MT

2014 ___ ___ 1100 **Seat No.** [| | | | | |]

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 5 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY FIVE of the following : **5**

(i) O is the centre of the circle. AB is the longest chord of the circle. If AB = 8.6 cm, what is the radius of the circle ?

(ii) A cylinder and a cone have equal radii and equal heights. If the volume of the cylinder is 300 cm^3, what is the volume of the cone ?

(iii) If $\sec \theta = 2$, what is the value of $\tan^2 \theta$?

(iv) What is the equation of a line whose slope is -2 and y-intercept is 3 ?

(v) The dimensions of a cuboid are 5 cm, 4 cm and 3 cm. Find its volume.

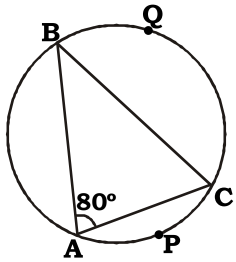
(vi) If x co-ordinate of point A is negative and y co-ordinate is positive, then in which quadrant point A lies ?

Q.2. Solve ANY FOUR of the following : **8**

(i) A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

(ii) Sides of a triangle are 40, 20 and 30. Determine whether they are sides of a right angled triangle.

(iii) In the adjoining figure, if m (arc APC) = 60° and m ∠BAC = 80° Find (a) ∠ABC (b) m (arc BQC).



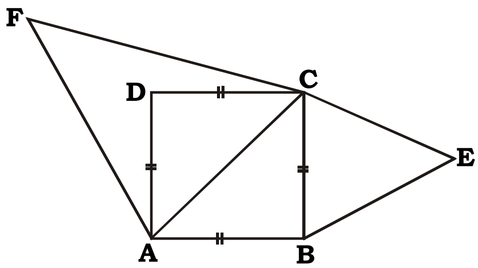
(iv) Draw a circle of radius 2.6 cm. Draw tangent to the circle from any point on the circle using centre of the circle.

(v) If $\tan A + \frac{1}{\tan A} = 2$, show that $\tan^2 A + \frac{1}{\tan^2 A} = 2$.

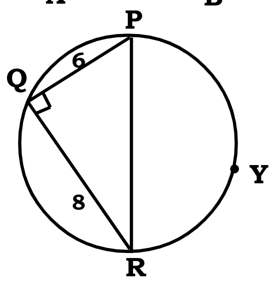
(vi) If $x = a \sin \theta$, $y = b \tan \theta$ then prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

Q.3. Solve ANY THREE of the following : 9

(i) In the adjoining figure, □ABCD is a square. The ΔBCE on side BC and ΔACF on the diagonal AC are similar to each other. Then show that A (ΔBCE) = $\frac{1}{2}$ A (ΔACF)



(ii) Find the radius of the circle passing through the vertices of a right angled triangle when lengths of perpendicular sides are 6 and 8.



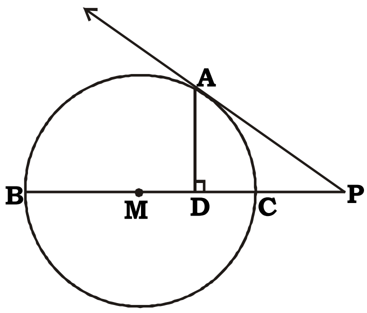
(iii) Construct the incircle of ΔSRN, such that RN = 5.9 cm, RS = 4.9 cm, ∠R = 95°.

(iv) Find the value of k, if (– 3, 11), (6, 2) and (k, 4) are collinear points.

(v) A building has 8 right cylindrical pillars whose cross sectional diameter is 1 m and whose height is 4.2 m. Find the expenditure to paint those pillars at the rate of Rs. 24 per m^2.

Q.4. Solve ANY TWO of the following : 8

(i) In the adjoining figure, BC is a diameter of the circle with centre M. PA is a tangent at A from P which is a point on line BC. AD ⊥ BC. Prove that $DP^2 = BP \times CP - BD \times CD$.



