MT

2014 ___ ___ 1100 **Seat No.** | | | | | | | |

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 6 (E)

Time : 2 Hours **(Pages 3)** **Max. Marks : 40**

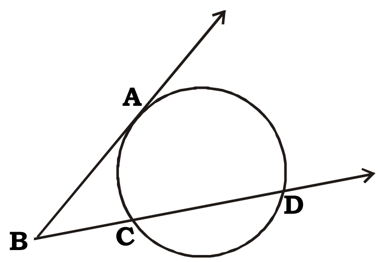
Note :

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q.1. Solve ANY FIVE of the following : **5**

(i) Line AB is a tangent and line BCD is a secant. If AB = 6 units, BC = 4 units, find BD.



(ii) If the radius is 2 cm and length of corresponding arc is 3.14 cm, find the area of a sector.

(iii) What is the directed angle, whose terminal arm lies along the coordinate axes, called ?

(iv) A line has the equation y = 3x – 2. State its y-intercept.

(v) Find the area of a circle with radius 7 cm.

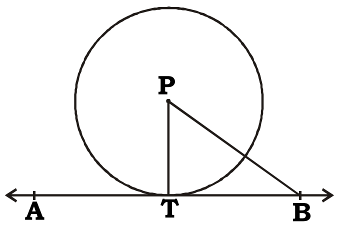
(vi) If $\sec \theta = \frac{2}{\sqrt{3}}$, find the value of θ ?

Q.2. Solve ANY FOUR of the following : **8**

(i) Find the side of square whose diagonal is $16\sqrt{2}$ cm .

(ii) In ΔPQR, seg PM is the median. If PM = 9 and $PQ^2 + PR^2 = 290$. Find QR.

(iii) In the adjoining figure, point P is centre of the circle and line AB is the tangent to the circle at T. The radius of the circle is 6 cm. Find PB if $\angle TPB = 60^\circ$.



(iv) Draw a circle of radius 3.6 cm, take a point M on it. Draw a tangent to the circle at M without using centre of the circle.

(v) Prove : $\sec^2 \theta + \operatorname{cosec}^2 \theta = \sec^2 \theta \cdot \operatorname{cosec}^2 \theta$

(vi) If the angle $\theta = -60^\circ$, find the value of $\sin \theta$, $\cos \theta$, $\sec \theta$ and $\tan \theta$.

Q.3. Solve ANY THREE of the following : 9

(i) □ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

(ii) Two circles which are not congruent touch externally. The sum of their areas is $130\pi cm^2$ and the distance between their centers is 14 cm. Find radii of circles.

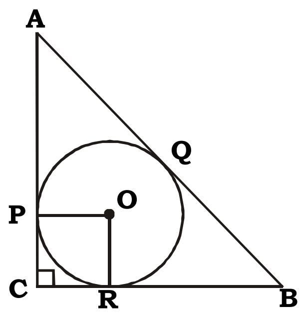
(iii) Construct the incircle of ΔRST in which RS = 6 cm, ST = 7 cm and RT = 6.5 cm.

(iv) Find the equation of the line which passes through (2, 7) and whose y-intercept is 3.

(v) The curved surface area of the frustum of a cone is 180 sq. cm and the perimeters of its circular bases are 18 cm and 6 cm respectively. Find the slant height of the frustum of a cone.

Q.4. Solve ANY TWO of the following : 8

(i) In a right angled triangle ABC, $\angle ACB = 90^\circ$ a circle is inscribed in the triangle with radius r. a, b, c are the lengths of the sides BC, AC and AB respectively. Prove that $2r = a + b - c$.



(ii) Show that (– 2, 1), (0, 3), (2, 1) and (0, – 1) are the vertices of a parallelogram.

(iii) Two poles of height 18 metres and 7 metres are erected on the ground. A wire of length 22 metres tied to the top of the poles. Find the angle made by the wire with the horizontal.

