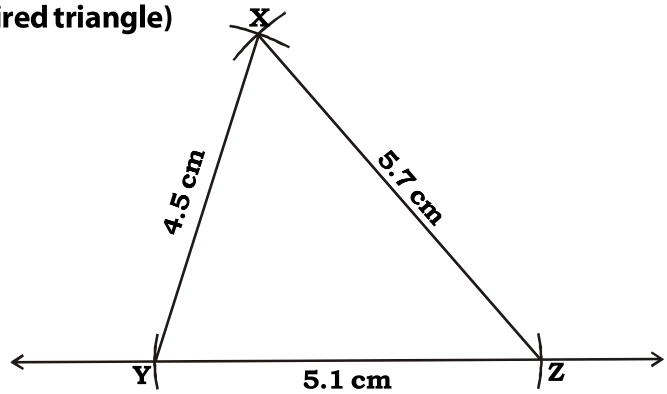
	(iii) Points A and B are the points of contact of the two tangents from P to the circle. To Prove : PA = PB **[½ mark for figure]** 	½
	Construction : Draw seg OA, seg OB and seg OP.	½
	Proof : In ΔPAO and ΔPBO,	
	m ∠PAO = m ∠PBO = 90° [Radius is perpendicular to the tangent]	½
	Hypotenuse OP ≅ Hypotenuse OP [Common side]	½
	seg OA ≅ seg OB [Radii of same circle]	½
	∴ ΔPAO ≅ ΔPBO [By hypotenuse - side theorem]	½
	seg PA ≅ seg PB [c.s.c.t]	½
	∴ PA = PB	
(ii)	Analysis : ΔLMN ~ ΔXYZ [Given]	
	$\therefore \frac{LM}{XY} = \frac{MN}{YZ} = \frac{LN}{XZ} = \frac{4}{3}$ ...... (i) [c.s.s.t.]	
	$\therefore \frac{LM}{XY} = \frac{4}{3}$ [From (i)] $\therefore \frac{MN}{YZ} = \frac{4}{3}$ [From (i)]	
	$\therefore \frac{LN}{XZ} = \frac{4}{3}$ [From (i)]	
	$\therefore \frac{6}{XY} = \frac{4}{3}$ $\therefore \frac{6.8}{YZ} = \frac{4}{3}$ $\therefore \frac{7.6}{XZ} = \frac{4}{3}$	
	$\therefore \frac{18}{4} = XY$ $\therefore \frac{20.4}{4} = YZ$ $\therefore \frac{22.8}{4} = XZ$	
	∴ XY = 4.5 cm ∴ YZ = 5.1 cm ∴ XZ = 5.7 cm	
	Information for constructing ΔXYZ is complete.	

2 marks for Analysis
½ mark for drawing ΔLMN
1½ mark for ΔXYZ

(Given triangle)



(Required triangle)



(iii) Diameter of the roller = 0.9 m

$\therefore$ its radius (r) = $\frac{0.9}{2}$

= 0.45 m ½

its length (h) = 1.8 m ½

Curved surface area of the roller = $2\pi rh$ ½

= $2 \times 3.14 \times 0.45 \times 1.8$

= 6.28×0.81

= 5.0868 m^2 ½

Area of the ground pressed by the roller in 1 revolution = curved surface area of roller

$\therefore$ Area of the ground pressed in one revolution = 5.0868 m^2 ½

Area of the ground pressed in 500 revolution = 500×5.0868 ½

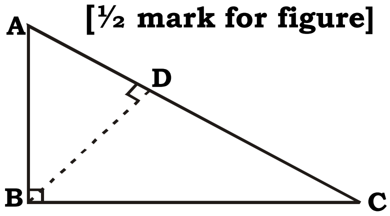
= $500 \times \frac{50868}{10000}$ ½

= 2543.4 m^2

$\therefore$ Area of the ground pressed by the roller is 2543.4 m^2. ½

Q.5. Solve the following : (Any 2)

[½ mark for figure] ½



(i) Given : In ΔABC, $m\angle ABC = 90^\circ$

To Prove : $AC^2 = AB^2 + BC^2$ ½

Construction : Draw seg $BD \perp$ side AC such that A - D - C.

Proof : In ΔABC,

$m\angle ABC = 90^\circ$ [Given]

seg $BD \perp$ hypotenuse AC [Construction]

$\therefore$ $\Delta ABC \sim \Delta ADB \sim \Delta BDC$ .....(i) [Similarity in right angled triangles]

$\Delta ABC \sim \Delta ADB$ [From (i)] 1

$\therefore \dfrac{AB}{AD} = \dfrac{AC}{AB}$ [Corresponding sides of similar triangles]

$\therefore AB^2 = AC \times AD$ ....(ii) ½

$\Delta ABC \sim \Delta BDC$ [From (i)]

$\therefore \dfrac{BC}{DC} = \dfrac{AC}{BC}$ [Corresponding sides of similar triangles]