(ii) Lines x = 5 and y = 4 form a rectangle with co-ordinate axes. Find the equation of the diagonals.

(iii) If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Q.5. Solve ANY TWO of the following : 10

(i) Prove : In a triangle, the angle bisector divides the side opposite to the angle in the ratio of the remaining sides.

(ii) $\Delta SHR \sim \Delta SVU$, In ΔSHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and $\frac{SH}{SV} = \frac{3}{5}$; construct ΔSVU.

(iii) The diameter of the base of metallic cone is 2 cm and height is 10 cm. 900 such cones are molten to form 1 right circular cylinder whose radius is 10 cm. Find total surface area of the right circular cylinder so formed. (Given $\pi = 3.14$)

Best Of Luck

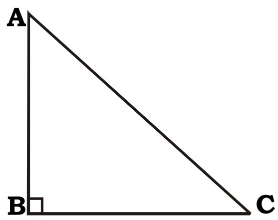
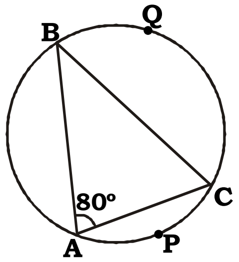
MT

2014 ___ ___ 1100 **Seat No.**

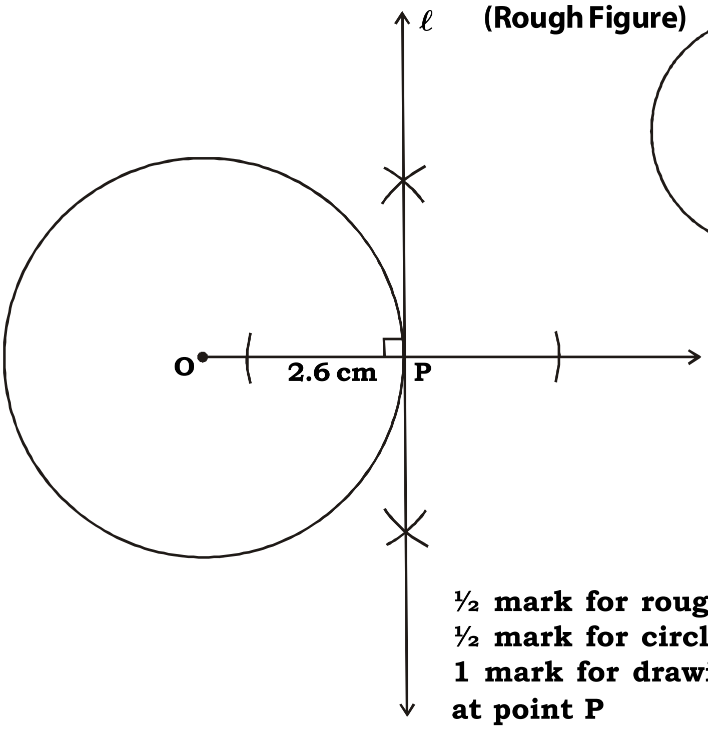
MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 5 (E)

