MT

2014 ___ ___ 1100 **Seat No.**

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 1 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

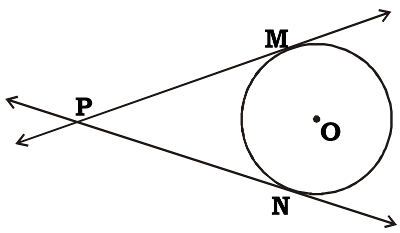
Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY FIVE of the following : **5**

(i) Lines PM and PN are tangents to the circle with centre O.
If PM = 7 cm, find PN.



(ii) Using Euler's formula, find F, if V = 6 and E = 12.

(iii) If x-coordinate of point A is negative and y-coordinate is positive, then in which quadrant point A lies ?

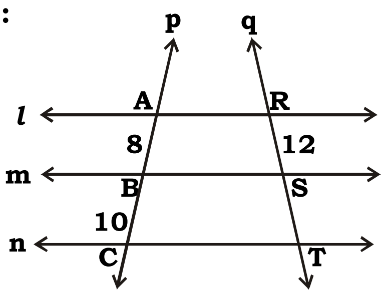
(iv) If m = 5 and c = – 3, then write the equation of the line.

(v) The area of a circle is 314 cm^2 and the area of its minor sector is 31.4 cm^2. Find the area of its major sector.

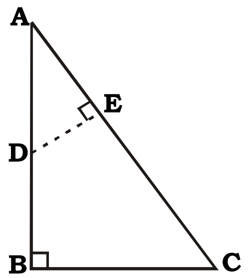
(vi) Find the value of $3\sin^2\theta + 3\cos^2\theta$.

Q.2. Solve ANY FOUR of the following : **8**

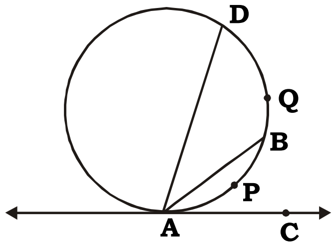
(i) In the adjoining figure,
line l || line m ||line n.
Lines p and q are transversals.
From given information
find ST.



(ii) ΔABC is a right angled at B. D is any point on AB. $DE \perp AC$. If AD = 6 cm, AB = 12 cm, AC = 18 cm, find AE.



(iii) In the adjoining figure, seg AB and seg AD are chords of the circle. C be a point on tangent to the circle at point A. If m (arc APB) = 80° and $\angle BAD$ = 30°, then find (i) $\angle BAC$ (ii) m (arc BQD)



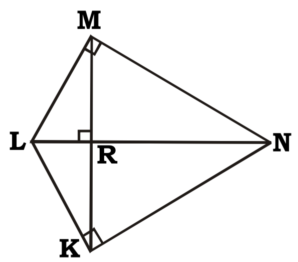
(iv) Draw a tangent at any point 'M' on the circle of radius 2.9 cm and centre 'O'.

(v) Eliminate θ, if, $x = a \sec \theta$, $y = b \tan \theta$

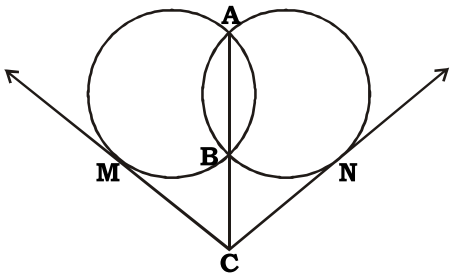
(vi) If $\sin \theta + \sin^2 \theta = 1$, prove that $\cos^2 \theta + \cos^4 \theta = 1$.

Q.3. Solve ANY THREE of the following : 9

(i) In the adjoining figure, $\angle LMN = 90°$ and $\angle LKN = 90°$, seg MK $\perp$ seg LN. Prove that R is the midpoint of seg MK.



(ii) In the adjoining figure, two circles intersect each other in two points A and B. Seg AB is the chord of both circles. Point C is the exterior point of both the circles on the line AB. From the point C tangents are drawn to the circles touching at M and N. Prove that CM = CN.



