MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 4 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY FIVE of the following : **5**

(i) Two circles with centres P and Q having diameter 25 cm and 15 cm respectively touch each other externally at A. Find the distance between P and Q.

(ii) The corresponding central angle of an arc is 90°. What is the length of this arc, if the radius of the circle is 14 cm ?

(iii) For the angle in standard position, if the initial arm rotates 340° in the anticlockwise direction, state the quadrant in which the terminal arm lies.

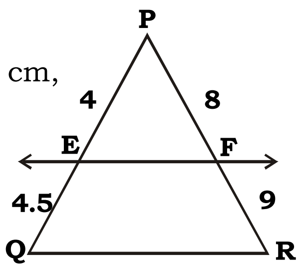
(iv) Find the slope of a line having inclination 60°.

(v) What is the volume of a cube with side 5 cm ?

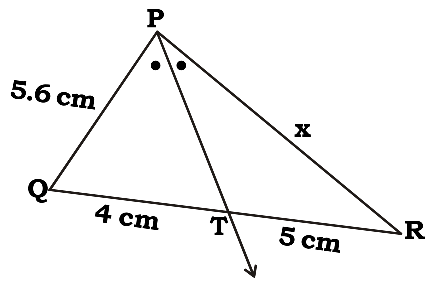
(vi) If $3 \sin \theta - 4 \cos \theta = 0$, what is the value of $\tan \theta$?

Q.2. Solve ANY FOUR of the following : **8**

(i) If PE = 4 cm, QE = 4.5 cm, PF = 8 cm, and RF = 9 cm, then state whether EF || QR.



(ii) Ray PT is the angle bisector of $\angle QPR$. Find the value of x and the perimeter of ΔPQR.



(iii) If two circles touch externally then show that the distance between their centers is equal to the sum of their radii.

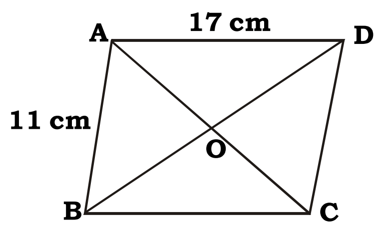
(iv) Draw a circle of radius 3.6 cm, take a point M on it. Draw a tangent to the circle at M without using centre of the circle.

(v) If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$.

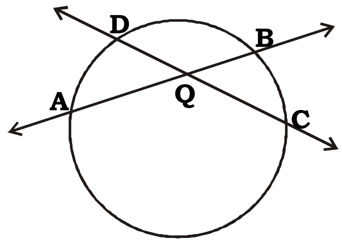
(vi) Eliminate θ, if $x = a \sec\theta$, $y = b \tan\theta$

Q.3. Solve ANY THREE of the following : 9

(i) Adjacent sides of a parallelogram are 11 cm and 17 cm. If the length of one of its diagonals is 26 cm. Find the length of the other.



(ii) Chords AB and CD of a circle intersect in point Q in the interior of a circle as shown in figure, if m (arc AD) = 25° and m (arc BC) = 31°, then find $\angle BQC$.



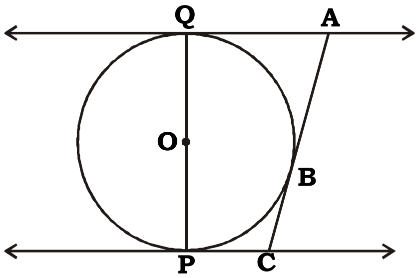
(iii) Construct incircle of ΔSGN such that SG = 6.7 cm, $\angle S = 70°$, $\angle G = 50°$ and draw incircle of ΔSGN.

(iv) Using slope concept, Check whether the points A (7, 8), B (– 5, 2) and C (3, 6) are collinear.

(v) The length, breadth and height of a cuboid are in the ratio 5:4:2. If the total surface area is 1216 cm^2, find the dimensions of the solid.

Q.4. Solve ANY TWO of the following : **8**

(i) In the adjoining figure, points P, B and Q are points of contact of the respective tangents. line QA is parallel to line PC. If QA = 7.2 cm, PC = 5 cm, find the radius of the circle.



(ii) If the points A (1, 2), B (4, 6), C (3, 5) are the vertices of a triangle ABC. Find the equation of the line passing through the mid points of AB and AC.