Q.5. Solve ANY TWO of the following : 10

(i) Prove : If a line parallel to a side of a triangle intersects other sides in two distinct points, then the line divides those sides in proportion.

(ii) Draw a triangle ABC, right angled at B such that, AB = 3 cm and BC = 4 cm. Now construct a triangle similar to ΔABC, each of whose sides is $\frac{7}{5}$ times the corresponding side of ΔABC.

(iii) A cylinder of radius 12 cm contains water upto depth of 20 cm. A spherical iron ball is dropped into the cylinder and thus water level is raised by 6.75 cm. what is the radius of the ball ?

Best Of Luck 👍

MT

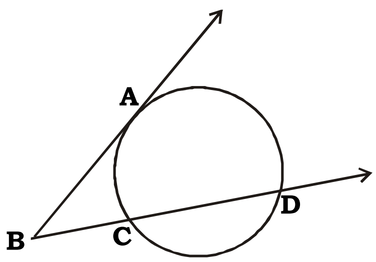
2014 ___ ___ 1100 **Seat No.**

MT - MATHEMATICS (71) GEOMETRY - PRELIM II - PAPER - 6 (E)

Time : 2 Hours **Prelim - II Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

(i) Line BCD is a secant intersecting the circle at points C and D and line BA is a tangent at A ½



∴ $AB^2 = BC \times BD$

∴ $6^2 = 4 \times BD$

∴ $36 = 4 \times BD$

∴ $BD = \frac{36}{4}$

∴ **BD = 9 units** ½

(ii) Radius (r) = 2 cm

Length of arc (l) = 3.14 cm

Area of sector $= l \times \frac{r}{2}$ ½

$= 3.14 \times \frac{2}{2}$

$= 3.14\ cm^2$

∴ **The area of a sector is $3.14\ cm^2$.** ½

(iii) If the terminal arm of a directed angle lies along the co-ordinate axes, then it is called a quadrantal angle. 1

(iv) Equation of the line is $y = 3x - 2$

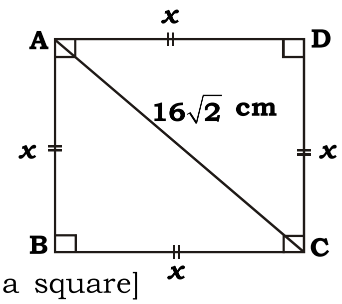
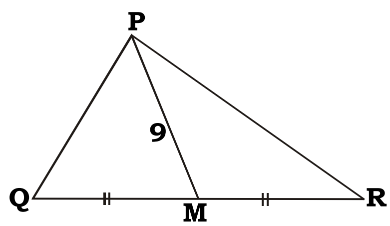
Comparing the given equation with slope-intercept form ½

$y = mx + c,\ c = -2$

∴ **y intercept of the line is – 2.** ½

(v) Radius of circle (r) = 7 cm

∴ Area of the circle $= \pi r^2$ ½

	$= \frac{22}{7} \times 7 \times 7$	
	$= 154$ cm²	
	∴ The area of a circle is 154 cm².	½
(vi)	$\sec\theta = \frac{2}{\sqrt{3}}$ [Given]	
	But, $\sec 30° = \frac{2}{\sqrt{3}}$	½
	∴ $\sec\theta = \sec 30°$	
	∴ $\theta = 30°$	½
A.2.	**Solve ANY FOUR of the following :**	
(i)	□ABCD is a square. $AC = 16\sqrt{2}$ cm To find : Side of a square □ABCD is a square [Given] Let the sides of the square be x cm In ΔABC, 	½
	m ∠ABC = 90° [Angle of a square]	
	∴ $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]	½
	∴ $(16\sqrt{2})^2 = x^2 + x^2$	
	∴ $256 \times 2 = 2x^2$	
	∴ $x^2 = \frac{256 \times 2}{2}$	½
	∴ $x^2 = 256$	
	∴ $x = 16$ [Taking square roots]	
	∴ The side of a square is 16 cm.	½
(ii)	In ΔPQR, seg PM is the median [Given]	
	∴ $PQ^2 + PR^2 = 2PM^2 + 2QM^2$ [By Appollonius theorem]	½
	∴ $290 = 2(9)^2 + 2QM^2$ [Given]	
	∴ $290 = 2(81) + 2QM^2$	
	∴ $290 = 162 + 2QM^2$ 	
	∴ $290 - 162 = 2QM^2$	
	∴ $128 = 2QM^2$	
	∴ $QM^2 = \frac{128}{2}$	
	∴ $QM^2 = 64$	
	∴ $QM = 8$ units [Taking square roots]	½