(iii) Construct the incircle of ΔSRN, such that RN = 5.9 cm, RS = 4.9 cm, $\angle R$ = 95°.

(iv) Write down the equation of a the line whose slope is $\frac{3}{2}$ and which passes through P where P divides the line segment joining A (– 2, 6) and B (3, – 4) in the ratio 2 : 3.

(v) The radius of a circle is 3.5 cm and area of the sector is 3.85 cm^2. Find the length of the corresponding arc and the measure of arc.

Q.4. Solve ANY TWO of the following : 8

(i) Suppose AB and AC are equal chords of a circle and a line parallel to the tangent at A intersects the chords at D and E. Prove that AD = AE.

(ii) Find the equation of the straight line passing through the origin and the point of intersection of the lines x + 2y = 7 and x – y = 4.

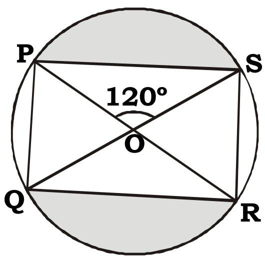
(iii) Eliminate θ, if $x = 2 \cos \theta - 3 \sin \theta$, $y = \cos \theta + 2 \sin \theta$

Q.5. Solve ANY TWO of the following : 10

(i) Prove : The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

(ii) $\Delta SHR \sim \Delta SVU$, In ΔSHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and $\frac{SH}{SV} = \frac{3}{5}$; construct ΔSVU.

(iii) In the adjoining figure,
PR and QS are two diameters of the circle.
If PR = 28 cm and PS = $14\sqrt{3}$ cm, find
(i) Area of triangle OPS
(ii) The total area of two shaded segments.
($\sqrt{3}$ = 1.73)



Best Of Luck

MT

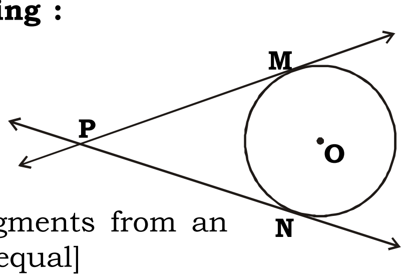
2014 ___ ___ 1100 **Seat No.**

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 1 (E)

Time : 2 Hours **Prelim - II Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i)



PM = PN
[Length of the two tangent segments from an external point to a circle are equal] ½
But, PM = 7 cm [Given]
∴ PN = 7 cm ½

(ii)
F + V = E + 2
∴ F + 6 = 12 + 2 ½
∴ F + 6 = 14
∴ F = 14 – 6
∴ F = 8 ½

(iii)
$\theta = -30^\circ$ [Given]
$\sin\theta = \sin(-30)$ ½
$= -\sin 30$
$= -\frac{1}{2}$
∴ $\sin(-30) = -\frac{1}{2}$ ½

(iv)
m = 5, c = – 3
∴ By slope point form, the equation of line is
y = mx + c ½
∴ y = 5x – 3
∴ 5x – y – 3 = 0 ½

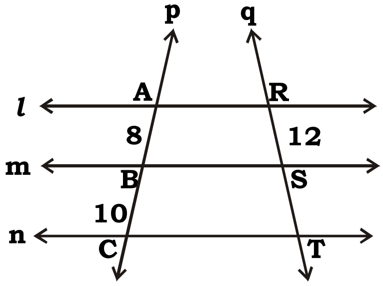
(v) Area of major sector = Area of circle – Area of minor sector ½
= 314 – 31.4
= 282.6 cm²

∴ The area of the major sector is 282.6 cm². ½

(vi) $3\sin^2\theta + 3\cos^2\theta$ = $3(\sin^2\theta + \cos^2\theta)$ ½
= 3 (1) [∵ $\sin^2\theta + \cos^2\theta = 1$]
= 3 ½

A.2. Solve ANY FOUR of the following :

(i)



line l || line m || line n [Given]

∴ On transversals p and q,

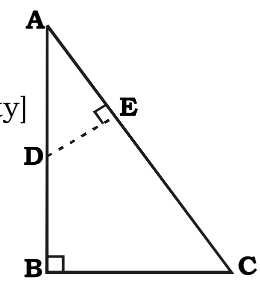
$$\frac{AB}{BC} = \frac{RS}{ST}$$ [By Property of Intercepts made by three parallel lines] 1