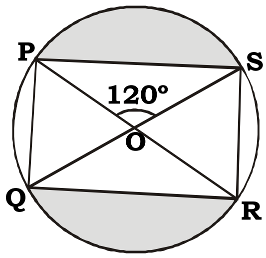
(iii) Two buildings are in front of each other on either side of a road of width 10 metres. From the top of the first building, which is 30 metres high, the angle of elevation of the top of the second is 45°. What is the height of the second building ?

Q.5. Solve ANY TWO of the following : **10**

(i) Prove : In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides.

(ii) Draw a triangle ABC, right angled at B such that, AB = 3 cm and BC = 4 cm. Now construct a triangle similar to ΔABC, each of whose sides is $\frac{7}{5}$ times the corresponding side of ΔABC.

(iii) In the adjoining figure,
PR and QS are two diameters of the circle.
If PR = 28 cm and PS = $14\sqrt{3}$ cm, find
(i) Area of triangle OPS
(ii) The total area of two shaded segments.
($\sqrt{3}$ = 1.73)



Best Of Luck

MT

2014 ___ ___ 1100 **Seat No.**

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 4 (E)

Time : 2 Hours **Prelim - II Model Answer Paper** **Max. Marks : 40**

A.1.	**Attempt ANY FIVE of the following :**	
(i)	Diameter of bigger circle = 25 cm	
	∴ Its radius (r_1) = 12.5 cm	
	Diameter of smaller circle = 15 cm	
	∴ Its radius (r_2) = 7.5 cm	½
	The two circles with centres P and Q touch each other externally at A.	
	∴ Distance between P and Q = $r_1 + r_2$	
	= 12.5 + 7.5	
	= 20 cm	
	∴ **The distance between P and Q is 20 cm.**	½
(ii)	Measure of central angle (θ) = 90°	
	Radius (r) = 14 cm	
	Length of the arc (l) = $\frac{\theta}{360} \times 2\pi r$	½
	∴ $l = \frac{90}{360} \times 2 \times \frac{22}{7} \times 14$	
	∴ l = 22	
	∴ **Length of the arc is 22 cm.**	½
(iii)	For the standard angle, if the initial arm rotates 340° in the anticlockwise direction then the terminal arm lies in the IV quadrant.	1
(iv)	Inclination of the line (θ) = 60°	
	∴ Slope of the line = tan θ	½
	= tan 60	
	= $\sqrt{3}$	
	∴ **Slope of the line is $\sqrt{3}$.**	½

(v)

Side of a cube (l) = 5 cm

Volume of cube = l^3

= $(5)^3$

= $125\ cm^3$ ½

$\therefore$ **Volume of the cube is $125\ cm^3$.** ½

(vi)

$3 \sin \theta - 4 \cos \theta = 0$

$\therefore\ 3 \sin \theta = 4 \cos \theta$ ½

$\therefore\ \dfrac{\sin \theta}{\cos \theta} = \dfrac{4}{3}$

$\therefore\ \boxed{\tan \theta = \dfrac{4}{3}}$ ½

A.2. Solve ANY FOUR of the following :

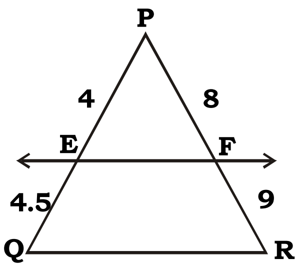
(i)

$\dfrac{PE}{EQ} = \dfrac{4}{4.5}$

$\therefore\ \dfrac{PE}{EQ} = \dfrac{4 \times 10}{4.5 \times 10}$

$\therefore\ \dfrac{PE}{EQ} = \dfrac{40}{45}$

$\therefore\ \dfrac{PE}{EQ} = \dfrac{8}{9}$ ......(i) ½



$\dfrac{PF}{FR} = \dfrac{8}{9}$ ......(ii) ½

In ΔPQR,

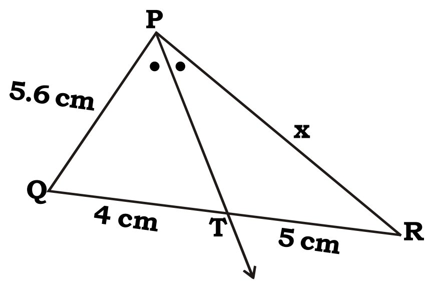
$\dfrac{PE}{EQ} = \dfrac{PF}{FR}$ [From (i) and (ii)] ½