$\therefore$ QM $= \frac{1}{2}$ QR [$\because$ M is midpoint of side QR] ½

$\therefore$ 8 $= \frac{1}{2}$ QR

$\therefore$ 8×2 = QR

$\therefore$ QR = 16 units ½

(iii) In ΔPTB,

m $\angle$TPB = 60° [Given]

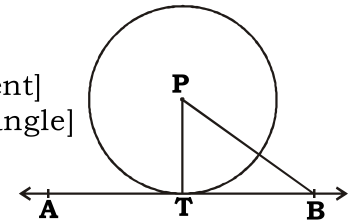
m $\angle$PTB = 90°

[Radius is perpendicular to tangent]

$\therefore$ m $\angle$PBT = 30° [Remaining angle]

$\therefore$ ΔPTB is a 30° - 60° - 90° triangle 1

$\therefore$ By 30° - 60° - 90° triangle theorem



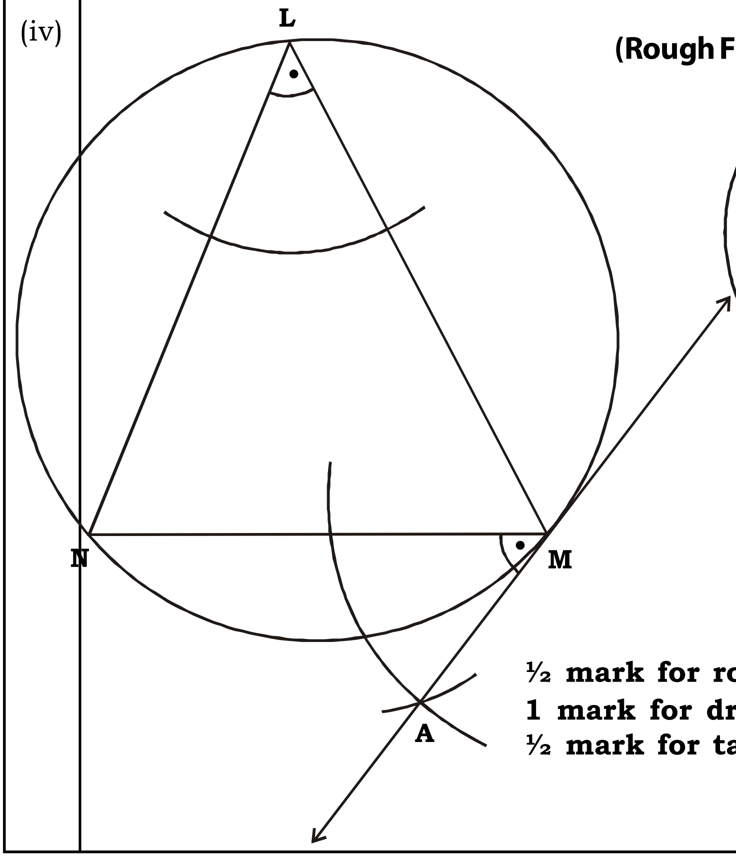
PT $= \frac{1}{2}$ PB [Side opposite to 30°] ½

$\therefore$ 6 $= \frac{1}{2}$ PB [Given]