∴ $$\frac{8}{10} = \frac{12}{ST}$$ [Given]

∴ $$ST = \frac{12 \times 10}{8}$$ ½

∴ ST = 15 units ½

(ii)



In ΔABC and ΔAED,

∠BAC ≅ ∠DAE [Common angle]

∠ABC ≅ ∠AED [∵ Each is 90°]

∴ ΔABC ~ ΔAED [By AA test of similarity] 1

∴ $$\frac{AB}{AE} = \frac{AC}{AD}$$ [c.s.s.t.] ½

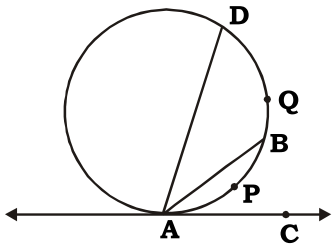
∴ $$\frac{12}{AE} = \frac{18}{6}$$ [Given]

∴ $$AE = \frac{12 \times 6}{18}$$

∴ AE = 4 units ½

(iii) m $\angle BAC = \frac{1}{2}$ m(arc APB)

[Tangent secant theorem] ½



$\therefore$ m $\angle BAC = \frac{1}{2} \times 80$

$\therefore$ **m $\angle BAC = 40^\circ$** ½

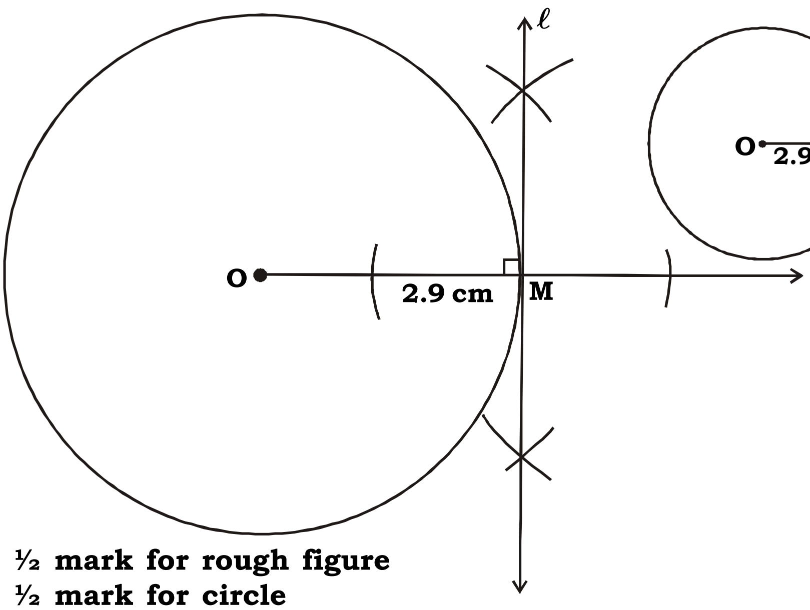
m $\angle BAD = \frac{1}{2}$ m (arc BQD) [Inscribed angle theorem] ½

$\therefore$ $30 = \frac{1}{2}$ m (arc BQD)

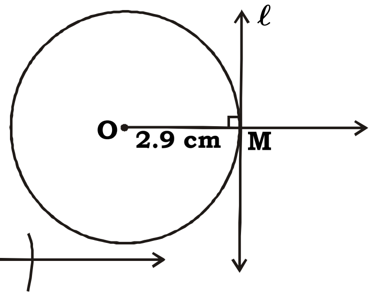
$\therefore$ m (arc BQD) $= 30 \times 2$

$\therefore$ **m (arc BQD) $= 60^\circ$** ½

(iv)



(Rough Figure)



½ mark for rough figure
½ mark for circle
1 mark for drawing perpendicular

(v) $x = a \sec \theta$

$\therefore$ $\sec \theta = \frac{x}{a}$ ......(i) ½

$y = b \tan \theta$

$\therefore$ $\tan \theta = \frac{y}{b}$ ......(ii) ½

$$1 + \tan^2\theta = \sec^2\theta$$

$\therefore \quad 1 + \left(\frac{y}{b}\right)^2 = \left(\frac{x}{a}\right)^2$ [From (i) and (ii)] ½