$\therefore$ line EF || side QR [By converse of B.P.T.] ½

(ii)

In ΔPQR,

ray PT bisects $\angle QPR$ [Given]



$\therefore\ \dfrac{PQ}{PR} = \dfrac{QT}{TR}$ ½

[Property of angle bisector of a triangle]

$\therefore\ \dfrac{5.6}{x} = \dfrac{4}{5}$

$\therefore\ x = \dfrac{5 \times 5.6}{4}$ ½

∴ x = 7

∴ PR = 7 cm

QR = QT + TR [∵ Q - T - R]

∴ QR = 4 + 5 ½

∴ QR = 9 cm

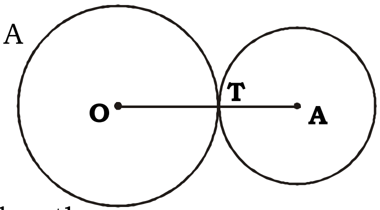
Perimeter of ΔPQR = PQ + QR + PR

= 5.6 + 9 + 7

∴ **Perimeter of ΔPQR = 21.6 cm** ½

(iii) Given : Two circles with centres O and A touch each other externally at point T.

(½ mark for figure) ½



To Prove : OA = OT + AT

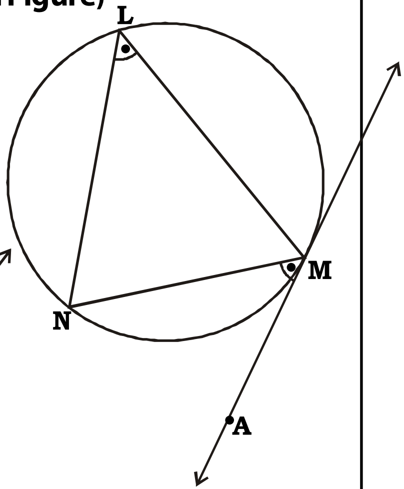
Proof : O - T - A

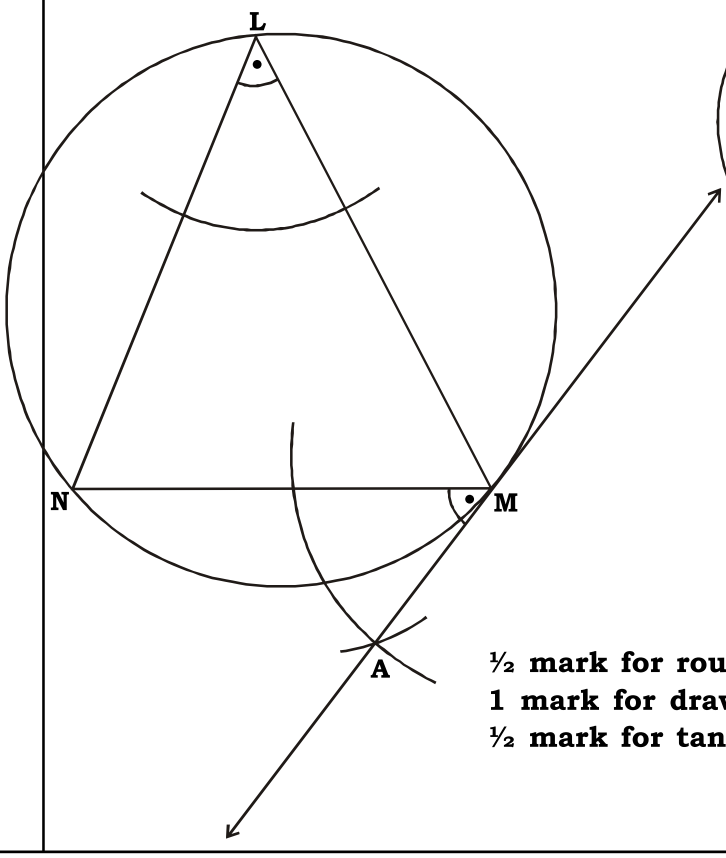
[If two circles are touching circles then the common point lies on the line joining their centres] ½

∴ OA = OT + AT [∵ O - T - A] ½

(iv)

(Rough Figure)





½ mark for rough figure

1 mark for drawing ∠NMA ≅ ∠NLM

½ mark for tangent at M.

