MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 2 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

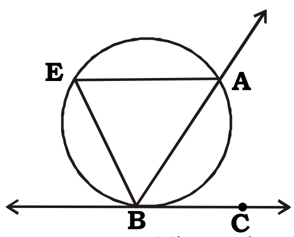
Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY FIVE of the following : **5**

(i) In the adjoining figure, if m $\angle ABC = 55°$, then what is m $\angle AEB$.



(ii) The dimensions of a cuboid are 3 cm × 9 cm × x cm. The volume of this cuboid is equal to the volume of a cube with side 6 cm. What is the value of x ?

(iii) If $\sin \theta = 1$, what is the value of θ ?

(iv) Find the slope of a line whose inclination is 45°.

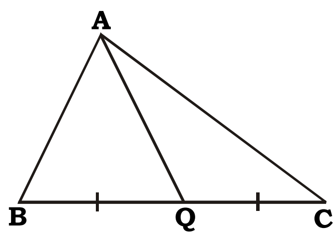
(v) The area of a circle with radius R is equal to the sum of the areas of circles with radii 6 cm and 8 cm. What is the value of R ?

(vi) What is the value of $\cot^2 \theta - \dfrac{1}{\sin^2 \theta}$?

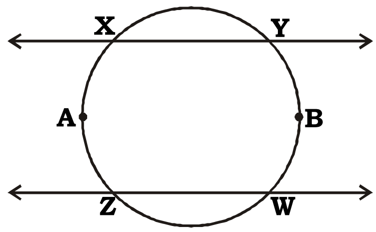
Q.2. Solve ANY FOUR of the following : **8**

(i) If $\Delta ABC \sim \Delta DEF$, and A (ΔABC) : A (ΔDEF) = 16 : 25, BC = 2.2 cm, then find EF.

(ii) In the adjoining figure, $AB^2 + AC^2 = 122$, BC = 10. Find the length of the median on side BC.



(iii) In the adjoining figure, m (arc XAZ) = m (arx YBW). Prove that XY || ZW



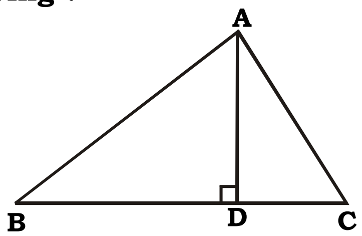
(iv) Draw a circle of radius 2.6 cm. Draw tangent to the circle from any point on the circle using centre of the circle.

(v) If the angle θ = – 60°, find the value of sin θ, cos θ, sec θ and tan θ.

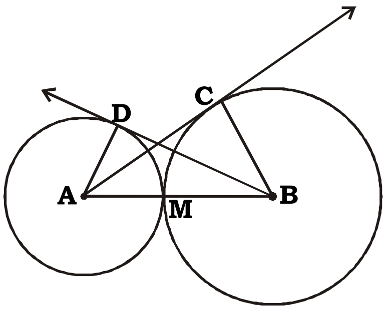
(vi) If x = a sin θ, y = b tan θ then prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

Q.3. Solve ANY THREE of the following : 9

(i) The perpendicular AD on the base BC of ΔABC intersects BC at D so that BD = 3 CD. Prove that $2AB^2 = 2AC^2 + BC^2$.



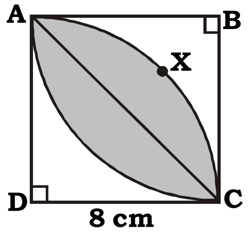
(ii) In the adjoining figure, A and B are centers of two circles touching each other at M. Line AC and line BD are tangents. If AD = 6 cm and BC = 9 cm then find the length of seg AC and seg BD.



(iii) Construct incircle of ΔSGN such that SG = 6.7 cm, ∠S = 70°,∠G = 50° and draw incircle of ΔSGN.

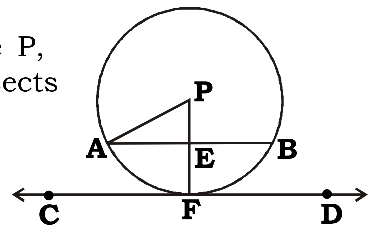
(iv) Find the value of k if (– 3, 11), (6, 2) and (k, 4) are collinear points.

(v) Calculate the area of the shaded region in the adjoining figure where □ABCD is a square with side 8 cm each.