Time : 2 Hours **Prelim - II Model Answer Paper** **Max. Marks : 40**

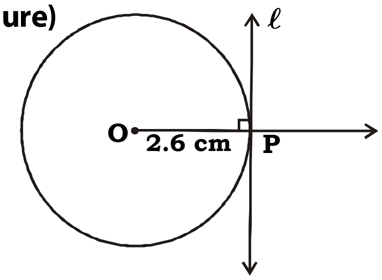
A.1.	**Attempt ANY FIVE of the following :**	
(i)	AB is the longest chord of the circle [Given]	
	But, diameter is the longest chord of the circle	
	∴ seg AB is the diameter of the circle	
	Diameter = 8.6 cm	
	∴ Radius = 4.3 cm	1
(ii)	A cylinder and cone have equal height and equal radii	
	∴ Volume of cone $= \frac{1}{3} \times$ volume of cylinder	½
	$= \frac{1}{3} \times 300$	
	$= 100\ cm^3$	
	∴ Volume of the cone is $100\ cm^3$.	½
(iii)	$\sec\theta = 2$ [Given]	
	But, $\sec 60 = 2$	½
	∴ $\sec\theta = \sec 60$	
	∴ $\theta = 60°$	½
(iv)	Slope (m) = – 2	
	y intercept (c) = 3	
	∴ Equation of the line by slope-intercept form is $y = mx + c$	½
	∴ $y = -2x + 3$	
	∴ $2x + y - 3 = 0$	½
(v)	Length of a cuboid (l) = 5 cm	
	Its breadth (b) = 4 cm	
	Its height (h) = 3 cm	
	Volume of a cuboid $= l \times b \times h$	½
	$= 5 \times 4 \times 3$	
	$= 60\ cm^3$	
	∴ Volume of the cuboid is $60\ cm^3$.	½

(vi)	If x co-ordinate of point A is negative and y co-ordinate is positive. Then, point A lies in the II quadrant.	1
A.2.	**Solve ANY FOUR of the following :**	
(i)	In the adjoining figure, seg AB represents the wall seg AC represents the ladder seg BC represents the distance of the foot of the ladder from the base of the wall AC = 10 m AB = 8 m In ΔABC, m $\angle ABC = 90^\circ$ [Given]	½
	 **(½ mark for figure)**	
	∴ $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]	½
	∴ $(10)^2 = (8)^2 + BC^2$ ∴ $100 = 64 + BC^2$ ∴ $BC^2 = 100 - 64$ ∴ $BC^2 = 36$ ∴ BC = 6 m [Taking square roots]	
	∴ The distance of the foot of the ladder from the base of the wall is 6 m.	½
(ii)	$(40)^2 = 1600$ ......(i)	½
	$(20)^2 + (30)^2 = 400 + 900$ $= 1300$ ......(ii)	½
	∴ $(40)^2 \neq (20)^2 + (30)^2$ [From (i) and (ii)]	½
	∴ The given sides do not form a right angled triangle. [By Converse of Pythagoras theorem]	½
(iii)	(a) m $\angle ABC = \frac{1}{2}$ m(arc APC) [Inscribed angle theorem]	½
	 	
	∴ m $\angle ABC = \frac{1}{2} \times 60$	
	∴ m $\angle ABC = 30^\circ$	½
	(b) m $\angle BAC = \frac{1}{2}$ m(arc BQC) [Inscribed angle theorem]	½
	∴ $80 = \frac{1}{2}$ m(arc BQC) ∴ m(arc BQC) = 80×2	
	∴ m(arc BQC) = 160°	½

(iv)



(Rough Figure)



½ mark for rough figure
½ mark for circle
1 mark for drawing perpendicular at point P

(v)

$$\tan A + \frac{1}{\tan A} = 2$$

$$\therefore \left(\tan A + \frac{1}{\tan A}\right)^2 = 4 \quad \text{[Squaring both sides]}$$ ½

$$\therefore \tan^2 A + 2 \tan A \cdot \frac{1}{\tan A} + \frac{1}{\tan^2 A} = 4$$ ½

$$\therefore \tan^2 A + 2 + \frac{1}{\tan^2 A} = 4$$

$$\therefore \tan^2 A + \frac{1}{\tan^2 A} = 4 - 2$$ ½

$$\therefore \tan^2 A + \frac{1}{\tan^2 A} = 2$$ ½

(vi)

$$x = a \sin \theta$$

$$\therefore \frac{1}{\sin \theta} = \frac{a}{x}$$

$$\therefore \operatorname{cosec} \theta = \frac{a}{x} \quad \text{......(i)} \quad \left[\because \operatorname{cosec} \theta = \frac{1}{\sin \theta}\right]$$ ½

$$y = b \tan \theta$$

$\therefore \quad \dfrac{1}{\tan\theta} = \dfrac{b}{y}$

$\therefore \quad \cot\theta = \dfrac{b}{y}$ .....(ii) $\left[\because \cot\theta = \dfrac{1}{\tan\theta}\right]$ ½