$\therefore \quad 1 + \frac{y^2}{b^2} = \frac{x^2}{a^2}$

$\therefore \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ½

(vi) $\sin\theta + \sin^2\theta = 1$ [Given]

$\therefore \quad \sin\theta = 1 - \sin^2\theta$ ½

$\therefore \quad \sin\theta = \cos^2\theta$ $\left[\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ \because 1 - \sin^2\theta = \cos^2\theta \end{array}\right]$ ½

$\therefore \quad \sin^2\theta = \cos^4\theta$ [Squaring both sides] ½

$\therefore \quad 1 - \cos^2\theta = \cos^4\theta$ $\left[\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ \because 1 - \cos^2\theta = \sin^2\theta \end{array}\right]$ ½

$\therefore \quad \cos^2\theta + \cos^4\theta = 1$

A.3. Solve ANY THREE of the following :

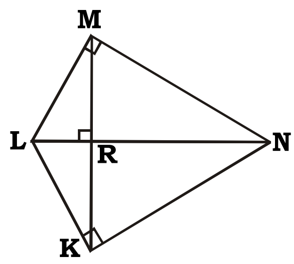
(i) In ΔLMN,

$m\angle LMN = 90^\circ$ [Given]

seg $MR \perp$ hypotenuse LN [Given]

$\therefore \quad MR^2 = LR \times RN$.....(i)

[By property of geometric mean] 1



In ΔLKN,

$m\angle LKN = 90^\circ$ [Given]

seg $KR \perp$ hypotenuse LN [Given]

$\therefore \quad KR^2 = LR \times RN$.....(ii) [By property of geometric mean] 1

$\therefore \quad MR^2 = KR^2$ [From (i) and (ii)]

$\therefore \quad MR = KR$ [Taking square roots]

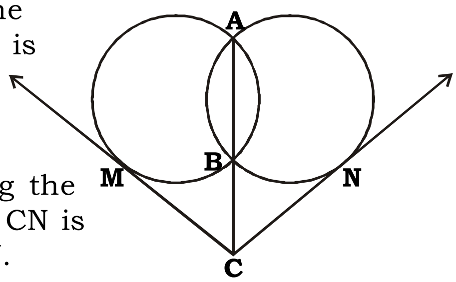
$\therefore$ R is the midpoint of seg MK 1

(ii) Line CBA is a secant intersecting the circle at points B and A and line CM is a tangent to the circle at point M.

$\therefore \quad CM^2 = CB \times CA$ ....(i)

[Tangent secant property] 1

Line CBA is a secant intersecting the circle at points B and A and line CN is a tangent to the circle at point N.

