MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 3 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

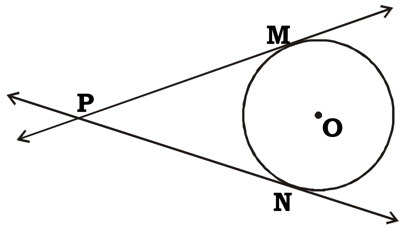
Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY FIVE of the following : **5**

(i) Lines PM and PN are tangents to the circle with centre O. If PM = 7 cm, find PN.



(ii) The radius of the base of a cone is 7 cm and its height is 24 cm. What is its slant height ?

(iii) State the value of tan (– 60).

(iv) The slope of line AB is $\frac{2}{3}$. What is the slope of line DE which is parallel to line AB ?

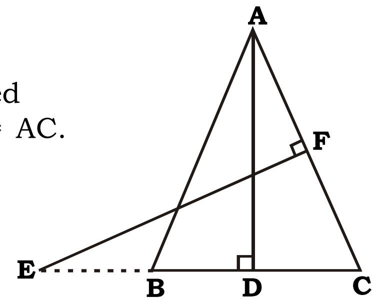
(v) Radius of a circle is 10 cm. The length of an arc of this circle is 25 cm. What is the area of the sector ?

(vi) Find tan θ, for the angle θ, whose terminal arm passes through (3, 4).

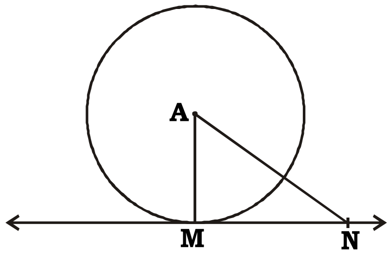
Q.2. Solve ANY FOUR of the following : **8**

(i) The ratio of the areas of two triangles with the common base is 6 : 5. Height of the larger triangle is 9 cm. Then find the corresponding height of the smaller triangle.

(ii) In the adjoining figure, E is a point on side CB produced of an isosceles ΔABC with AB = AC. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.



(iii) In the adjoining figure, point A is the centre of the circle. AN = 10 cm. Line NM is tangent at M. Determine the radius of the circle if MN = 5 cm.



(iv) Draw a circle of radius 3.6 cm, take a point M on it. Draw a tangent to the circle at M without using centre of the circle.

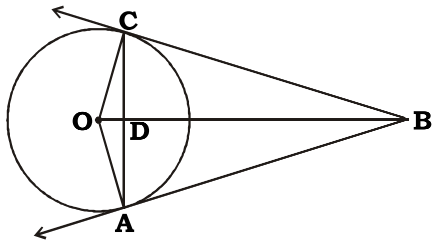
(v) If the terminal arm passes through the point (1, –1) making an angle find the value of sec θ.

(vi) Eliminate θ, if $x = a \sec \theta$, $y = b \tan \theta$

Q.3. Solve ANY THREE of the following : 9

(i) Find the length of the altitude of an equilateral triangle, each side measuring 'a' units.

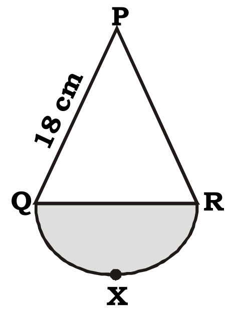
(ii) In the adjoining figure, BC and BA are tangents to circle. Prove that OD is perpendicular bisector of AC, where O is the centre of the circle.



(iii) Construct the incircle of ΔRST in which RS = 6 cm, ST = 7 cm and RT = 6.5 cm.

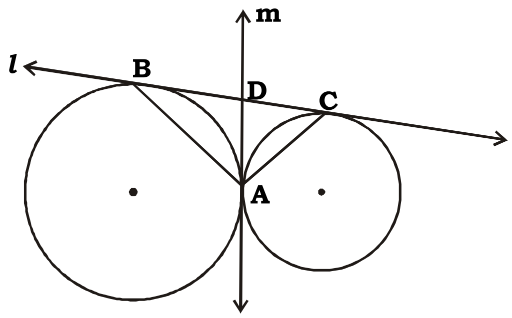
(iv) Find x if the slope of line joining (x, – 2) and (8, – 11) is $\frac{-3}{4}$.