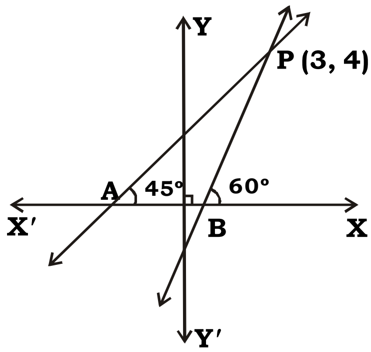


Q.4. Solve ANY TWO of the following : **8**

(i) In the adjoining figure, in a circle with centre P, a chord AB is parallel to a tangent and intersects the radius drawn from a point of contact at the midpoint of the radius. If AB = 12, find the radius of the circle.



(ii) In the adjoining figure, two lines are intersecting at point (3, 4). Find the equation of line PA and line PB.



(iii) A pilot in an aeroplane observes that Vashi bridge is on one side of the plane and Worli sea-link is just on the opposite side. The angle of depression of Vashi bridge and Worli sea-link are 60° and 30° respectively. If the aeroplane is at a height of $5500\sqrt{3}$ m at that time, what is the distance between Vashi bridge and Worli sea-link ?

Q.5. Solve ANY TWO of the following : **10**

(i) **Prove :** If a line parallel to a side of a triangle intersects other sides in two distinct points, then the line divides those sides in proportion.

(ii) Draw a triangle ABC, right angled at B such that, AB = 3 cm and BC = 4 cm. Now construct a triangle similar to ΔABC, each of whose sides is $\frac{7}{5}$ times the corresponding side of ΔABC.

(iii) An ink container of cylindrical shape is filled with ink upto 91%. Ball pen refills of length 12 cm and inner diameter 2 mm are filled upto 84%. If the height and radius of the ink container are 14 cm and 6 cm respectively, find the number of refills that can be filled with this ink.

Best Of Luck

MT

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MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 2 (E)

Time : 2 Hours **Prelim - II Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i)

$\angle AEB \cong \angle ABC$ [Angles in alternate segments] ½

But, $\angle ABC = 55^\circ$ [Given]

$\therefore$ $\boxed{\angle AEB = 55^\circ}$ ½

(ii)

Volume of cuboid = Volume of cube [Given]

$\therefore$ $3 \times 9 \times x = (6)^3$ ½

$\therefore$ $3 \times 9 \times x = 6 \times 6 \times 6$

$\therefore$ $x = \dfrac{6 \times 6 \times 6}{3 \times 9}$

$\therefore$ $\boxed{x = 8}$ ½

(iii)

$\sin \theta = 1$ [Given]

But, $\sin 90 = 1$ ½

$\therefore$ $\sin \theta = \sin 90$

$\therefore$ $\boxed{\theta = 90^\circ}$ ½

(iv)

Inclination of the line $(\theta) = 45^\circ$

$\therefore$ Slope of the line $= \tan \theta$ ½

$= \tan 45^\circ$

$= 1$

$\therefore$ $\boxed{\text{Slope of the line is 1.}}$ ½

(v)

According to given condition,

$\pi R^2 = \pi \times (6)^2 + \pi \times (8)^2$ ½

$\therefore$ $\pi R^2 = \pi [6^2 + 8^2]$

$\therefore$ $R^2 = 36 + 64$

$\therefore$ $R^2 = 100$

$\therefore$ $R = 10$ [Taking square roots]

$\therefore$ $\boxed{\text{The value of R is 10.}}$ ½

(vi) $\cot^2\theta - \dfrac{1}{\sin^2\theta}$

$= \cot^2\theta - \operatorname{cosec}^2\theta$ ½

$= -1$ [$\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\therefore \cot^2\theta - \operatorname{cosec}^2\theta = -1$] ½

A.2. Solve ANY FOUR of the following :

(i) $\Delta ABC \sim \Delta DEF$ [Given]

$\therefore \dfrac{A(\Delta ABC)}{A(\Delta DEF)} = \dfrac{BC^2}{EF^2}$ [Areas of similar triangles] ½

$\therefore \dfrac{16}{25} = \dfrac{(2.2)^2}{EF^2}$ [Given]