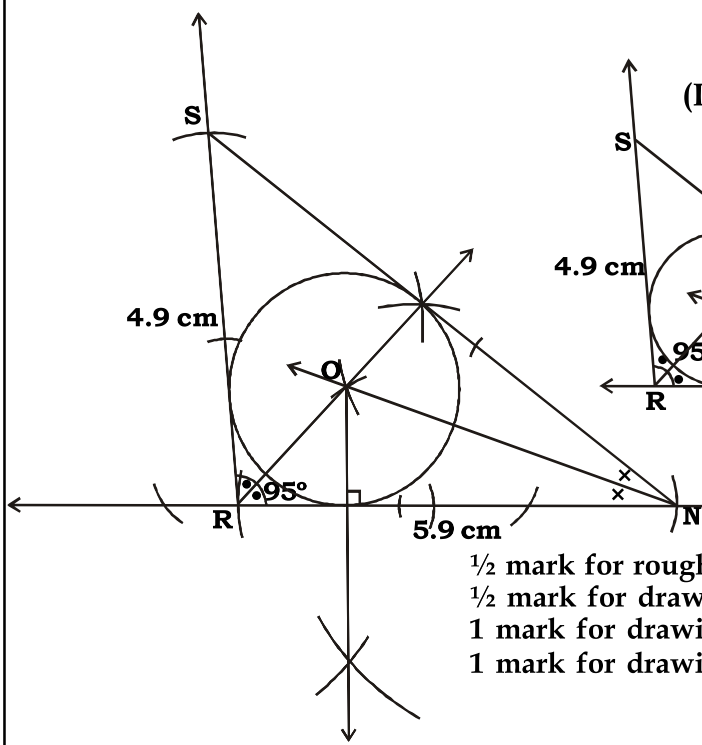


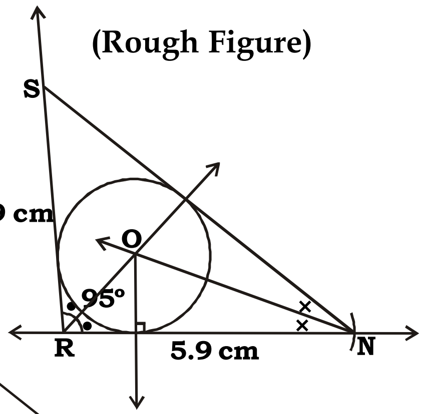
∴ $CN^2 = CB \times CA$ ......(ii) [Tangent secant property] **1**

∴ $CM^2 = CN^2$ [From (i) and (ii)]

∴ $CM = CN$ [Taking square roots] **1**

(iii)





(Rough Figure)

½ mark for rough figure
½ mark for drawing ΔSRN
1 mark for drawing the angle bisectors
1 mark for drawing the incircle

(iv) $A \equiv (-2, 6)$, $B \equiv (3, -4)$

Point P divides seg AB internally in the ratio 2 : 3 **½**

Let, $P \equiv (x, y)$

By section formula for internal division,

$$x = \frac{mx_2 + nx_1}{m + n} = \frac{2(3) + 3(-2)}{2 + 3} = \frac{6 - 6}{5} = \frac{0}{5} = 0$$

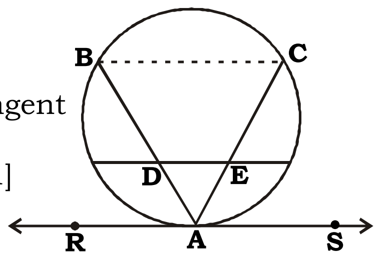
$$y = \frac{my_2 + ny_1}{m + n} = \frac{2 \times (-4) + 3 \times 6}{2 \times 3} = \frac{-8 + 18}{5} = \frac{10}{5} = 2$$

∴ $P \equiv (0, 2)$ **1**

The line having slope $\frac{3}{2}$ passes through the point P (0, 2)

∴ The equation of the line by slope point form is,

	$(y - y_1) = m\,(x - x_1)$	½
	$\therefore\ (y - 2) = \frac{3}{2}(x - 0)$	
	$\therefore\ 2(y - 2) = 3x$	
	$\therefore\ 2y - 4 = 3x$	½
	$\therefore\ 3x - 2y + 4 = 0$	
	∴ The equation of the required line is $3x - 2y + 4 = 0$	½
(v)	Radius of a circle (r) = 3.5 cm	
	Area of the sector = 3.85 cm^2	
	Area of sector $= \frac{r}{2} \times l$	½
	$\therefore\ 3.85 = \frac{3.5}{2} \times l$	
	$\therefore\ \frac{3.85 \times 2}{3.5} = l$	
	$\therefore\ \frac{385 \times 2 \times 10}{100 \times 35} = l$	½
	$\therefore\ l = \frac{22}{10}$	
	$\therefore\ l = 2.2$ cm	½
	Area of sector $= \frac{\theta}{360} \times \pi r^2$	½
	$\therefore\ 3.85 = \frac{\theta}{360} \times \frac{22}{7} \times 3.5 \times 3.5$	
	$\therefore\ 3.85 = \frac{\theta}{360} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}$	½
	$\therefore\ \frac{385}{100} = \frac{\theta}{360} \times 11 \times \frac{35}{10}$	
	$\therefore\ \frac{385 \times 360 \times 10}{100 \times 11 \times 35} = \theta$	
	$\therefore\ \theta = 36°$	½
	∴ Length of arc is 2.2 cm and measure of an arc is 36°.	
A.4.	**Solve ANY TWO of the following :**	
(i)	Construction : Draw seg BC.	½
	Proof : Take points R and S on the tangent at A as shown in the figure	
	line DE || line RS [Given]	
	∴ On transversal AD,	
	∠EDA ≅ ∠DAR	
	[Converse of alternate angles test] **(½ marks for figure)**	½