(v) In the adjoining figure, P is the centre of the circle with radius 18 cm. If the area of the ΔPQR is 100 cm^2 and area of the segment QXR is 13.04 cm^2. Find the central angle θ. (π = 3.14)



Q.4. Solve ANY TWO of the following : 8

(i) In the adjoining figure, point A is a common point of contact of two externally touching circles and line l is a common tangent to both circles touching at B and C. Line m is another common tangent at A and it intersects BC at D. Prove that (i) $\angle BAC = 90°$ (ii) Point D is the midpoint of seg BC.



(ii) If P (– 2, 4), Q (4, 8), R (10, 5) and S (4, 1) are the vertices of a quadrilateral show that it is a parallelogram.

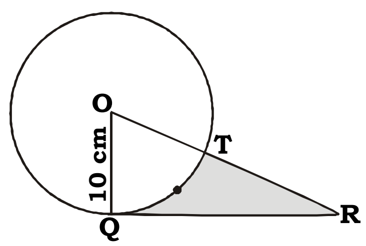
(iii) Eliminate θ, if $x = 3 \operatorname{cosec} \theta + 4 \cot \theta$, $y = 4 \operatorname{cosec} \theta - 3 \cot \theta$

Q.5. Solve ANY TWO of the following : 10

(i) **Prove :** In a triangle, the angle bisector divides the side opposite to the angle in the ratio of the remaining sides.

(ii) $\Delta SHR \sim \Delta SVU$, In ΔSHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and $\frac{SH}{SV} = \frac{3}{5}$; construct ΔSVU.

(iii) In the adjoining figure, seg QR is a tangent to the circle with centre O. Point Q is the point of contact. Radius of the circle is 10 cm. OR = 20 cm. Find the area of the shaded region. (π = 3.14, $\sqrt{3}$ = 1.73)



Best Of Luck

MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 3 (E)

Time : 2 Hours **Prelim - II Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i)
PM = PN [Length of the two tangent segments from an external point to a circle are equal] ½
But, PM = 7 cm [Given]
∴ PN = 7 cm ½

(ii)
Radius of base of cone (r) = 7 cm
its height (h) = 24 cm ½
$l^2 = r^2 + h^2$
∴ $l^2 = 7^2 + 21^2$
∴ $l^2 = 49 + 576$
∴ $l^2 = 625$
∴ $l = 25$ [Taking square roots]
∴ Slant height of cone is 25 cm ½

(iii)
tan (– 60)
= – tan 60 ½
= $-\sqrt{3}$
∴ tan (– 60) = $-\sqrt{3}$ ½

(iv)
line DE || line AB [Given]
∴ Slope of line DE = slope of line AB ½
But, slope of line AB = $\frac{2}{3}$ [Given]
∴ Slope of line DE = $\frac{2}{3}$ ½

(v)
Radius of circle (r) = 10 cm
Length of arc (l) = 25 cm
Area of sector = $l \times \frac{r}{2}$ ½

$= 25 \times \frac{10}{2}$

$= 25 \times 2$

$= 125 \text{ cm}^2$

$\therefore$ The area of the sector is 125 cm^2. ½

(vi) The terminal arm passes through (3, 4)

$\therefore$ $x = 3$

$y = 4$ ½

$\tan\theta = \frac{y}{x}$

$\therefore$ $\tan\theta = \frac{4}{3}$ ½

A.2. Solve ANY FOUR of the following :

(i) Let the areas of the larger and the smaller triangle be A_1 and A_2 respectively.

Let their heights be h_1 and h_2 respectively.

$\frac{A_1}{A_2} = \frac{6}{5}$ and $h_1 = 9$ cm [Given] ½

The two triangles have a common base [Given]

$\therefore$ $\frac{A_1}{A_2} = \frac{h_1}{h_2}$ [Triangles with common base] ½

$\therefore$ $\frac{6}{5} = \frac{9}{h_2}$

$\therefore$ $h_2 = \frac{5 \times 9}{6}$ ½

$\therefore$ $h_2 = \frac{15}{2}$

$\therefore$ $h_2 = 7.5$

$\therefore$ The corresponding height of the smaller triangle is 7.5 cm. ½

(ii) In ΔABC,

seg AB $\cong$ seg AC [Given]

$\therefore$ $\angle ABC \cong \angle ACB$ ......(i)