$\therefore \dfrac{4}{5} = \dfrac{2.2}{EF}$ [Taking square roots] ½

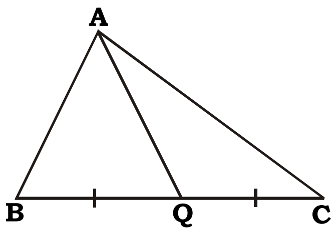
$\therefore EF = \dfrac{2.2 \times 5}{4}$ ½

$\therefore EF = \dfrac{5.5}{2}$

$\therefore \boxed{EF = 2.75 \text{ cm}}$ ½

(ii) In ΔABC,

seg AQ is the median [Given]



$\therefore BQ = QC = \dfrac{1}{2} \times BC$ ½

$\therefore BQ = QC = \dfrac{1}{2} \times 10$ [Given]

$\therefore BQ = QC = 5$ units ……(i) ½

$AB^2 + AC^2 = 2AQ^2 + 2BQ^2$ [By Appollonius theorem] ½

$\therefore 122 = 2AQ^2 + 2(5)^2$ [From (i) and given]

$\therefore 122 = 2AQ^2 + 2(25)$

$\therefore 122 = 2AQ^2 + 50$

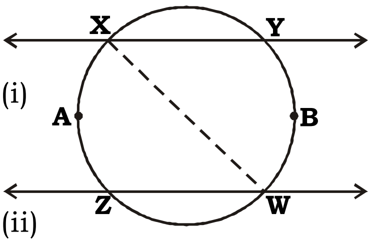
$\therefore 2AQ^2 = 122 - 50$

$\therefore 2AQ^2 = 72$

$\therefore AQ^2 = 36$

$\therefore \boxed{AQ = 6 \text{ units}}$ [Taking square roots] ½

(iii) Construction : Draw seg XW ½



Proof : m ∠XWZ = $\frac{1}{2}$ m (arc XAZ) ....(i)

[Inscribed angle theorem] ½

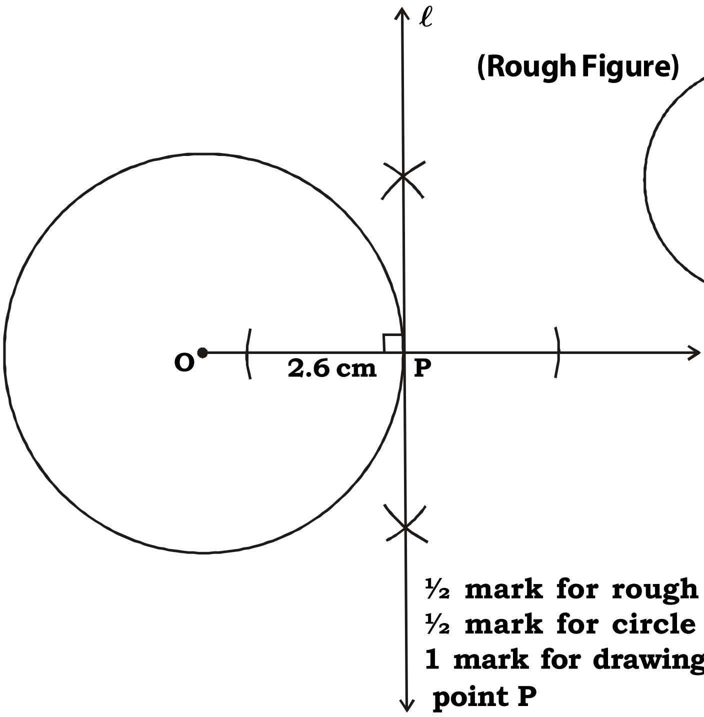
m ∠WXY = $\frac{1}{2}$ m (arc YBW) ....(ii) ½

But, m (arc XAZ) = m (arc YBW) ....(iii) [Given]

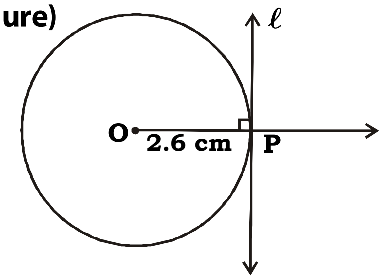
∴ m ∠XWZ = m ∠WXY [From (i), (ii) and (iii)]

∴ line XY || line ZW [Alternate angles test] ½

(iv)



(Rough Figure)



½ mark for rough figure
½ mark for circle
1 mark for drawing perpendicular at point P