We know,

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ ½

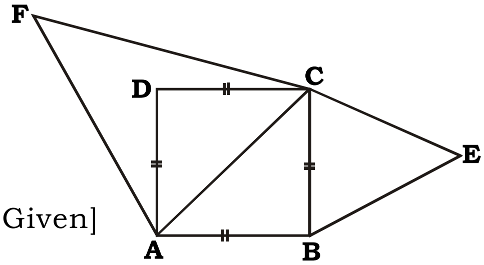
$\therefore \quad \operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\therefore \quad \left(\dfrac{a}{x}\right)^2 - \left(\dfrac{b}{y}\right)^2 = 1$ [From (i) and (ii)]

$\therefore \quad \dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = 1$ ½

A.3. Solve ANY THREE of the following :

(i)



$\Delta BCE \sim \Delta ACF$ [Given]

$\therefore \quad \dfrac{A(\Delta BCE)}{A(\Delta ACF)} = \dfrac{BC^2}{AC^2}$ ......(i) [Areas of similar triangles] ½

□ABCD is a square [Given]

$\therefore \quad AB = BC = CD = AD$ ......(ii) [Sides of a square] ½

In ΔABC,

$m\angle ABC = 90^\circ$ [Angle of a square]

$\therefore \quad AC^2 = AB^2 + BC^2$ [By Pythagoras theorem] ½

$\therefore \quad AC^2 = BC^2 + BC^2$ [From (i)] ½

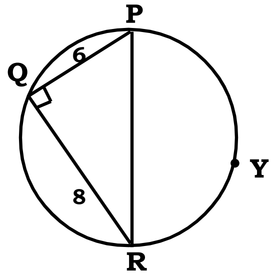
$\therefore \quad AC^2 = 2BC^2$ .....(iii)

$\therefore \quad \dfrac{A(\Delta BCE)}{A(\Delta ACF)} = \dfrac{BC^2}{2BC^2}$ [From (i) and (iii)] ½

$\therefore \quad \dfrac{A(\Delta BCE)}{A(\Delta ACF)} = \dfrac{1}{2}$

$\therefore \quad A(\Delta BCE) = \dfrac{1}{2}\, A(\Delta ACF)$ ½

(ii)



In ΔPQR,

$m\angle PQR = 90^\circ$ [Given]

$\therefore \quad PR^2 = PQ^2 + QR^2$ [By Pythagoras theorem] ½

$\therefore \quad PR^2 = 6^2 + 8^2$

$\therefore \quad PR^2 = 36 + 64$

$\therefore \quad PR^2 = 100$

$\therefore$ PR = 10 units [Taking square roots] ½

m $\angle$PQR = 90° [Given]

m $\angle$PQR = $\frac{1}{2}$ m(arc PYR) [Inscribed angle theorem] ½

$\therefore$ 90° = $\frac{1}{2}$ m(arc PYR)

$\therefore$ m (arc PYR) = 180° ½

$\therefore$ arc PYR is a semicircle

$\therefore$ seg PR is the diameter.