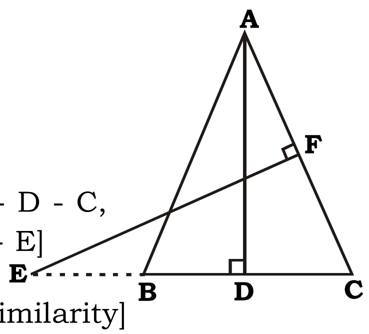
[Isosceles triangle theorem] 1

In ΔABD and ΔECF,

$\angle ABD \cong \angle FCE$ [From (i) and B - D - C, A - F - C, C - B - E]

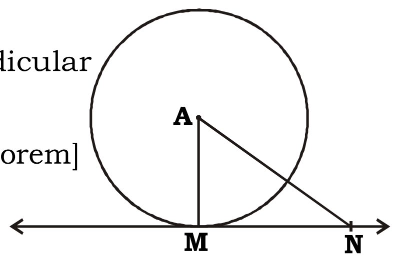
$\angle ADB \cong \angle EFC$ [$\because$ each is 90°]

$\therefore$ $\Delta ABD \sim \Delta ECF$ [By AA test of similarity] 1



(iii) In ΔAMN,

m $\angle AMN = 90^\circ$ [Radius is perpendicular to the tangent] ½



$\therefore$ $AN^2 = AM^2 + MN^2$ [By Pythagoras theorem] ½

$\therefore$ $10^2 = AM^2 + 5^2$ [Given]

$\therefore$ $100 = AM^2 + 25$

$\therefore$ $AM^2 = 100 - 25$ ½

$\therefore$ $AM^2 = 75$

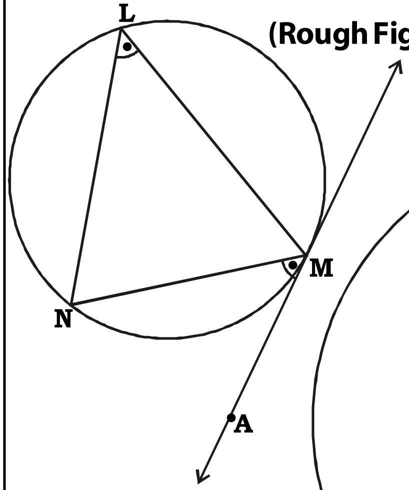
$\therefore$ $AM = \sqrt{75}$

$\therefore$ $AM = \sqrt{25 \times 3}$

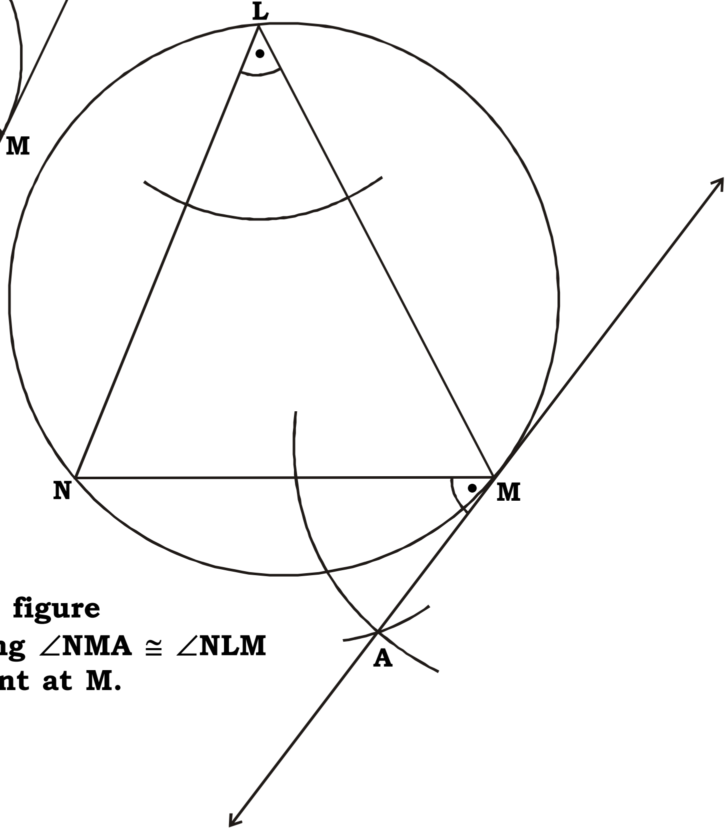
$\therefore$ $AM = 5\sqrt{3}$ cm.