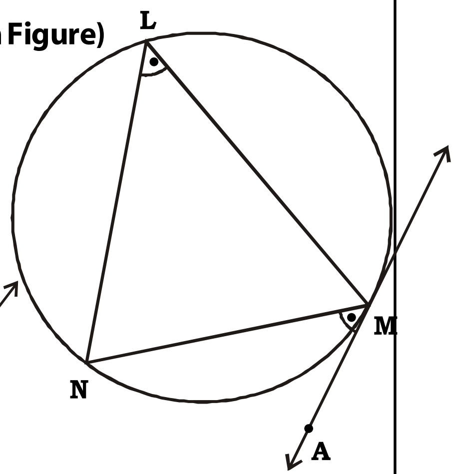
$\therefore$ PB = 6×2

$\therefore$ PB = 12 cm ½

(iv)



(Rough Figure)



½ mark for rough figure

1 mark for drawing $\angle$NMA $\cong$ $\angle$NLM

½ mark for tangent at M.

(v)

L.H.S. = $\sec^2 \theta + \text{cosec}^2 \theta$

$= \dfrac{1}{\cos^2 \theta} + \dfrac{1}{\sin^2 \theta}$ $\left[\because \sec\theta = \dfrac{1}{\cos\theta}, \text{cosec}\,\theta = \dfrac{1}{\sin\theta}\right]$ ½

$= \dfrac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \,.\, \sin^2 \theta}$ ½

$= \dfrac{1}{\cos^2 \theta \,.\, \sin^2 \theta}$ $[\because \sin^2 \theta + \cos^2 \theta = 1]$ ½

$= \sec^2 \theta \,.\, \text{cosec}^2 \theta$ ½

= R.H.S.

$\therefore$ $\sec^2 \theta + \text{cosec}^2 \theta = \sec^2 \theta \,.\, \text{cosec}^2 \theta$

(vi)

$\theta = -60°$

$\sin(-\theta) = -\sin\theta$

$\therefore$ $\sin(-60) = -\sin 60$

$\therefore$ $\boxed{\sin(-60) = -\dfrac{\sqrt{3}}{2}}$ ½

$\sec(-\theta) = \sec\theta$

$\therefore$ $\sec(-60) = \sec 60$

$\therefore$ $\boxed{\sec(-60) = 2}$ ½

$\cos(-\theta) = \cos\theta$

$\therefore$ $\cos(-60) = \cos 60$

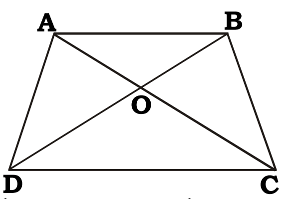
$\therefore$ $\boxed{\cos(-60) = \dfrac{1}{2}}$ ½

$\tan(-\theta) = -\tan\theta$

$\therefore$ $\tan(-60) = -\tan 60$

$\therefore$ $\boxed{\tan(-60) = -\sqrt{3}}$ ½

A.3. Solve ANY THREE of the following :



(i)

□ABCD is a trapezium

side AB || side DC [Given]

$\therefore$ On transversal AC,

$\angle BAC \cong \angle DCA$ [Converse of alternate angles test]

$\therefore$ $\angle BAO \cong \angle DCO$ ......(i) $[\because$ A - O - C] 1

In ΔAOB and ΔCOD,

$\angle BAO \cong \angle DCO$ [From (i)]

$\angle AOB \cong \angle COD$ [Vertically opposite angles]

$\therefore$ $\Delta AOB \sim \Delta COD$ [By AA test of similarity] 1

$\therefore \quad \frac{AO}{CO} = \frac{BO}{DO}$ [c.s.s.t.]