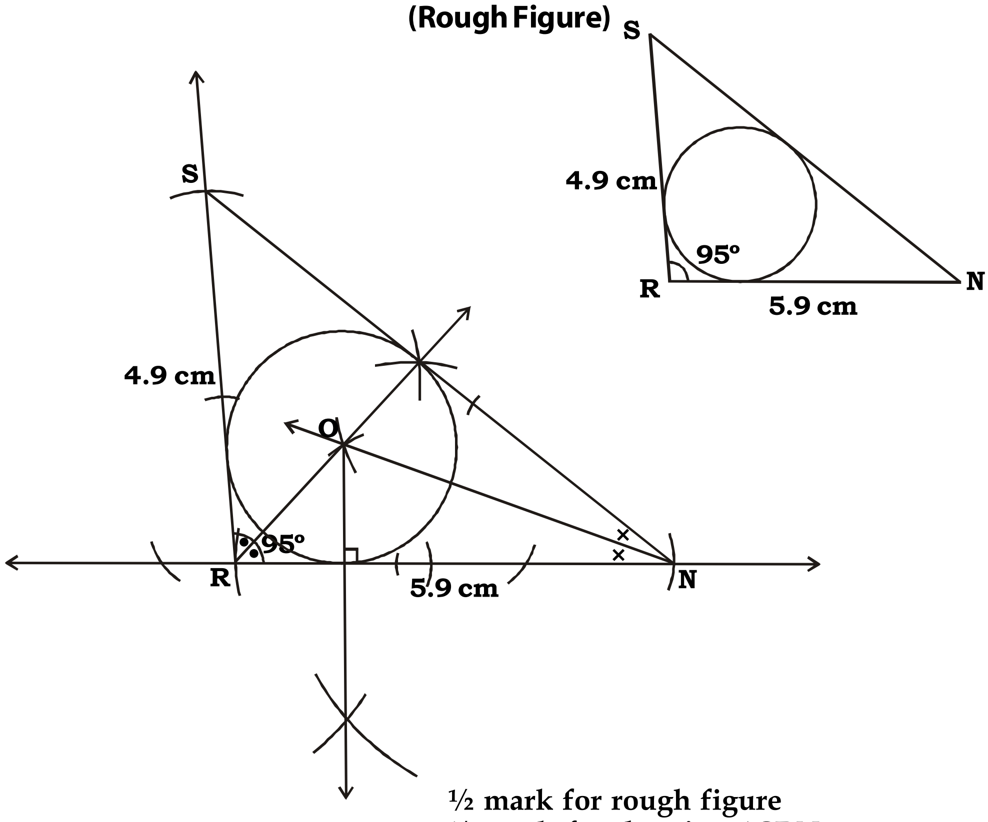
$\therefore$ Diameter = 10 units. ½

$\therefore$ Radius = 5 units [$\because$ Radius is half of the diameter]

$\therefore$ **Radius of the circle is 5 units.** ½

(iii)

(Rough Figure)

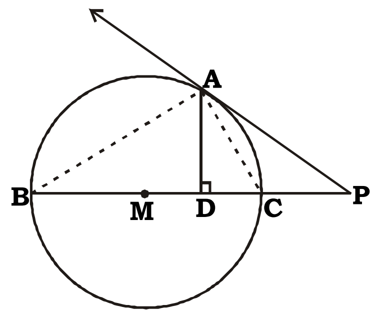


½ mark for rough figure
½ mark for drawing ΔSRN
1 mark for drawing the angle bisectors
1 mark for drawing the incircle

(iv)	Let, A ≡ (– 3, 11) ≡ (x_1, y_1)	½
	B ≡ (6, 2) ≡ (x_2, y_2)	
	C ≡ (k, 4) ≡ (x_3, y_3)	
	∵ Points A, B and C are collinear	
	∴ Slope of line AB = Slope of line BC	½
	∴ $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_3 - y_2}{x_3 - x_2}$	½
	∴ $\dfrac{2 - 11}{6 - (-3)} = \dfrac{4 - 2}{k - 6}$	
	∴ $\dfrac{-9}{6 + 3} = \dfrac{2}{k - 6}$	½
	∴ $\dfrac{-9}{9} = \dfrac{2}{k - 6}$	
	∴ $-1 = \dfrac{2}{k - 6}$	½
	∴ $-(k - 6) = 2$	
	∴ $-k + 6 = 2$	
	∴ $-k = 2 - 6$	
	∴ $-k = -4$	
	∴ $k = 4$	½
	∴ **The value of k is 4**	
(v)	Diameter of a pillar = 1 m	
	∴ Its radius (r) = $\left(\dfrac{1}{2}\right)$ m	½
	Its height (h) = 4.2 m	
	Curved surface area of a pillar = $2\pi rh$	½
	$= 2 \times \dfrac{22}{7} \times \dfrac{1}{2} \times 4.2$	
	$= 13.2\ m^2$	½
	∴ Curved surface area of 8 pillars = 8×13.2	
	$= 105.6\ m^2$	½
	Rate of painting = Rs. 24 per m^2	
	∴ Total expenditure = Area to be painted × Rate of painting	½
	= 105.6 × 24	
	= 2534.40	
	∴ **Total expenditure to paint the pillars is Rs. 2534.40.**	½