(v)

$\sin\theta + \sin^2\theta = 1$ [Given]

$\therefore \sin\theta = 1 - \sin^2\theta$ ½

$\therefore \sin\theta = \cos^2\theta$ $\left[\begin{array}{l}\sin^2\theta + \cos^2\theta = 1\\ \therefore 1 - \sin^2\theta = \cos^2\theta\end{array}\right]$ ½

$\therefore \sin^2\theta = \cos^4\theta$ [Squaring both sides] ½

$\therefore 1 - \cos^2\theta = \cos^4\theta$ $\left[\begin{array}{l}\sin^2\theta + \cos^2\theta = 1\\ \therefore 1 - \cos^2\theta = \sin^2\theta\end{array}\right]$ ½

$\therefore \cos^2\theta + \cos^4\theta = 1$

(vi)

$x = a\sec\theta$

$\therefore \sec\theta = \dfrac{x}{a}$ ......(i) ½

$y = b\tan\theta$

$\therefore \tan\theta = \dfrac{y}{b}$ ......(ii) ½

$1 + \tan^2\theta = \sec^2\theta$

$\therefore 1 + \left(\dfrac{y}{b}\right)^2 = \left(\dfrac{x}{a}\right)^2$ [From (i) and (ii)] ½

$\therefore 1 + \dfrac{y^2}{b^2} = \dfrac{x^2}{a^2}$

$\therefore \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ ½

A.3. Solve ANY THREE of the following :

(i)

□ABCD is a parallelogram [Given]

$OB = OD = \dfrac{1}{2} \times BD$ .....(i) [∵ Diagonals of parallelogram bisect each other] ½

$\therefore OB = OD = \dfrac{1}{2} \times 26$ [Given]

$\therefore OB = OD = 13$ cm ½

In ΔADB,

seg AO is the median [From (i) and by definition]

$\therefore AB^2 + AD^2 = 2AO^2 + 2OB^2$ [By Appollonius theorem] ½

$\therefore (11)^2 + (17)^2 = 2AO^2 + 2\,(13)^2$

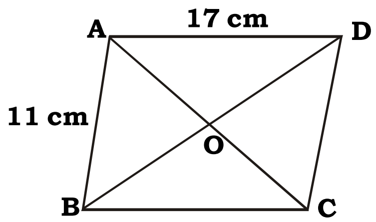
$\therefore 121 + 289 = 2AO^2 + 2\,(169)$

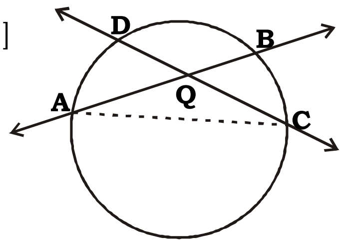
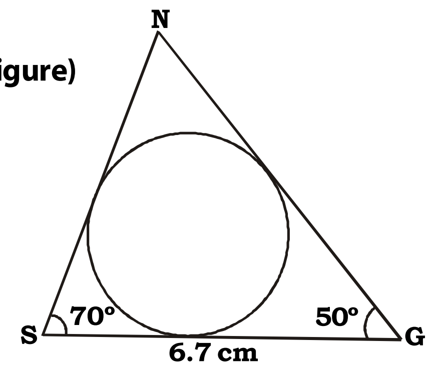
$\therefore 410 = 2AO^2 + 338$