$\therefore \quad \frac{AO}{BO} = \frac{CO}{DO}$ [By Alternendo] **1**

(ii) Let the radius of first circle be r_1 and that of second circle be r_2
Circles are touching externally

$\therefore \quad r_1 + r_2 = 14$ cm

$\therefore \quad r_2 = 14 - r_1$ ....(i) **½**

According to given information

A (I Circle) + A (II Circle) = $130\ \pi$ cm²

$\therefore \quad \pi r_1^2 + \pi r_2^2 = 130\ \pi$ **½**

$\therefore \quad \pi\ (r_1^2 + r_2^2) = 130\ \pi$

$\therefore \quad r_1^2 + r_2^2 = 130$

$\therefore \quad r_1^2 + (14 - r_1)^2 = 130$ [From (i)] **½**

$\therefore \quad r_1^2 + 196 - 28r_1 + r_1^2 = 130$

$\therefore \quad 2r_1^2 - 28r_1 + 196 - 130 = 0$

$\therefore \quad 2r_1^2 - 28r_1 + 66 = 0$

$\therefore \quad 2\ (r_1^2 - 14r_1 + 33) = 0$

$\therefore \quad r_1^2 - 14r_1 + 33 = 0$

$\therefore \quad r_1^2 - 11r_1 - 3r_1 + 33 = 0$

$\therefore \quad r_1\ (r_1 - 11) - 3\ (r_1 - 11) = 0$

$(r_1 - 3)\ (r_1 - 11) = 0$ **½**

$r_1 - 3 = 0$ or $r_1 - 11 = 0$

$\therefore \quad r_1 = 3$ cm or $r_1 = 11$ cm

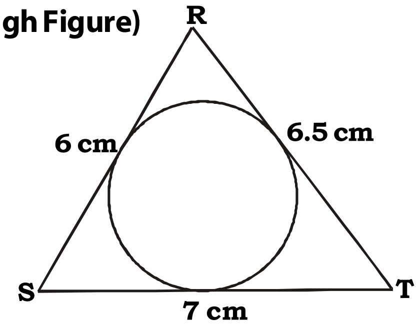
If $r_1 = 3$ If $r_1 = 11$

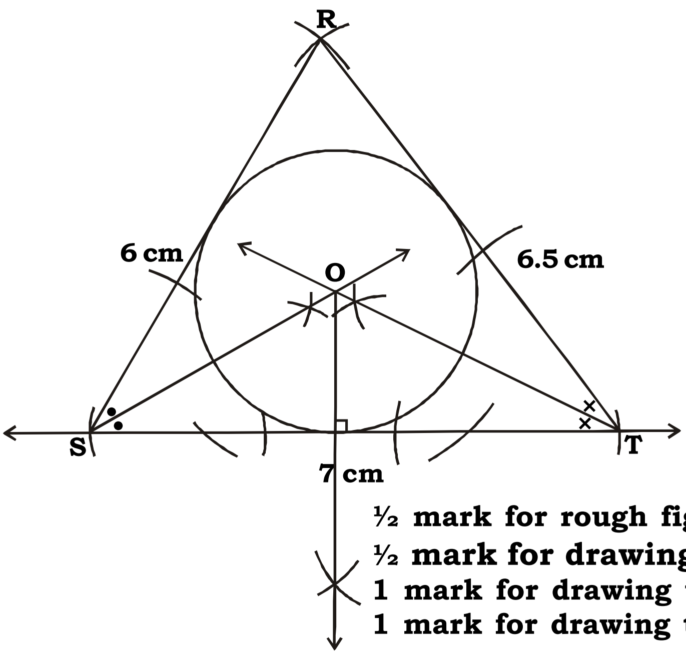
then, $r_2 = 14 - 3$ then, $r_2 = 14 - 11$

$r_2 = 11$ cm $r_2 = 3$ cm

$\therefore$ The radii of the circles is 11 cm and 3 cm. **1**

(iii) **(Rough Figure)**





½ mark for rough figure
½ mark for drawing DRST
1 mark for drawing the angle bisectors
1 mark for drawing the incircle

(iv) Let A ≡ (2, 7)
The y intercept of the line is 3
∴ The line intersects the y-axis at point (0, 3)
Let B ≡ (0, 3) ½
The line passes through point A and B
∴ The equation of the line AB
By two point form