(v) θ = – 60°

sin (– θ) = – sin θ

∴ sin (– 60) = – sin 60

∴ $\boxed{\sin(-60) = -\frac{\sqrt{3}}{2}}$ ½

sec (– θ) = sec θ

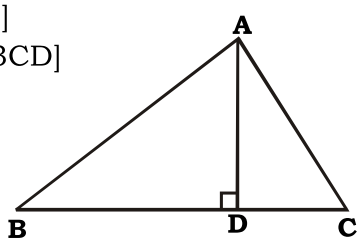
∴ sec (– 60) = sec 60

∴ $\boxed{\sec(-60) = 2}$ ½

cos (– θ) = cos θ

∴ cos (– 60) = cos 60

	$\therefore \boxed{\cos(-60) = \frac{1}{2}}$	½
	$\tan(-\theta) = -\tan\theta$	
	$\therefore \tan(-60) = -\tan 60$	
	$\therefore \boxed{\tan(-60) = -\sqrt{3}}$	½
(vi)	$x = a\sin\theta$	
	$\therefore \frac{1}{\sin\theta} = \frac{a}{x}$	
	$\therefore \operatorname{cosec}\theta = \frac{a}{x}$ .....(i) $\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$	½
	$y = b\tan\theta$	
	$\therefore \frac{1}{\tan\theta} = \frac{b}{y}$	
	$\therefore \cot\theta = \frac{b}{y}$ .....(ii) $\left[\because \cot\theta = \frac{1}{\tan\theta}\right]$	½
	We know,	
	$1 + \cot^2\theta = \operatorname{cosec}^2\theta$	½
	$\therefore \operatorname{cosec}^2\theta - \cot^2\theta = 1$	
	$\therefore \left(\frac{a}{x}\right)^2 - \left(\frac{b}{y}\right)^2 = 1$ [From (i) and (ii)]	
	$\therefore \frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$	½
A.3.	**Solve ANY THREE of the following :**	
(i)	$BC = BD + CD$ [B - D - C]	
	$\therefore BC = 3CD + CD$ [$\because BD = 3CD$]	½
	$\therefore BC = 4CD$ .....(i)	
	In ΔADB,	
	$m\angle ADB = 90^\circ$ [Given]	
	$\therefore AB^2 = AD^2 + BD^2$	
	[By Pythagoras theorem]	½
	$\therefore AB^2 = AD^2 + (3CD)^2$ [$\because BD = 3CD$]	
	$\therefore AB^2 = AD^2 + 9CD^2$	
	$\therefore AB^2 = AD^2 + CD^2 + 8CD^2$ .....(ii)	½



In ΔADC,
m $\angle ADC = 90^\circ$ [Given]
$\therefore$ $AC^2 = AD^2 + CD^2$ .....(iii) [By Pythagoras theorem] ½
$AB^2 = AC^2 + 8CD^2$ [From (ii) and (iii)]

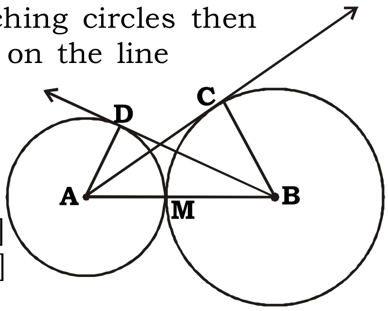
$\therefore$ $AB^2 = AC^2 + 8\left(\frac{BC}{4}\right)^2$ [From (i)] ½