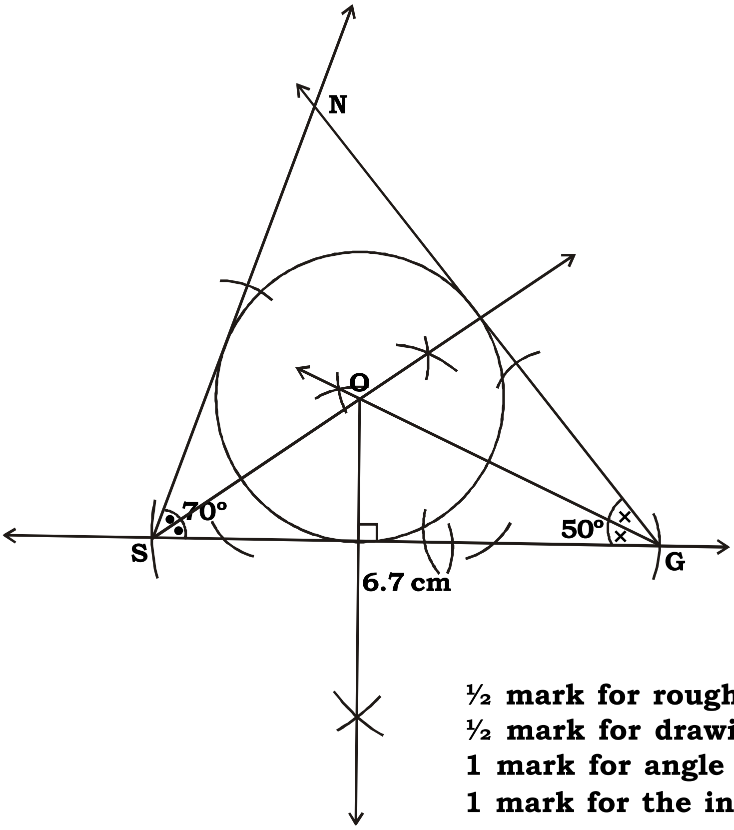
$\therefore 410 - 338 = 2AO^2$

$\therefore 72 = 2AO^2$

$\therefore AO^2 = 36$



	$\therefore$ AO = 6 cm [Taking square roots]	½
	AO = $\frac{1}{2}$ × AC [$\because$ Diagonals of parallelogram bisect each other]	½
	$\therefore$ 6 = $\frac{1}{2}$ × AC	
	$\therefore$ AC = 12 cm	
	$\therefore$ Length of other diagonal is 12 cm.	½
(ii)	Construction : Draw seg AC	½
	m (arc AD) = 25° [Given]	
	m $\angle$ACD = $\frac{1}{2}$ m(arc AD) [Inscribed angle theorem]	½
		
	$\therefore$ m $\angle$ACD = $\frac{1}{2}$ × 25°	
	$\therefore$ m $\angle$ACD = 12.5°	
	$\therefore$ m $\angle$ACQ = 12.5° [$\because$ D - Q - C]	
	m (arc BC) = 31° [Given]	½
	m $\angle$BAC = $\frac{1}{2}$ m(arc BC) [Inscribed angle theorem]	
	$\therefore$ m $\angle$BAC = $\frac{1}{2}$ × 31°	
	$\therefore$ m $\angle$BAC = 15.5°	
	$\therefore$ m $\angle$QAC = 15.5° [$\because$ A - Q - B]	½
	$\angle$BQC is an exterior angle of ΔAQC,	
	$\therefore$ m $\angle$BQC = m $\angle$QAC + m $\angle$ACQ [Remote interior angle theorem]	½
	$\therefore$ m $\angle$BQC = 15.5° + 12.5°	
	$\therefore$ m $\angle$BQC = 28°	½
(iii)	**(Rough Figure)** 	



½ mark for rough figure
½ mark for drawing ΔSGN
1 mark for angle bisectors
1 mark for the incircle

(iv)