$\therefore$ Radius of the circle is $5\sqrt{3}$ cm. ½

(iv)



(Rough Figure)



½ mark for rough figure
1 mark for drawing $\angle NMA \cong \angle NLM$
½ mark for tangent at M.

(v) The terminal arm passes through point (1, – 1)

$\therefore$ x = 1 and y = – 1

$r = \sqrt{x^2 + y^2}$ ½

$\therefore r = \sqrt{(1)^2 + (-1)^2}$

$\therefore r = \sqrt{1 + 1}$

$\therefore r = \sqrt{2}$ units ½

Let the angle be θ

$\sec\theta = \frac{r}{x}$ ½

$\therefore \sec\theta = \frac{\sqrt{2}}{1}$

$\therefore \boxed{\sec\theta = \sqrt{2}}$ ½

(vi) $x = a\sec\theta$

$\therefore \sec\theta = \frac{x}{a}$ ......(i) ½

$y = b\tan\theta$

$\therefore \tan\theta = \frac{y}{b}$ ......(ii) ½

$1 + \tan^2\theta = \sec^2\theta$

$\therefore 1 + \left(\frac{y}{b}\right)^2 = \left(\frac{x}{a}\right)^2$ [From (i) and (ii)] ½

$\therefore 1 + \frac{y^2}{b^2} = \frac{x^2}{a^2}$

$\therefore \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ½

A.3. Solve ANY THREE of the following :

(i) Given : ΔABC is an equilateral triangle.

AB = BC = AC = a ½

seg AD $\perp$ side BC

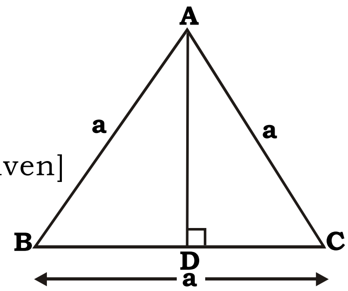
To find : AD

Sol. ΔABC is an equilateral triangle

AB = BC = AC = a ......(i) [Given] ½

In ΔADB,

m $\angle ADB = 90°$ [Given]



m ∠ABD = 60° [Angle of an equilateral triangle] ½

∴ m ∠BAD = 30° [Remaining angle]

∴ ΔADB is a 30° - 60° - 90° triangle

∴ By 30° - 60° - 90° triangle theorem, ½

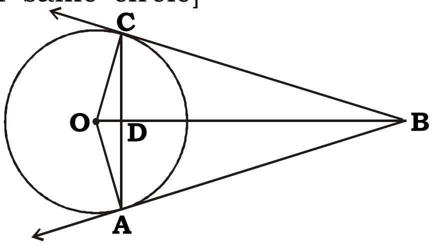
$AD = \frac{\sqrt{3}}{2} \times AB$ [Side opposite to 60°] ½

∴ $AD = \frac{\sqrt{3}}{2} \times a$ [From (i)]

∴ $\boxed{AD = \frac{\sqrt{3}}{2} a \text{ units.}}$ ½

(ii) OA = OC ....(i) [Radii of same circle] ½

BC = BA ....(ii)



[The lengths of the two tangent segments to a circle drawn from an external point are equal] ½

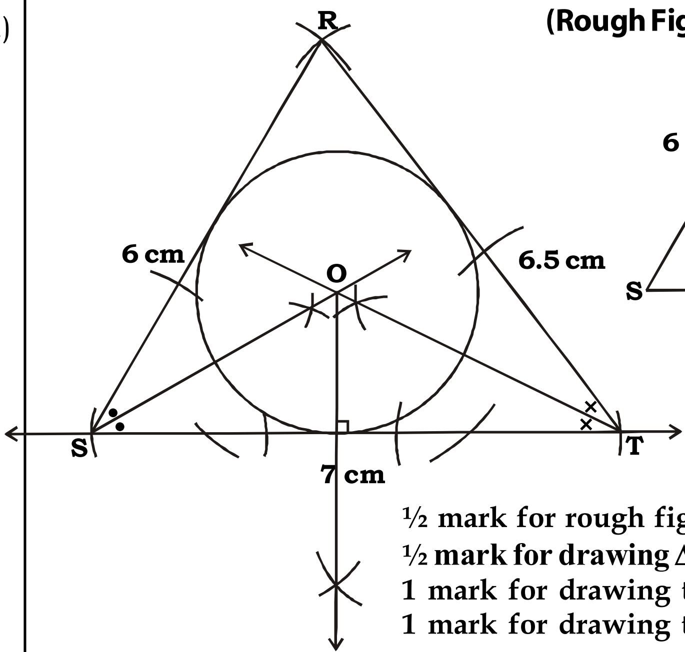
∴ Points O and B are equidistant from the end points A and C of seg AC. [From (i) and (ii)] ½