$\therefore$ $AB^2 = AC^2 + 8 \times \frac{BC^2}{16}$

$\therefore$ $AB^2 = AC^2 + \frac{BC^2}{2}$

$\therefore$ $2AB^2 = 2AC^2 + BC^2$ [Multiplying throughout by 2] ½

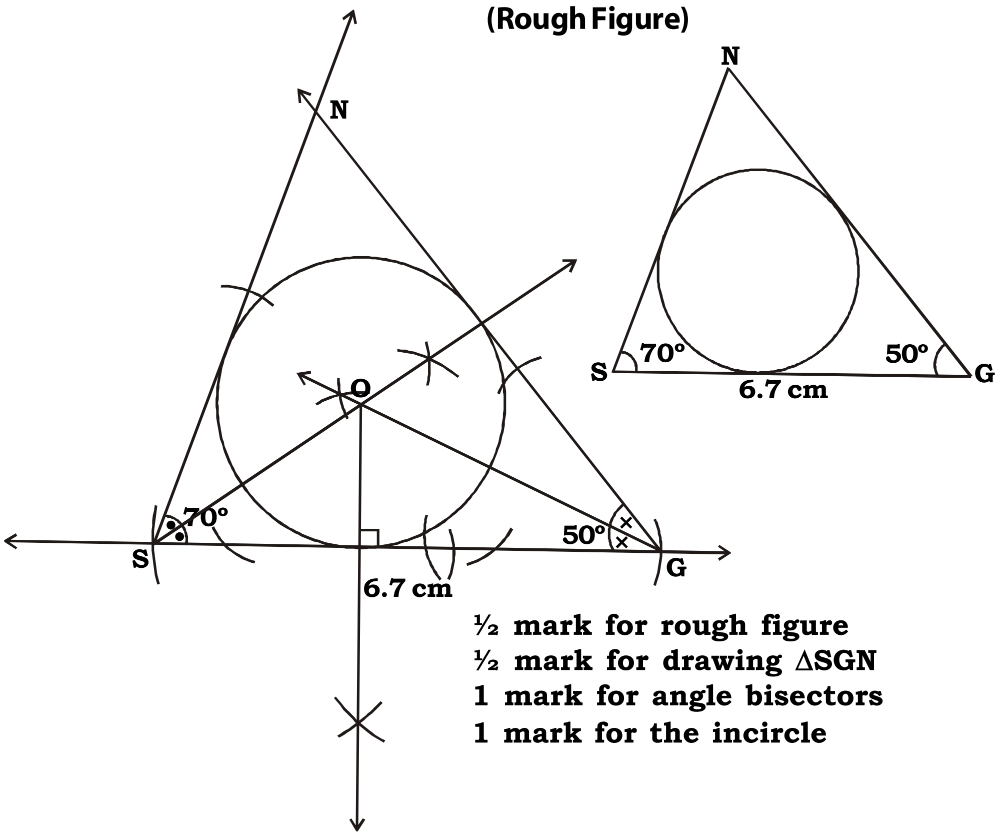
(ii) A - M - B [If two circles are touching circles then the common point lies on the line joining their centres] ½



AM = AD = 6 cm........(i)
[Radii of the same circle]
BM = BC = 9 cm ....(ii)
AB = AM + MB [$\because$ A - M - B]
$\therefore$ AB = 6 + 9 [From (i) and (ii)]
$\therefore$ AB = 15 cm ....(iii) ½
In ΔABC,
m $\angle ACB = 90^\circ$ [Radius is perpendicular to the tangent] ½
$\therefore$ $AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]
$\therefore$ $15^2 = AC^2 + 9^2$ [From (ii) and (iii)]
$\therefore$ $225 = AC^2 + 81$
$\therefore$ $AC^2 = 225 - 81$
$\therefore$ $AC^2 = 144$
$\therefore$ AC = 12 cm [Taking square roots] ½
In ΔADB,
m $\angle ADB = 90^\circ$ [Radius is perpendicular to the tangent] ½
$\therefore$ $AB^2 = AD^2 + BD^2$ [By Pythagoras theorem]
$\therefore$ $15^2 = 6^2 + BD^2$ [From (i) and (iii)]
$\therefore$ $225 = 36 + BD^2$
$\therefore$ $BD^2 = 225 - 36$
$\therefore$ $BD^2 = 189$
$\therefore$ $BD = \sqrt{9 \times 21}$
$\therefore$ $BD = 3\sqrt{21}$ cm. [Taking square roots] ½

$\therefore$ The lengths of seg AC and seg BD are 12 cm and $3\sqrt{21}$ cm respectively.