A ≡ (7, 8) ≡ (x_1, y_1)
B ≡ (– 5, 2) ≡ (x_2, y_2)
C ≡ (3, 6) ≡ (x_3, y_3) ½

Slope of line AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$ ½

$= \dfrac{2 - 8}{-5 - 7}$

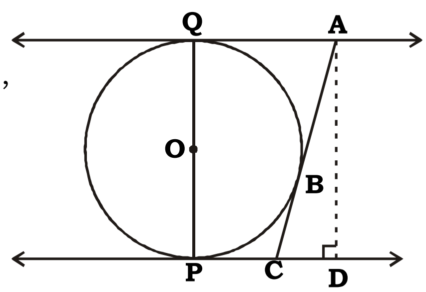
$= \dfrac{-6}{-12}$

$= \dfrac{1}{2}$ ½

Slope of line BC $= \dfrac{y_3 - y_2}{x_3 - x_2}$ ½

$= \dfrac{6 - 2}{3 - (-5)}$

$= \dfrac{4}{3 + 5}$

	$= \frac{4}{8}$ $= \frac{1}{2}$ ∴ Slope of line AB and slope of line BC are equal and point B is a common point for both the lines ∴ Points A, B and C are collinear.	½ ½
(v)	Ratio of the length, breadth and height of a cuboid is 5 : 4 : 2 Let the common multiple be 'x' ∴ Length of the cuboid = 5x cm its breadth = 4x cm and its height = 2x cm Total surface area of a cuboid = 1216 cm^2 Total surface area of a cuboid = $2 (lb + bh + lh)$ ∴ $1216 = 2 [(5x) (4x) + (4x) (2x) + (5x) (2x)]$ ∴ $\frac{1216}{2} = 20x^2 + 8x^2 + 10x^2$ ∴ $608 = 38x^2$ ∴ $\frac{608}{38} = x^2$ ∴ $x^2 = 16$ ∴ $x = 4$ [Taking square roots] Length of the cuboid = 5x = 5 (4) = 20 cm its Breadth = 4x = 4 (4) = 16cm and its height = 2x = 2 (4) = 8 cm ∴ Dimensions of the cuboid are 20 cm, 16 cm and 8 cm.	 ½ ½ ½ ½ ½ ½
A.4. (i)	**Solve ANY TWO of the following :** Construction : Draw seg AD ⊥ line PC, P - C - D  AQ = AB = 7.2 cm .....(i) CP = CB = 5 cm ....(ii) [The lengths of the two tangent segments to a circle drawn from an external point are equal]	 ½ ½

	$AC = AB + BC$ [A - B - C] $\therefore$ $AC = 7.2 + 5$ [From (i) and (ii)] $\therefore$ $AC = 12.2$ cm .....(iii) In $\square$AQPD, m $\angle AQP = 90^\circ$ m $\angle QPD = 90^\circ$ } [Radius is perpendicular to the tangent] m $\angle ADP = 90^\circ$ [Construction] $\therefore$ m$\angle QAD = 90^0$ [Remaining angle] $\therefore$ $\square$AQPD is a rectangle [By definition] $\therefore$ QA = PD = 7.2 cm ....(iv) [Opposite sides of a rectangle are congruent] PQ = AD .....(v) PD = PC + CD [P - C - D] $\therefore$ $7.2 = 5 + CD$ [From (ii) and (iv)] $\therefore$ $CD = 7.2 - 5$ $\therefore$ $CD = 2.2$ cm ....(vi) In ΔACD, m $\angle ADC = 90^\circ$ [Construction] $\therefore$ $AC^2 = AD^2 + CD^2$ [By Pythagoras theorem] $\therefore$ $(12.2)^2 = AD^2 + (2.2)^2$ [From (iii) and (vi)] $\therefore$ $AD^2 = (12.2)^2 - (2.2)^2$ $\therefore$ $AD^2 = (12.2 + 2.2)(12.2 - 2.2)$ $\therefore$ $AD^2 = 14.4 \times 10$ $\therefore$ $AD^2 = 144$ $\therefore$ $AD = 12$ cm [Taking square roots] $\therefore$ $PQ = 12$ cm [From (v)] PQ is a diameter of the circle $\therefore$ **Radius of the circle is 6 cm**	 ½ ½ ½ ½ ½ ½
(ii)	A ≡ (1, 2), B ≡ (4, 6), C ≡ (3, 5) Let D and E be the midpoints of sides AB and AC respectively Point D is the midpoint of side AB $\therefore$ $D \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $\equiv \left(\frac{1 + 4}{2}, \frac{2 + 6}{2}\right)$ $\equiv \left(\frac{5}{2}, \frac{8}{2}\right)$ $\equiv \left(\frac{5}{2}, 4\right)$	 ½ ½