A.4. **Solve ANY TWO of the following :**

(i) Construction : Draw seg AB and seg AC ½



Proof : In ΔADP,

m ∠ADP = 90° [Given]

∴ $AP^2 = AD^2 + DP^2$ [By Pythagoras theorem]

∴ $DP^2 = AP^2 - AD^2$ ……(i) **1**

Line AP is a tangent to the circle at point A and line PCB is a secant to the circle at points C and B

∴ $AP^2 = BP \times CP$ ……(ii) **1**

In ΔBAC,

m ∠BAC = 90° [Angle subtended by semicircle]

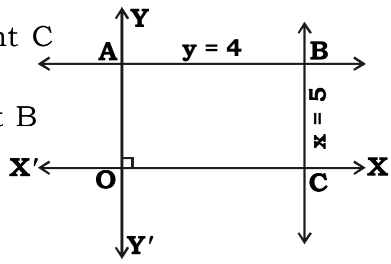
seg AD ⊥ hypotenuse BC

∴ $AD^2 = BD \times CD$ ……(iii) [Property of geometric mean] **1**

∴ $DP^2 = BP \times CP - BD \times CD$ [From (i), (ii) and (iii)] **½**

(ii) Let line y = 4 intersect the Y-axis at point A

∴ A ≡ (0, 4) **½**



Let line x = 5 intersect the X-axis at point C

∴ C ≡ (5, 0) **½**

Let line y = 4 and x = 5 intersect at point B

∴ B ≡ (5, 4) **½**

The origin O ≡ (0, 0)

□ABCO is a rectangle

seg AC and seg BO are the diagonals **½**

Equation of line AC by two point form,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ **½**

$$\therefore \quad \frac{x - 0}{0 - 5} = \frac{y - 4}{4 - 0}$$

$$\therefore \quad \frac{x}{-5} = \frac{y - 4}{4}$$

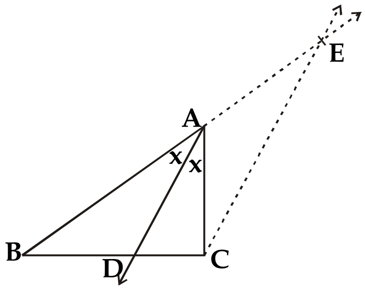
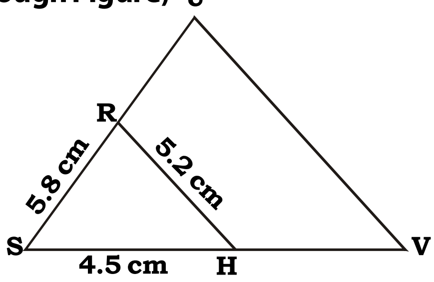
∴ $4x = -5(y - 4)$