(iii)



½ mark for rough figure
½ mark for drawing ΔSGN
1 mark for angle bisectors
1 mark for the incircle

(iv)

Let, $A \equiv (-3, 11) \equiv (x_1, y_1)$
$B \equiv (6, 2) \equiv (x_2, y_2)$ ½
$C \equiv (k, 4) \equiv (x_3, y_3)$

$\because$ Points A, B and C are collinear
Slope of line AB = Slope of line BC ½

$\therefore \quad \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_3 - y_2}{x_3 - x_2}$ ½

$\therefore \quad \dfrac{2 - 11}{6 - (-3)} = \dfrac{4 - 2}{k - 6}$

$\therefore \quad \dfrac{-9}{6 + 3} = \dfrac{2}{k - 6}$ ½

$\therefore \quad \dfrac{-9}{9} = \dfrac{2}{k - 6}$

$\therefore \quad -1 = \dfrac{2}{k - 6}$ ½

$\therefore \quad -(k - 6) = 2$

$\therefore \quad -k + 6 = 2$

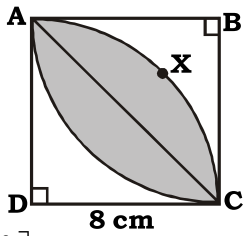
$\therefore \quad -k = 2 - 6$

$\therefore \quad -k = -4$

$\therefore$ k = 4

$\therefore$ **The value of k is 4** ½

(v) Mark point X as shown in the figure



□ABCD is a square [Given]

side = 8 cm

Radius (r) = side of a square

$\therefore$ r = 8 cm

Measure of arc (θ) = 90° [Angle of a square] ½

$$\text{Area of the segment AXC} = r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$$ ½

$$= 8^2\left[\frac{3.14 \times 90}{360} - \frac{\sin 90}{2}\right]$$

$$= 64\left[\frac{1.57}{2} - \frac{1}{2}\right]$$ ½

$$= 64\left[\frac{1.57 - 1}{2}\right]$$

$$= \frac{64 \times 0.57}{2}$$ ½

$$= \frac{36.48}{2}\ \text{cm}^2$$

Area of shaded region = 2 × Area of segment AXC ½

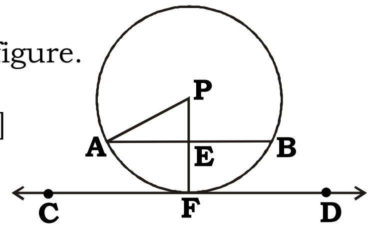
$$= 2 \times \frac{36.48}{2}$$

$$= 36.48\ \text{cm}^2$$

$\therefore$ **Area of shaded region is 36.48 cm².** ½

A.4. Solve ANY TWO of the following :

(i) Take points E and F as shown in the figure.



m ∠PFC = 90° ......(i)

[Radius is perpendicular to the tangent] ½

line CD || chord AB [Given]

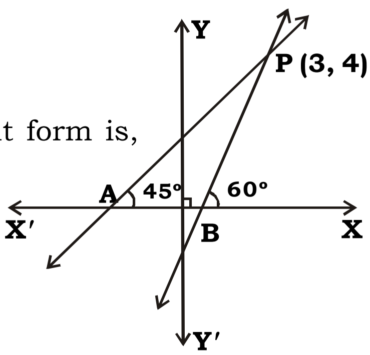
$\therefore$ On transversal PF,

∠PFC ≅ ∠PEA ......(ii) [Converse of corresponding angle test] ½

$\therefore$ m ∠PEA = 90° ......(iii) [From (i) and (ii)]

seg PE ⊥ chord AB [From (iii)]

$\therefore$ $AE = \frac{1}{2} \times AB$ [The perpendicular drawn from the centre of the circle to the chord bisects the chord] ½

	∴ $AE = \frac{1}{2} \times 12$ [Given]	
	∴ AE = 6 units	½
	Let the radius of the circle be 2x units	
	∴ PA = PF = 2x units [Radii of same circle]	
	∴ $PE = \frac{1}{2} PF$ [∵ E is the midpoint of seg PF]	
	∴ $PE = \frac{1}{2} \times 2x$	
	∴ PE = x units	½
	In ΔPEA,	
	m ∠PEA = 90° [From (iii)]	
	∴ $PA^2 = PE^2 + AE^2$ [By Pythagoras theorem]	½
	∴ $(2x)^2 = x^2 + 6^2$	
	∴ $4x^2 - x^2 = 36$	
	∴ $3x^2 = 36$	
	∴ $x^2 = 12$	
	∴ $x = \sqrt{4 \times 3}$	
	∴ $x = 2\sqrt{3}$ [Taking square roots]	½
	∴ $PA = 2 \times 2\sqrt{3} = 4\sqrt{3}$ units	
	∴ Radius of the circle is $4\sqrt{3}$ units.	½
(ii)	Inclination of line PA is 45°	
	Slope of line PA = tan θ = tan 45° = 1	½
	Line PA passes through point P (3, 4)	
	∴ The equation of the line by slope point form is,	½
	$(y - y_1) = m (x - x_1)$	½
	∴ $(y - 4) = 1 (x - 3)$	
	∴ $y - 4 = x - 3$	
	∴ $x - y - 3 + 4 = 0$	
	∴ $x - y + 1 = 0$	
		