	∴ ∠EDA ≅ ∠BAR ....(i) [∵ B - D - A] ∠BAR ≅ ∠BCA ....(ii) [Angles in alternate segment] ∴ ∠EDA ≅ ∠BCA ......(iii) [From (i) and (ii)] Similarly, we can prove that ∠DEA ≅ ∠CBA ......(iv) In ΔABC, seg AB ≅ seg AC [Given] ∴ ∠BCA ≅ ∠CBA ....(v) [Isosceles triangle theorem] In ΔDEA, ∠EDA ≅ ∠DEA [From (iii), (iv) and (v)] ∴ seg AD ≅ seg AE [Converse of isosceles triangle theorem] ∴ AD = AE	½ ½ ½ 1
(ii)	Let line x + 2y = 7 and x – y = 4 intersect at point A x + 2y = 7 ....(i) x – y = 4 ......(ii) Subtracting (ii) from (i), $x + 2y = 7$ $x - y = 4$ (–) (+) (–) $3y = 3$ ∴ $y = 1$ Substituting y = 1 in equation (ii), $x - 1 = 4$ ∴ $x = 4 + 1$ ∴ $x = 5$ ∴ A ≡ (5, 1) The straight line passes through A ≡ (5, 1) and O ≡ (0, 0) ∴ The equation of the line by two point form, $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$ ∴ $\frac{x - 5}{5 - 0} = \frac{y - 1}{1 - 0}$ ∴ $\frac{x - 5}{5} = \frac{y - 1}{1}$ ∴ $x - 5 = 5(y - 1)$ ∴ $x - 5 = 5y - 5$ ∴ $x - 5y - 5 + 5 = 0$ ∴ $x - 5y = 0$ ∴ **The equation of the line passing through the origin and the point of intersection of the lines x + 2y = 7 and x – y = 4 is x – 5y = 0.**	½ ½ ½ ½ ½ ½ ½ ½

(iii)

$$x = 2\cos\theta - 3\sin\theta \quad \text{......(i)}$$
$$y = \cos\theta + 2\sin\theta \quad \text{......(ii)}$$

Multiplying (ii) by 2,

$$2y = 2\cos\theta + 4\sin\theta \quad \text{.....(iii)}$$ ½

Subtracting (iii) from (i),

$$x - 2y = 2\cos\theta - 3\sin\theta - (2\cos\theta + 4\sin\theta)$$
$$\therefore \quad x - 2y = 2\cos\theta - 3\sin\theta - 2\cos\theta - 4\sin\theta$$
$$\therefore \quad x - 2y = -7\sin\theta$$
$$\therefore \quad \sin\theta = \frac{-(x-2y)}{7} \quad \text{......(iv)}$$ ½

Substituting $\sin\theta = -\dfrac{(x-2y)}{7}$ in equation (ii)

$$y = \cos\theta + 2\left[-\frac{(x-2y)}{7}\right]$$ ½
$$\therefore \quad y = \cos\theta - \frac{2(x-2y)}{7}$$
$$\therefore \quad y + \frac{2(x-2y)}{7} = \cos\theta$$
$$\therefore \quad \frac{7y + 2(x-2y)}{7} = \cos\theta$$ ½
$$\therefore \quad \frac{7y + 2x - 4y}{7} = \cos\theta$$
$$\therefore \quad \cos\theta = \frac{2x+3y}{7}$$ ½

We know,

$$\sin^2\theta + \cos^2\theta = 1$$ ½
$$\therefore \quad \left[\frac{-(x-2y)}{7}\right]^2 + \left[\frac{2x+3y}{7}\right]^2 = 1$$
$$\therefore \quad \frac{(x-2y)^2}{49} + \frac{(2x+3y)^2}{49} = 1$$ ½