∴ $-5y + 20 = 4x$

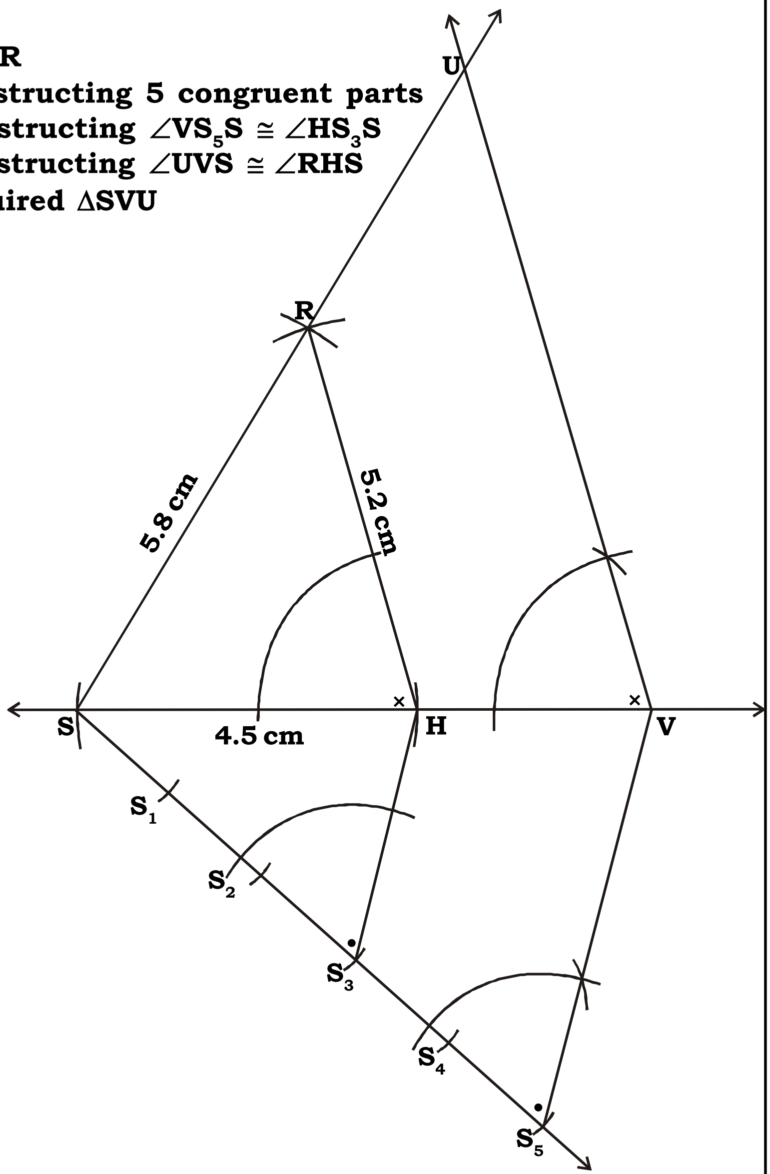
∴ $4x + 5y - 20 = 0$

∴ Equation of diagonal AC is $4x + 5y - 20 = 0$. **½**

	Equation of line BD by two point from,	
	$\therefore \quad \dfrac{x - x_1}{x_1 - x_2} = \dfrac{y - y_1}{y_1 - y_2}$	½
	$\therefore \quad \dfrac{x - 5}{5 - 0} = \dfrac{y - 4}{4 - 0}$	
	$\therefore \quad \dfrac{x - 5}{5} = \dfrac{y - 4}{4}$	
	$\therefore \quad 4(x - 5) = 5(y - 4)$	
	$\therefore \quad 5y - 20 = 4x - 20$	
	$\therefore \quad 4x - 5y = 0$	
	$\therefore$ **Equation of diagonal BD is 4x – 5y = 0.**	½
(iii)	$\tan\theta + \sin\theta = m$	
	$\tan\theta - \sin\theta = n$	
	$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$	
	$= \tan^2\theta + 2\tan\theta.\sin\theta + \sin^2\theta - [\tan^2\theta - 2\tan\theta\sin\theta + \sin^2\theta]$	½
	$= \tan^2\theta + 2\tan\theta.\sin\theta + \sin^2\theta - \tan^2\theta + 2\tan\theta.\sin\theta - \sin^2\theta]$	
	$= 4\tan\theta.\sin\theta$ ....(i)	½
	$4\sqrt{mn} = 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$	
	$= 4\sqrt{\tan^2\theta - \sin^2\theta}$	½
	$= 4\sqrt{\dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$	
	$= 4\sqrt{\sin^2\theta\left(\dfrac{1}{\cos^2\theta} - 1\right)}$	½
	$= 4\sqrt{\sin^2\theta\left(\sec^2\theta - 1\right)}$	½
	$= 4\sqrt{\tan^2\theta\,.\,\sin^2\theta}$ $\left[\begin{array}{l}1 + \tan^2\theta = \sec^2\theta \\ \therefore \tan^2\theta = \sec^2\theta - 1\end{array}\right]$	½
	$= 4 \times \sin\theta \times \tan\theta$ .....(ii)	½
	From (i) and (ii),	
	$m^2 - n^2 = 4\sqrt{mn}$	½