	∴ The equation of line PA is $x - y + 1 = 0$	½
	Inclination of line PB is 60°	
	∴ Slope of line PB = tan θ = tan 60° = $\sqrt{3}$	½
	Line PB passes through point P (3, 4)	
	∴ The equation of the line by slope point form is,	½
	∴ $(y - y_1) = m (x - x_1)$	
	∴ $(y - 4) = \sqrt{3} (x - 3)$	½
	∴ $y - 4 = \sqrt{3}x - 3\sqrt{3}$	

$\therefore \quad \sqrt{3}x - y + 4 - 3\sqrt{3} = 0$

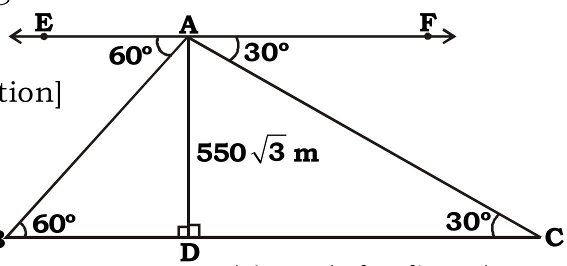
$\therefore$ **The equation of PB is $\sqrt{3}x - y + 4 - 3\sqrt{3} = 0$** ½

(iii) Let A be the point of observer
Let B and C represent the positions of Vashi bridge and Worli sea-link respectively.
AD represents the height of a plane from the ground

AD = $5500\sqrt{3}$ m 1

$\angle EAB$ and $\angle FAC$ are angles of depression
$m\angle EAB = m\angle ABD = 60^\circ$
$m\angle FAC = m\angle ACD = 30^\circ$ } [Converse of alternate angle test]

In right angled ΔADB,



(½ mark for figure)

$\tan 60^\circ = \frac{AD}{BD}$ [By definition] ½

$\therefore \sqrt{3} = \frac{5500\sqrt{3}}{BD}$

$\therefore$ BD = 5500 m ½

In right angled ΔADC,

$\tan 30^0 = \frac{AD}{DC}$ [By definition] ½

$\therefore \frac{1}{\sqrt{3}} = \frac{5000\sqrt{3}}{DC}$

$\therefore$ DC = 5500 × 3

$\therefore$ DC = 16500 m

BC = BD + DC [B - D - C] ½

$\therefore$ BC = 5500 + 16500

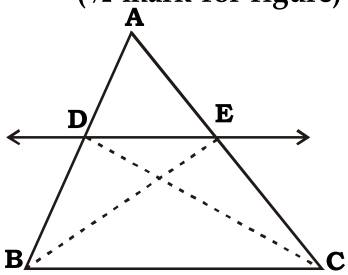
$\therefore$ BC = 22000 m

$\therefore$ **Distance between Vashi bridge and Worli sea-link is 22 km.** ½

A.5. Solve ANY TWO of the following :

(i) Given : In ΔABC,
(i) line DE || side BC ½
(ii) Line DE intersects sides AB and AC at points D and E respectively.

(½ mark for figure)



To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw seg BE and seg CD. ½

Proof : ΔADE and ΔBDE have a common vertex E and their bases AD and BD lie on the same line AB.

∴ Their heights are equal

∴ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{AD}{DB}$ ....(i) [Triangles having equal heights] ½

ΔADE and ΔCDE have a common vertex D and their bases AE and EC lie on the same line AC.

∴ Their heights are equal.

∴ $\frac{A(\Delta ADE)}{A(\Delta CDE)} = \frac{AE}{CE}$ .....(ii) [Triangles having equal heights] ½

line DE || side BC [Given]

ΔBDE and ΔCDE are between the same two parallel lines DE and BC. ½

∴ Their heights are equal.