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

$$\therefore \quad \frac{x - 2}{2 - 0} = \frac{y - 7}{7 - 3}$$

$$\therefore \quad \frac{x - 2}{2} = \frac{y - 7}{4}$$ ½

$\therefore \quad 4(x - 2) = 2(y - 7)$

$\therefore \quad 4x - 8 = 2y - 14$ ½

$\therefore \quad 4x - 2y - 8 + 14 = 0$

$\therefore \quad 4x - 2y + 6 = 0$

$\therefore \quad 2x - y + 3 = 0$ [Dividing throughout by 2] ½

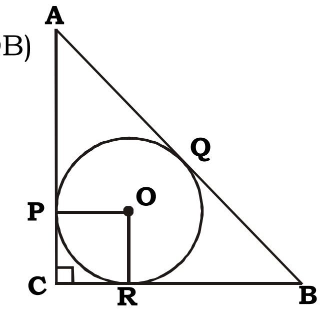
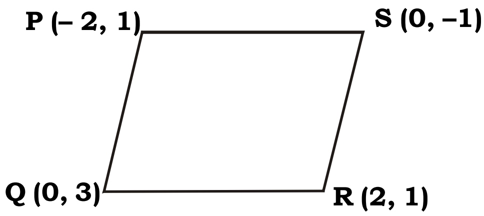
∴ The required equation of the line is $2x - y + 3 = 0$ ½

(v) Curved surface area of the frustum of a cone = 180 cm²
Perimeters of circular bases are 18 cm and 6 cm

$\therefore \quad 2\pi r_1 = 18$ ....(i) **1**

$2\pi r_2 = 6$ ....(ii)

Adding (i) and (ii), we get

	$2\pi r_1 + 2\pi r_2 = 18 + 6$	½
	$\therefore 2\pi (r_1 + r_2) = 24$	
	$\therefore \pi (r_1 + r_2) = \frac{24}{2}$	
	$\therefore \pi (r_1 + r_2) = 12$ ....(iii)	½
	Curved surface area of the frustum of a cone = $\pi(r_1 + r_2)\, l$	½
	$\therefore 180 = \pi(r_1 + r_2)\, l$	
	$\therefore 180 = 12 \times l$ [From (iii)]	
	$\therefore l = 15$ cm	
	∴ Slant height of the frustum of a cone is 15 cm.	½
A.4.	**Solve ANY TWO of the following :**	
(i)	Let the centre of the inscribed circle be 'O'	
	Let AP = AQ = x ....(i); CP = CR = y ....(ii); BR = BQ = z ......(iii) [The lengths of the two tangent segments to a circle drawn from an external point are equal]	1
	a + b – c = BC + AC – AB	
	∴ a + b – c = CR + RB + AP + PC – (AQ + QB) [B - R - C, A - P - C , A - Q - B]	
	∴ a + b – c = y + z + x + y – (x + z) [From (i), (ii) and (iii)]	
	∴ a + b – c = y + z + x + y – x – z	
	∴ a + b – c = 2y	
	∴ a + b – c = 2y	
	∴ a + b – c = 2CP ....(iv) [From (ii)]	1
		
	In □ PCRO	
	m ∠OPC = m ∠ORC = 90° [Radius is perpendicular to tangent]	
	m ∠PCR = 90° [Given]	
	∴ m ∠POR = 90° [Remaining angle]	
	∴ □PCRO is a rectangle [By definition]	1
	∴ CP = OR ....(v) [Opposite sides of a rectangle]	½
	∴ a + b – c = 2 OR [From (iv) and (v)]	
	∴ a + b – c = 2r	½
(ii)	Let, P ≡ (– 2, 1), Q ≡ (0, 3), R ≡ (2, 1), S ≡ (0, – 1)	
	Slope of a line = $\frac{y_2 - y_1}{x_2 - x_1}$	½
	Slope of line PQ = $\frac{3 - 1}{0 - (-2)}$	
	= $\frac{2}{0 + 2}$	
	= $\frac{2}{2}$	
		