A.5.	**Solve ANY TWO of the following :**	
(i)	Given : In ΔABC, ray AD is the bisector of ∠BAC such that B - D - C.	½
	To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$	
	 **(½ mark for figure)**	½
	Construction : Draw a line passing through C, parallel to line AD and intersecting line BA at point E, B - A - E.	
	Proof : In ΔBEC,	
	line AD || side CE [Construction]	
	∴ $\frac{BD}{DC} = \frac{AB}{AE}$ ....(i) [By B.P.T.]	½
	line CE || line AD [Construction]	
	∴ On transversal BE,	
	∠BAD ≅ ∠AEC ....(ii) [Converse of corresponding angles test]	½
	Also, On transversal AC,	
	∠DAC ≅ ∠ACE ....(iii) [Converse of alternate angles test]	½
	But, ∠BAD ≅ ∠DAC ....(iv) [∵ ray AD bisects ∠BAC]	½
	In ΔAEC,	
	∠AEC ≅ ∠ACE [From (ii), (iii) and (iv)]	
	∴ seg AC ≅ seg AE [Converse of Isosceles triangle theorem]	
	∴ AC = AE ....(v)	1
	∴ $\frac{BD}{DC} = \frac{AB}{AC}$ [From (i) and (v)]	½
(ii)	**(Rough Figure)** 	

1 mark for ΔSHR
1 mark for constructing 5 congruent parts
1 mark for constructing $\angle VS_5S \cong \angle HS_3S$
1 mark for constructing $\angle UVS \cong \angle RHS$
1 mark for required ΔSVU



(iii) Diameter of the base of metallic cone = 2 cm

$\therefore$ Its radius (r) = $\frac{2}{2}$ = 1 cm ½

Its height (h) = 10 cm

Volume of a metallic cone = $\frac{1}{3}\pi r^2 h$ ½

$= \frac{1}{3} \times \pi \times 1 \times 1 \times 10$

$= \frac{10\pi}{3}$ cm³ ½

$\therefore$ Volume of 900 metallic cones $= 900 \times \frac{10\pi}{3}$ ½

$= 3000\pi\ cm^3$

900 cones are melted to form a right circular cylinder

$\therefore$ Volume of a cylinder $= 3000\ \pi$ ½

For a cylinder, Radius (r_2) = 10 cm and height be h_2

Volume of a cylinder $= \pi r_1^2 h_1$ ½

$\therefore$ $3000\pi = \pi \times 10 \times 10\ h_2$

$\therefore$ $h_1 = 30$ cm ½

Total surface area of cylinder $= 2\pi r_1\ (r_1 + h_1)$ ½

$= 2 \times 3.14 \times 10\ (10 + 30)$

$= 6.28 \times 10 \times 40$ ½

$= 62.8 \times 40$

$= 2512\ cm^2$

$\therefore$ **Total surface area of the right circular cylinder is 2512 cm^2.** ½

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