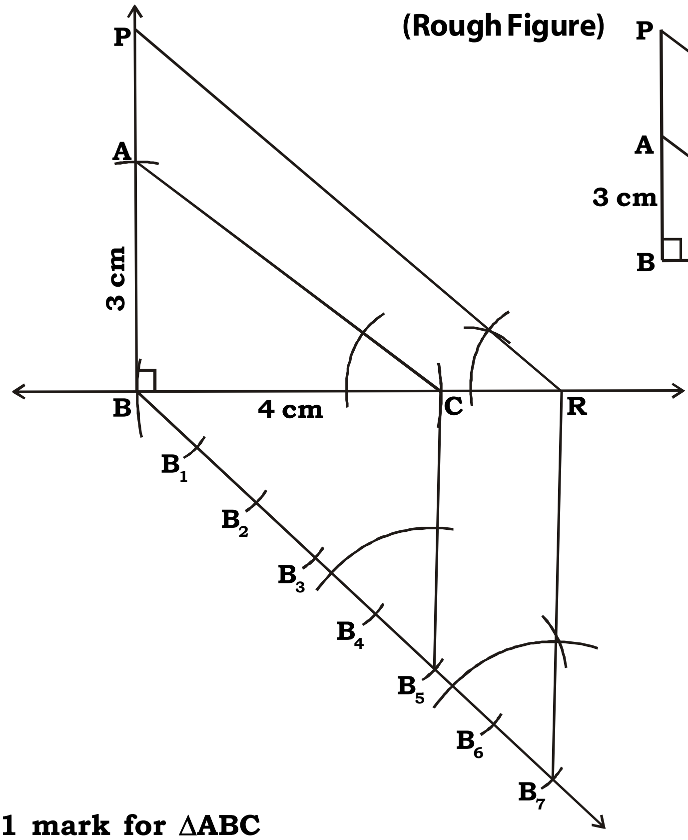
Also, they have same base DE. ½

∴ A(ΔBDE) = A(ΔCDE) .....(iii) [Areas of two triangles having equal bases and equal heights are equal] ½

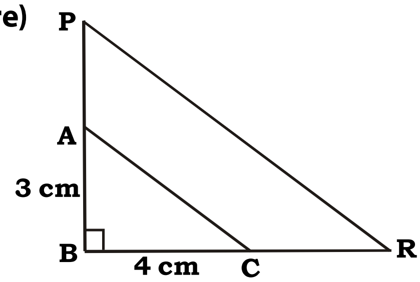
∴ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{A(\Delta ADE)}{A(\Delta CDE)}$ .....(iv) [From (i), (ii) and (iii)] ½

∴ $\frac{AD}{DB} = \frac{AE}{EC}$ [From (i), (ii) and (iv)] ½

(ii)



(Rough Figure)



1 mark for ΔABC
1 mark for constructing 7 congruent parts
1 mark for constructing $\angle CB_5B \cong \angle RB_7B$
1 mark for constructing $\angle ACB \cong \angle PRB$
1 mark for required ΔPRB

(iii)

Height of the cylindrical container (h) = 14cm

Its radius (r) = 6 cm

Volume of cylindrical container = $\pi r^2 h$

$= \pi \times 6 \times 6 \times 14$

$= 504\pi \text{ cm}^3$ ½

But, volume of ink filled in the cylindrical container = 91% of 504 π

$= \frac{91}{100} \times 504\pi \text{ cm}^3$ ½

Length of ball pen refill (h_1) = 12m

its inner diameter = 2 mm

$\therefore$ Its radius (r_1) = 1 mm ½

$= \frac{1}{10}$ cm

Volume of the refill = $\pi r_1^2 h_1$ ½

$= \pi \times \frac{1}{10} \times \frac{1}{10} \times 12$

$= \frac{12\pi}{100}$ cm³ ½

∴ But, volume of ink filled $= 84\%$ of $\frac{12\pi}{100}$ ½

$= \frac{84}{100} \times \frac{12\pi}{100}$ cm³ ½

Number of refills that can be filled with ink

$= \frac{\text{Volume of ink filled in the cylindrical container}}{\text{Volume of ink filled in each refill}}$ ½

$= \dfrac{\frac{91 \times 504\pi}{100}}{\frac{84 \times 12\pi}{100 \times 100}}$

$= \frac{91 \times 504\pi}{100} \times \frac{100 \times 100}{84 \times 12\pi}$ ½

$= 4550$

∴ Number of refills that can be filled with this ink is 4550. ½

❖❖❖❖