Multiplying throughout by 49,

$$(x-2y)^2 + (2x+3y)^2 = 49$$ ½

A.5. Solve ANY TWO of the following :

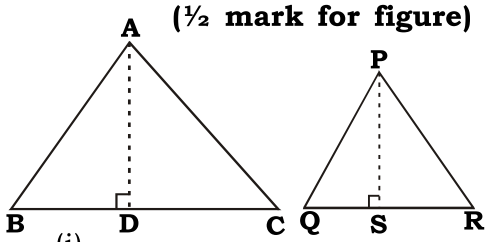
(i) Given : $\Delta ABC \sim \Delta PQR$.

To Prove : $\dfrac{A(\Delta ABC)}{A(\Delta PQR)} = \dfrac{BC^2}{QR^2} = \dfrac{AB^2}{PQ^2} = \dfrac{AC^2}{PR^2}$ ½

Construction : (½ mark for figure) ½

(i) Draw seg AD ⊥ side BC, B - D - C

(ii) Draw seg PS ⊥ side QR, Q - S - R



Proof : $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ ......(i)

[The ratio of the areas of two triangles is equal to ratio of the products of a base and its corresponding height] ½

$\Delta ABC \sim \Delta PQR$ [Given]

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ......(ii) [c.s.s.t.] ½

Also, $\angle B \cong \angle Q$ .....(iii) [c.a.s.t.]

In ΔADB and ΔPSQ,

$\angle ADB \cong \angle PSQ$ [Each is a right angle]

$\angle B \cong \angle Q$ [From (ii)]

$\therefore \Delta ADB \sim \Delta PSQ$ [By A-A test of similarity] **1**

$\therefore \frac{AD}{PS} = \frac{BD}{QS} = \frac{AB}{PQ}$ .....(iv) [c.s.s.t.]

$\therefore \frac{AD}{PS} = \frac{BC}{QR}$ ......(v) [From (ii) and (iv)] ½

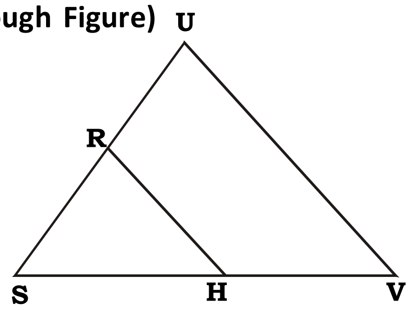
$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ [From (i)]

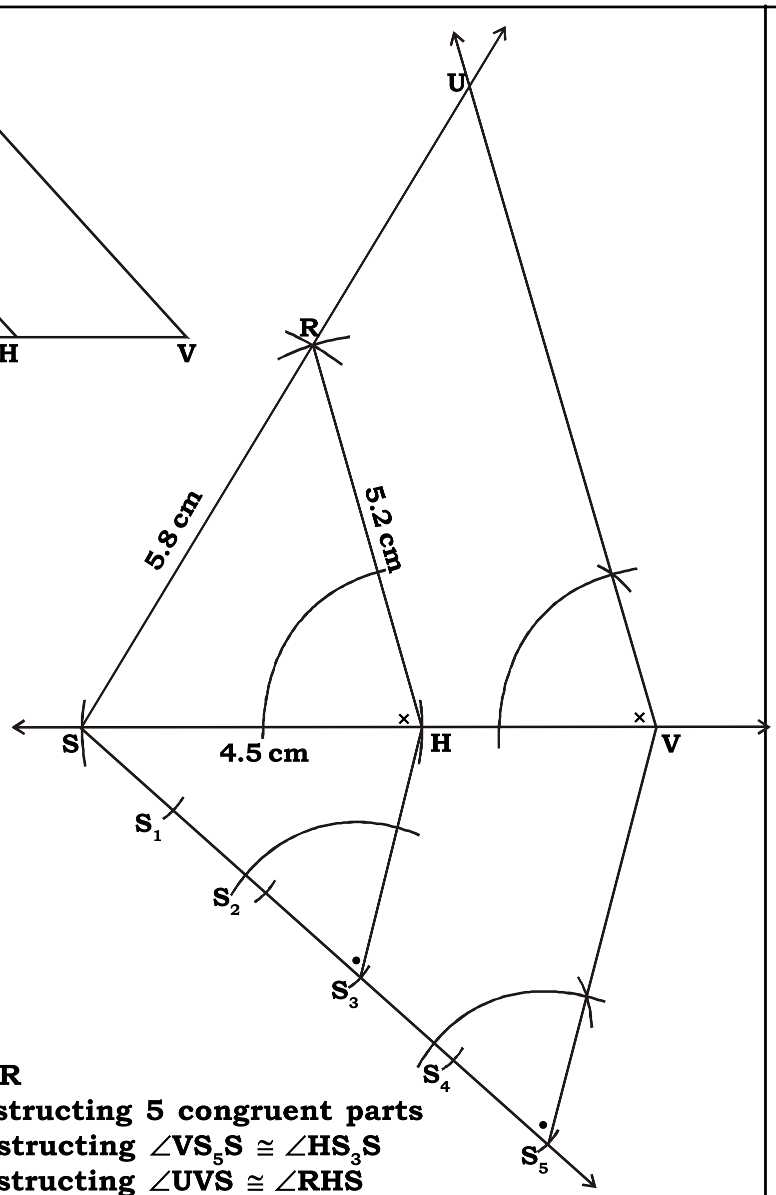
$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR}$ [From (v)] ½