∴ Points O and B lie on the perpendicular bisector of seg AC. [By perpendicular bisector theorem] ½

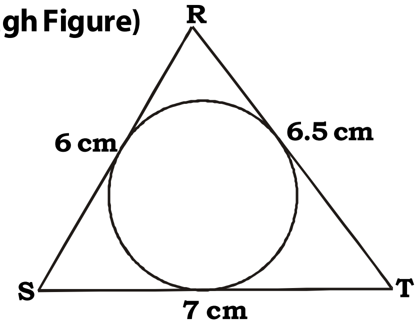
∴ seg OB is the perpendicular bisector of seg AC. ½

∴ seg OD is the perpendicular bisector of seg AC. [∵ O - D - B] ½

(iii)



(Rough Figure)



½ mark for rough figure
½ mark for drawing ΔRST
1 mark for drawing the angle bisectors
1 mark for drawing the incircle

(iv)

Let, A ≡ (x, – 2) ≡ (x_1, y_1) ½

B ≡ (8, – 11) ≡ (x_2, y_2)

Slope of line AB = $\frac{-3}{4}$ [Given]

Slope of line AB = $\frac{y_2 - y_1}{x_2 - x_1}$ ½

$\therefore \quad \frac{-3}{4} = \frac{-11-(-2)}{8-x}$

$\therefore \quad \frac{-3}{4} = \frac{-11+2}{8-x}$ ½

$\therefore \quad \frac{-3}{4} = \frac{-9}{8-x}$

$\therefore \quad 3\,(8 - x) = 9 \times 4$ ½

$\therefore \quad 24 - 3x = 36$

$\therefore \quad 3x = 24 - 36$

$\therefore \quad 3x = -12$

$\therefore \quad x = \frac{-12}{3}$ ½

$\therefore \quad x = -4$

∴ **The value of x is – 4.** ½

(v)

Radius of a circle (r) = 18 cm

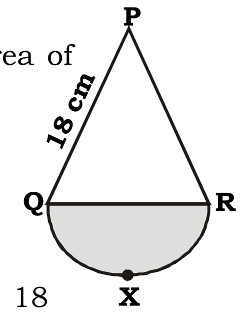
Area of ΔPQR = 100 cm^2

Area of the segment QXR = 13.04 cm^2

Area of sector P-QXR = Area of ΔPQR + Area of segment QXR 1

= 100 + 13.04

Area of sector P-QXR = 113.04 cm^2



Area of sector = $\frac{\theta}{360} \times \pi r^2$ ½

$\therefore \quad 113.04 = \frac{\theta}{360} \times 3.14 \times 18 \times 18$

$\therefore \quad 11304 = \frac{\theta}{360} \times 314 \times 18 \times 18$ ½

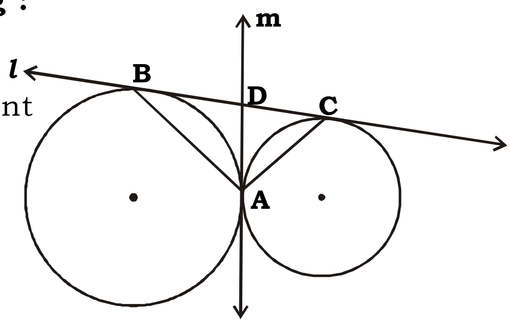
$\therefore \quad \frac{11304 \times 360}{314 \times 18 \times 18} = \theta$ ½

$\therefore \quad \theta = 40$

∴ **Central angle is 40°.** ½

A.4. Solve ANY TWO of the following :

(i)



In ΔBDA,

$DB = DA$ ....(i)

[The lengths of the two tangent segments from an external point to a circle are equal] ½

∴ $\angle DBA \cong \angle DAB$

[Isosceles triangle theorem]