Point E is the midpoint of side AC

$\therefore$ E $\equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ ½

$\equiv \left(\frac{1 + 3}{2}, \frac{2 + 5}{2}\right)$ ½

$\equiv \left(\frac{4}{2}, \frac{7}{2}\right)$

$\equiv \left(2, \frac{7}{2}\right)$ ½

The required line passes through points D and E

$\therefore$ The equation of the line by two point form,

$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$ ½

$\therefore \frac{x - 5/2}{5/2 - 2} = \frac{y - 4}{4 - 7/2}$

$\therefore \frac{(2x - 5)/2}{(5 - 4)/2} = \frac{y - 4}{(8 - 7)/2}$ ½

$\therefore \frac{2x - 5}{5 - 4} = \frac{(y - 4)}{1/2}$

$\therefore 2x - 5 = 2\ (y - 4)$

$\therefore 2x - 5 = 2y - 8$

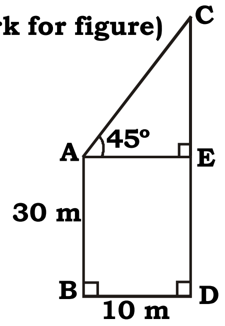
$\therefore 2x - 2y - 5 + 8 = 0$

$\therefore 2x - 2y + 3 = 0$

$\therefore$ **The equation of the required line is $2x - 2y + 3 = 0$** ½

(iii)

(½ mark for figure)



seg AB and seg CD represents the two buildings

AB = 30 m

seg BD represents the width of the road

BD = 10 m

A represents the position of observer. 1

$\angle$CAE is the angle of elevation

m $\angle$CAE = 45°

☐ABDE is a rectangle

AB = DE = 30 m

BD = AE = 10 m } [Opposite sides of rectangle]

In right angled ΔCEA,

$\tan 45° = \frac{CE}{AE}$ [By definition] ½

$\therefore \quad 1 = \dfrac{CE}{10}$

$\therefore \quad CE = 10$ m ½

$CD = CE + ED \quad [\because C - E - D]$ ½

$\therefore \quad CD = 10 + 30$

$\therefore \quad CD = 40$ m ½

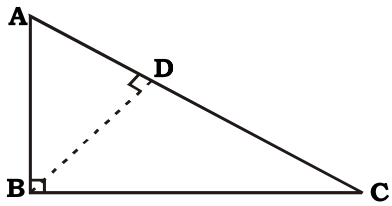
$\therefore$ **The height of second building is 40 m.** ½

A.5. Solve ANY TWO of the following :

(i) Given : In ΔABC, $m\angle ABC = 90°$ ½

To Prove : $AC^2 = AB^2 + BC^2$

Construction : Draw seg BD $\perp$ side AC such that A - D - C. ½



(½ mark for figure)

Proof : In ΔABC,

$m\angle ABC = 90°$ [Given]

seg BD $\perp$ hypotenuse AC [Construction]

$\therefore \quad \Delta ABC \sim \Delta ADB \sim \Delta BDC$ .....(i)

[Similarity in right angled triangles] 1

$\Delta ABC \sim \Delta ADB$ [From (i)]

$\therefore \quad \dfrac{AB}{AD} = \dfrac{AC}{AB}$ [Corresponding sides of similar triangles] ½

$\therefore \quad AB^2 = AC \times AD$ ....(ii)

$\Delta ABC \sim \Delta BDC$ [From (i)]

$\therefore \quad \dfrac{BC}{DC} = \dfrac{AC}{BC}$ [Corresponding sides of similar triangles]

$\therefore \quad BC^2 = AC \times DC$ ....(iii) ½

Adding (ii) and (iii) we get,

$AB^2 + BC^2 = AC \times AD + AC \times DC$ ½

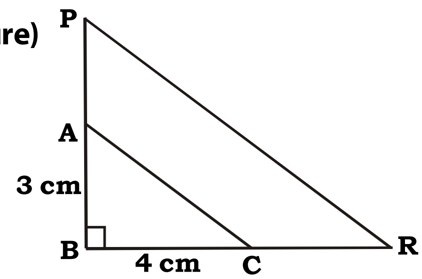
$\therefore \quad AB^2 + BC^2 = AC\,(AD + DC)$