$\therefore BC^2 = AC \times DC$ ....(iii) ½

Adding (ii) and (iii) we get,

$AB^2 + BC^2 = AC \times AD + AC \times DC$ ½

$\therefore AB^2 + BC^2 = AC\,(AD + DC)$

$\therefore AB^2 + BC^2 = AC \times AC$ [$\because$ A - D - C] ½

$\therefore AB^2 + BC^2 = AC^2$

$\therefore AC^2 = AB^2 + BC^2$ ½

(ii) Let seg AB represents the tree ½

seg BC represents width of river

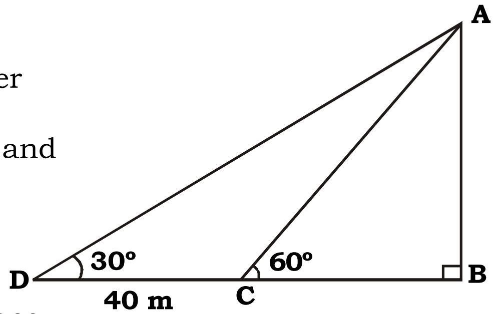
Let BC = x m

C and D represents the initial and final positions of the observer

DC = 40 m

$\angle ACB$ and $\angle ADB$ are the angles of elevation

$m\angle ACB = 60^\circ$ and $m\angle ADB = 30^\circ$



[½ mark for figure]

In right angled ΔACB,

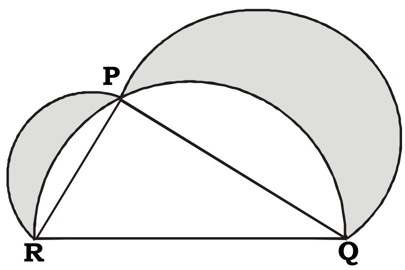
$\tan 60^\circ = \dfrac{AB}{BC}$ [By definition] ½

$\therefore \sqrt{3} = \dfrac{AB}{x}$

$\therefore AB = \sqrt{3}\,x$ m .....(i) ½

In right angled ΔADB,

$\tan 30^\circ = \dfrac{AB}{DB}$ [By definition] ½

	$\therefore \quad \frac{1}{\sqrt{3}} = \frac{AB}{40 + x}$	
	$\therefore \quad AB = \frac{40 + x}{\sqrt{3}}$ m .....(ii)	½
	From (i) and (ii) we get,	
	$\sqrt{3}x = \frac{40 + x}{\sqrt{3}}$	½
	$\therefore \quad 3x = 40 + x$	
	$\therefore \quad 3x - x = 40$	
	$\therefore \quad 2x = 40$	
	$\therefore \quad x = 20$	
	$\therefore \quad BC = 20$ m	½
	$\therefore \quad AB = 20\sqrt{3}$ m [From (i)]	
	$\therefore \quad AB = 20 \times 1.73$	½
	$\therefore \quad AB = 34.6$ m	
	∴ **Height of tree is 34.6 m and width of river is 20 m.**	½
(iii)	Diameter PR = 6 units	
	∴ Its radius (r_1) = 3 units	½
	Diameter PQ = 8 units	
	∴ Its radius (r_2) = 4 units	
		
	In ΔPQR,	
	m ∠RPQ = 90° ......(i) [Angle subtended by a semicircle]	½
	$QR^2 = PR^2 + PQ^2$ [By Pythagoras theorem]	
	$\therefore \quad QR^2 = 62 + 82$	
	$\therefore \quad QR^2 = 36 + 64$	
	$\therefore \quad QR = 100$	
	$\therefore \quad QR = 10$ units [Taking square roots]	½
	Diameter QR = 10 units	
	∴ Its radius (r_3) = 5 units	
	ΔPQR is a right angled triangle [From (i)]	
	A (ΔPQR) = $\frac{1}{2}$ × product of perpendicular sides	½
	$= \frac{1}{2} \times PR \times PQ$	
	$= \frac{1}{2} \times 6 \times 8$	
	= 24 sq. units.	½

	Area of shaded portion = Area of semicircle with diameter PR + Area of semicircle with diameter PQ + Area of ΔPQR – Area of semicircle with diameter QR	1
	$= \frac{1}{2}\pi r_1^2 + \frac{1}{2}\pi r_2^2 + 24 - \frac{1}{2}\pi r_3^2$	½
	$= \left(\frac{1}{2}\pi r_1^2 + \frac{1}{2}\pi r_2^2 - \frac{1}{2}\pi r_3^2\right) + 24$	
	$= \frac{1}{2}\pi (r_1^2 + r_2^2 - r_3^2) + 24$	
	$= \frac{1}{2} \times 3.14\ (3^2 + 4^2 - 5^2) + 24$	
	$= \frac{1}{2} \times 3.14\ (9 + 16 - 25) + 24$	½
	$= \frac{1}{2} \times 3.14\ (25 - 25) + 24$	
	$= \frac{1}{2} \times 3.14\ (0) + 24$	
	$= 0 + 24$	
	$= 24$ sq. units	
	Area of the shaded portion is 24 sq. units.	½
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