Let,

$m\angle DBA = m\angle DAB = x^\circ$......(ii) ½

In ΔDAC,

$DA = DC$ ......(iii) [The lengths of the two tangent segments from an external point to a circle are equal] ½

∴ $\angle DAC \cong \angle DCA$ [Isosceles triangle theorem]

Let,

$m\angle DAC = m\angle DCA = y^\circ$ ......(iv) ½

$m\angle BAC = m\angle DAB + m\angle DAC$ [Angle Addition Property] ½

∴ $m\angle BAC = (x + y)^\circ$ ....(v) [From (ii) and (iv)]

In ΔABC,

$m\angle ABC + m\angle ACB + m\angle BAC = 180^\circ$

[∵ Sum of the measures of the angles of a triangle is 180°] ½

∴ $x + y + x + y = 180$ [From (ii), (iv), (v) and B - D - C]

∴ $2x + 2y = 180$

∴ $2(x + y) = 180$

∴ $x + y = \frac{180}{2}$

∴ $x + y = 90$

∴ $m\angle BAC = 90^\circ$ [From (v)] ½

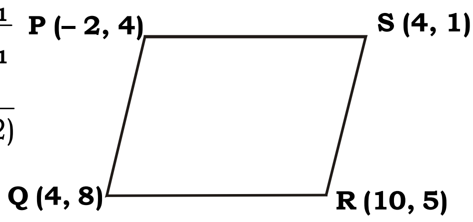
From (i) and (iii) we get,

$DB = DC$

∴ D is the midpoint of seg BC. ½

(ii)

$P \equiv (-2, 4)$, $Q \equiv (4, 8)$, $R \equiv (10, 5)$, $S \equiv (4, 1)$



Slope of a line $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

Slope of line PQ $= \frac{8 - 4}{4 - (-2)}$

$= \frac{4}{4 + 2}$

$= \frac{4}{6}$

	$\therefore$ Slope of line PQ $= \frac{2}{3}$	½
	Slope of line RS $= \frac{1-5}{4-10}$	
	$= \frac{-4}{-6}$	
	$\therefore$ Slope of line RS $= \frac{2}{3}$	½
	$\therefore$ Slope of line PQ = Slope of line RS	
	$\therefore$ line PQ || line RS ....(i)	
	Slope of line QR $= \frac{5-8}{10-4}$	
	$= \frac{-3}{6}$	
	$\therefore$ Slope of line QR $= \frac{-1}{2}$	½
	Slope of line PS $= \frac{1-4}{4-(-2)}$	
	$= \frac{-3}{4+2}$	
	$= \frac{-3}{6}$	
	$\therefore$ Slope of line PS $= \frac{-1}{2}$	½
	$\therefore$ Slope of line QR = Slope of line PS	
	$\therefore$ line QR || line PS ....(ii)	½
	In □PQRS,	
	side PQ || side RS [From (i)]	
	side QR || side PS [From (ii)]	
	$\therefore$ □PQRS is a parallelogram [By definition]	**1**
(iii)	$x = 3 \operatorname{cosec} \theta + 4 \cot \theta$ ......(i)	
	$y = 4 \operatorname{cosec} \theta - 3 \cot \theta$ .....(ii)	
	Multiplying (i) by 4,	
	$\therefore$ $4x = 12 \operatorname{cosec} \theta + 16 \cot \theta$ .....(iii)	½
	Multiplying (ii) by 3,	
	$\therefore$ $3y = 12 \operatorname{cosec} \theta - 9 \cot \theta$ .....(iv)	½
	Subtracting (iv) from (iii),	
	$4x - 3y = 12 \operatorname{cosec} \theta + 16 \cot \theta - (12 \operatorname{cosec} \theta - 9 \cot \theta)$	½
	$\therefore$ $4x - 3y = 12 \operatorname{cosec} \theta + 16 \cot \theta - 12 \operatorname{cosec} \theta + 9 \cot \theta$	
	$\therefore$ $4x - 3y = 25 \cot \theta$	