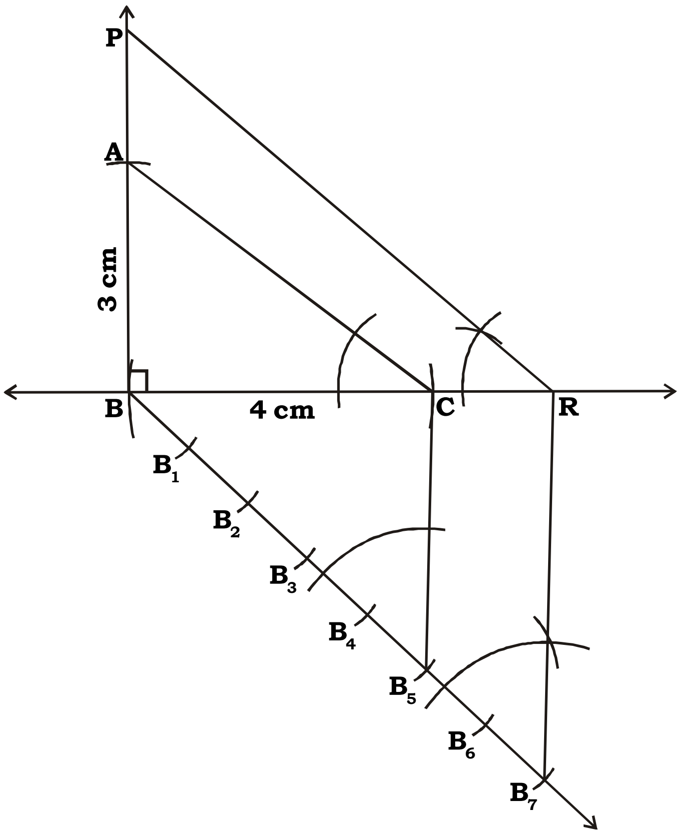
$\therefore \quad AB^2 + BC^2 = AC \times AC \quad [\because A - D - C]$ ½

$\therefore \quad AB^2 + BC^2 = AC^2$

$\therefore \quad AC^2 = AB^2 + BC^2$ ½

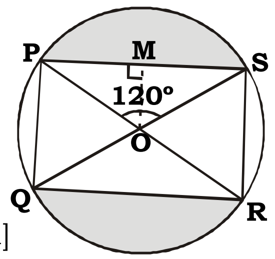
(ii) **(Rough Figure)**





1 mark for ΔABC
1 mark for constructing 7 congruent parts
1 mark for constructing $\angle CB_5B \cong \angle RB_7B$
1 mark for constructing $\angle ACB \cong \angle PRB$
1 mark for required ΔPRB

(iii) (i) Draw seg OM $\perp$ side PS ½



OP = $\frac{1}{2} \times$ PR [Radius is half of diameter]

$\therefore$ OP = $\frac{1}{2} \times 28$

$\therefore$ OP = 14 cm ½

seg OM $\perp$ chord PS [By construction]

$\therefore$ PM = $\frac{1}{2} \times$ PS [The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\therefore$ PM = $\frac{1}{2} \times 14\sqrt{3}$

$\therefore$ PM = $7\sqrt{3}$ cm ½

In ΔOMP,

$\angle$OMP = 90° [By construction]

$\therefore$ $OM^2 + PM^2 = OP^2$ [By Pythagoras theorem]

$\therefore$ $OM^2 + (7\sqrt{3})^2 = 14^2$

$\therefore$ $OM^2 = 196 - 147$

$\therefore$ $OM^2 = 49$

$\therefore$ OM = 7 cm [Taking square roots] ½

Area of ΔOPS $= \frac{1}{2} \times$ base $\times$ height ½

Area of ΔOPS $= \frac{1}{2} \times PS \times OM$

$= \frac{1}{2} \times 14\sqrt{3} \times 7$

$= 49\sqrt{3}$

$= 49\ (1.73)$

$\therefore$ Area of ΔOPS = 84.77 cm² ½

Area of sector (O-PS) $= \frac{\theta}{360} \times \pi r^2$ ½

$= \frac{120}{360} \times \frac{22}{7} \times 14 \times 14$

$= \frac{616}{3}$

$= 205.33$ cm² ½

(ii) Area of segment PS = (Area of sector O-PS) – (Area of ΔOPS)

= 205.33 – 84.77

= 120.56 cm² ½

Similarly we can prove,

Area of segment QR = 120.56 cm²

$\therefore$ Total area of two shaded segments

= 120.56 + 120.56 = 241.12 cm²

Area of ΔOPS is 84.77 cm² and total area of two shaded segments is 241.12 cm². ½

❖❖❖❖