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC^2}{QR^2}$ ....(vi)

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC^2}{QR^2} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2}$ [From (ii) and (vi)] ½

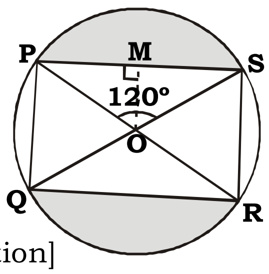
(ii) **(Rough Figure)**





1 mark for ΔSHR
1 mark for constructing 5 congruent parts
1 mark for constructing $\angle VS_5S \cong \angle HS_3S$
1 mark for constructing ∠UVS ≅ ∠RHS
1 mark for required ΔSVU

(iii) Draw seg OM ⊥ side PS ½



$OP = \frac{1}{2} \times PR$ [Radius is half of diameter]

$\therefore OP = \frac{1}{2} \times 28$

$\therefore OP = 14$ cm ½

seg OM ⊥ chord PS [By construction]

$\therefore PM = \frac{1}{2} \times PS$ [The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\therefore$ PM $= \frac{1}{2} \times 14\sqrt{3}$

$\therefore$ PM $= 7\sqrt{3}$ cm ½

In ΔOMP,

$\angle$OMP = 90° [By construction]

$OM^2 + PM^2 = OP^2$ [By Pythagoras theorem]

$\therefore$ $OM^2 + (7\sqrt{3})^2 = 14^2$

$\therefore$ $OM^2 = 196 - 147$

$\therefore$ $OM^2 = 49$

$\therefore$ OM = 7 cm [Taking square roots] ½

Area of ΔOPS $= \frac{1}{2} \times$ base $\times$ height ½

Area of ΔOPS $= \frac{1}{2} \times PS \times OM$

$= \frac{1}{2} \times 14\sqrt{3} \times 7$

$= 49\sqrt{3}$

$= 49\ (1.73)$ ½

$\therefore$ Area of ΔOPS $= 84.77\ cm^2$

Area of sector OPS $= \frac{\theta}{360} \times \pi r^2$ ½

$= \frac{120}{360} \times \frac{22}{7} \times 14 \times 14$

$= \frac{616}{3}$

$= 205.33\ cm^2$ ½

Area of segment PS = Area of sector OPS – Area of ΔOPS

$= 205.33 - 84.77$

$= 120.56\ cm^2$ ½

Similarly we can prove, Area of segment QR = $120.56\ cm^2$

$\therefore$ Total area of two shaded segments = 120.56 + 120.56 = $241.12\ cm^2$

Area of ΔOPS is $84.77\ cm^2$ and total area of two shaded segments is $241.12\ cm^2$. ½

❖❖❖❖