$\therefore \quad \cot\theta = \dfrac{4x - 3y}{25}$ ½

Substituting $\cot\theta = \dfrac{4x - 3y}{25}$ in equation (i)

$$x = 3\operatorname{cosec}\theta + 4\left(\frac{4x - 3y}{25}\right)$$

$\therefore \quad x = 3\operatorname{cosec}\theta + \dfrac{16x - 12y}{25}$ ½

$\therefore \quad x - \dfrac{16x - 12y}{25} = 3\operatorname{cosec}\theta$

$\therefore \quad \dfrac{25x - 16x + 12y}{25} = 3\operatorname{cosec}\theta$

$\therefore \quad \dfrac{9x + 12y}{25} = 3\operatorname{cosec}\theta$

$\therefore \quad \dfrac{3(3x + 4y)}{3 \times 25} = \operatorname{cosec}\theta$

$\therefore \quad \operatorname{cosec}\theta = \dfrac{3x + 4y}{25}$ ½

We know,

$\therefore \quad \operatorname{cosec}^2\theta - \cot^2\theta = 1$ ½

$\therefore \quad \left(\dfrac{3x + 4y}{25}\right)^2 - \left(\dfrac{4x - 3y}{25}\right)^2 = 1$

$\therefore \quad \dfrac{(3x + 4y)^2}{625} - \dfrac{(4x - 3y)^2}{625} = 1$

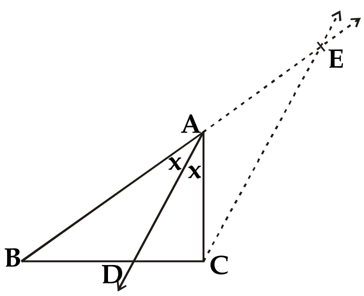
Multiplying throughout by 625,

$(3x + 4y)^2 - (4x - 3y)^2 = 625$ ½

A.5. Solve ANY TWO of the following :

(i) Given : In ΔABC, ray AD is the bisector of $\angle BAC$ such that B - D - C.

To Prove : $\dfrac{BD}{DC} = \dfrac{AB}{AC}$ ½



(½ marks for figure)

Construction : Draw a line passing through C, parallel to line AD and intersecting line BA at point E, B - A - E. ½

Proof : In ΔBEC,

line AD || side CE [Construction]

$\therefore \quad \dfrac{BD}{DC} = \dfrac{AB}{AE}$ ....(i) [By B.P.T.] ½

line CE || line AD [Construction]

$\therefore$ On transversal BE,

∠BAD ≅ ∠AEC ....(ii) [Converse of corresponding angles test] ½

Also, On transversal AC,

∠DAC ≅ ∠ACE ....(iii) [Converse of alternate angles test] ½

But, ∠BAD ≅ ∠DAC........(iv) [∵ ray AD bisects ∠BAC] ½

In ΔAEC,

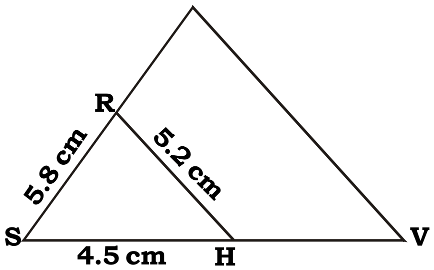
∠AEC ≅ ∠ACE [From (ii), (iii) and (iv)]

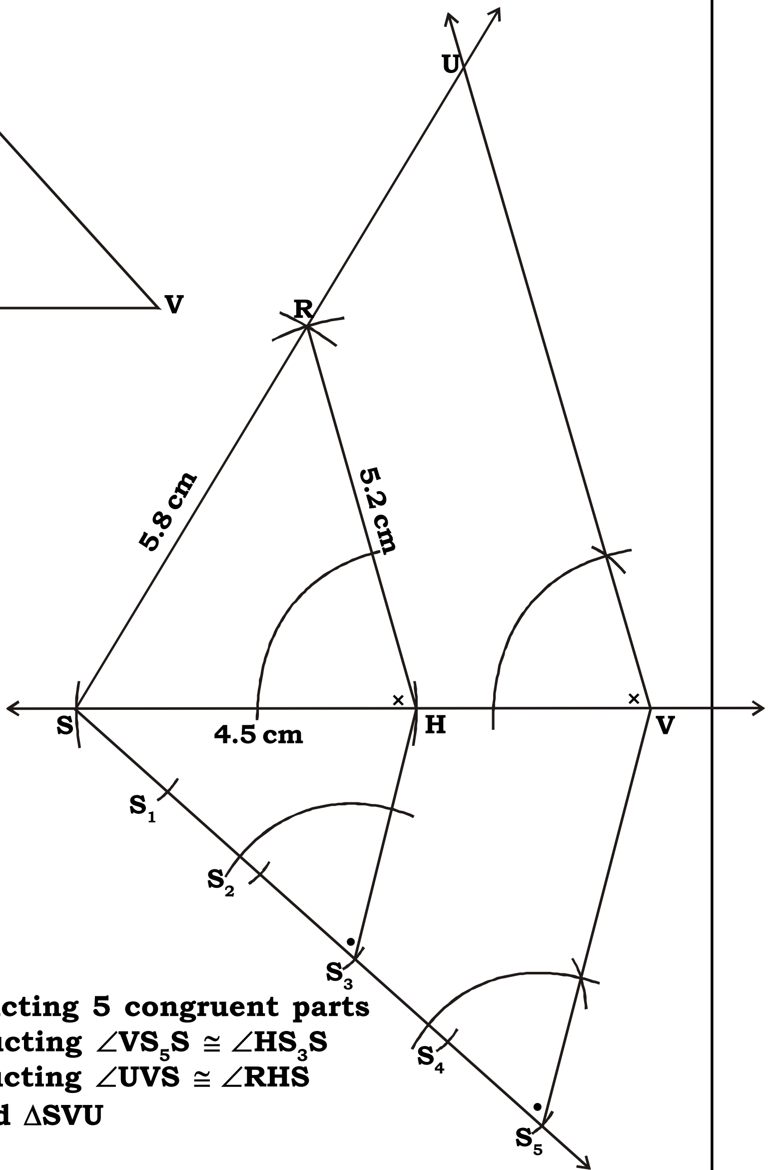
∴ seg AC ≅ seg AE [Converse of Isosceles triangle theorem]

∴ AC = AE ....(v) **1**

∴ $\frac{BD}{DC} = \frac{AB}{AC}$ [From (i) and (v)] ½

(ii) **(Rough Figure)**





1 mark for ΔSHR
1 mark for constructing 5 congruent parts
1 mark for constructing $\angle VS_5S \cong \angle HS_3S$
1 mark for constructing ∠UVS ≅ ∠RHS
1 mark for required ΔSVU

(iii) In ΔOQR,

$m\angle OQR = 90^\circ$ [Radius is perpendicular to the tangent] ½

$OQ^2 + QR^2 = OR^2$ [By Pythagoras theorem]

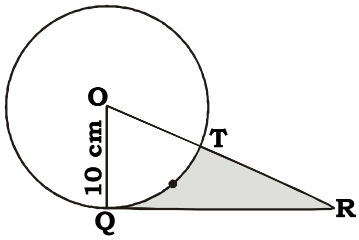
$\therefore 10^2 + QR^2 = 20^2$

$\therefore QR^2 = 400 - 100$

$\therefore QR^2 = 300$

$\therefore QR = \sqrt{300}$

$\therefore QR = \sqrt{100 \times 3}$ ½



$\therefore QR = 10\sqrt{3}$

$\therefore QR = 10\ (1.73)$

$\therefore QR = 17.3$ cm ½

Area of $\Delta OQR = \frac{1}{2} \times$ Product of Perpendicular sides

$= \frac{1}{2} \times OQ \times QR$

$= \frac{1}{2} \times 10 \times 17.3$

$= 86.5\ cm^2$ ½

In ΔOQR, $m\angle OQR = 90^\circ$

$OQ = 10$ cm

$OR = 20$ cm

$\therefore OQ = \frac{1}{2}\ OR$

$\therefore$ By converse of $30^\circ - 60^\circ - 90^\circ$ triangle theorem. ½

$\therefore m\angle ORQ = 30^\circ$

$\therefore m\angle QOR = 60^\circ$ [Remaining angle] ½

Now, For sector O-QXT

Measure of arc $(\theta) = 60^\circ$

Radius $(r) = 10$ cm

Area of Sector O-QXT $= \frac{\theta}{360} \times \pi r^2$ ½

$= \frac{60}{360} \times 3.14 \times 10 \times 10$

$= \frac{157}{3}$

Area of sector O-QXT $= 52.33\ cm^2$ ½

Area of shaded region = Area of ΔOQR – Area of sector O-QXT ½

$= 86.5 - 52.33$

$= 34.17\ cm^2$

$\therefore$ **Area of the shaded region is $34.17\ cm^2$** ½

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