$\therefore$ Slope of line PQ $= 1$ ½

Slope of line RS $= \dfrac{-1-1}{0-2}$

$= \dfrac{-2}{-2}$

$\therefore$ Slope of line RS $= 1$ ½

$\therefore$ Slope of line PQ $=$ Slope of line RS

$\therefore$ line PQ || line RS ....(i) ½

Slope of line QR $= \dfrac{1-3}{2-0}$

$= \dfrac{-2}{2}$

$\therefore$ Slope of line QR $= -1$ ½

Slope of line PS $= \dfrac{-1-1}{0-(-2)}$

$= \dfrac{-2}{0+2}$

$= \dfrac{-2}{2}$

$\therefore$ Slope of line PS $= -1$ ½

$\therefore$ Slope of line QR $=$ Slope of line PS

$\therefore$ line QR || line PS ......(ii) ½

In □PQRS,

side PQ || side RS [From (i)]

side QR || side PS [From (ii)]

$\therefore$ □PQRS is a parallelogram [By definition] ½

$\therefore$ The points (–2, 1), (0, 3), (2, 1) and (0, – 1) are the vertices of parallelogram.

(iii) seg AB and CD represents two poles. AB = 18 m, CD = 7 m

seg AC represent the length of the wire. AC = 22 m

□EBDC is a rectangle

$\therefore$ EB = CD = 7 m

[Opposite sides of rectangle]

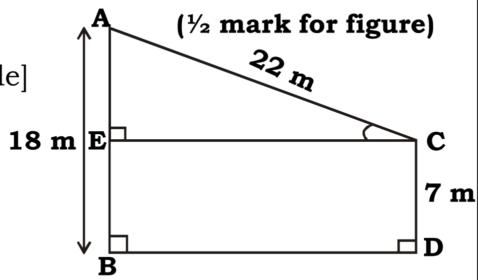
AB = AE + EB [$\because$ A - E - B]

$\therefore$ 18 = AE + 7

$\therefore$ 18 – 7 = AE

$\therefore$ AE = 11 m

(½ mark for figure) 1



In right angled ΔAEC,

$\sin C = \frac{AE}{AC}$ [By definition] ½

$\therefore \sin C = \frac{11}{22}$

$\therefore \sin C = \frac{1}{2}$ ......(i) ½

But,

$\sin 30^\circ = \frac{1}{2}$ ......(ii) ½

$\therefore \sin C = \sin 30^\circ$ ½

$\therefore \angle C = 30^\circ$

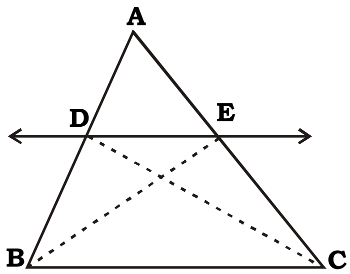
$\therefore$ **The angle made by the wire with horizontal is 30°.** ½

A.5. Solve ANY TWO of the following :

(i) Given : In ΔABC,

(i) line DE || side BC ½

(ii) Line DE intersects sides AB and AC at points D and E respectively.



(½ mark for figure)

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw seg BE and seg CD. ½

Proof : ΔADE and ΔBDE have a common vertex E and their bases AD and BD lie on the same line AB.

$\therefore$ Their heights are equal

$\therefore \frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{AD}{DB}$ ....(i) [Triangles having equal heights] ½

ΔADE and ΔCDE have a common vertex D and their bases AE and EC lie on the same line AC.

$\therefore$ Their heights are equal.

$\therefore \frac{A(\Delta ADE)}{A(\Delta CDE)} = \frac{AE}{CE}$ ....(ii) [Triangles having equal heights] ½

line DE || side BC [Given]

ΔBDE and ΔCDE are between the same two parallel lines DE and BC. ½

$\therefore$ Their heights are equal.

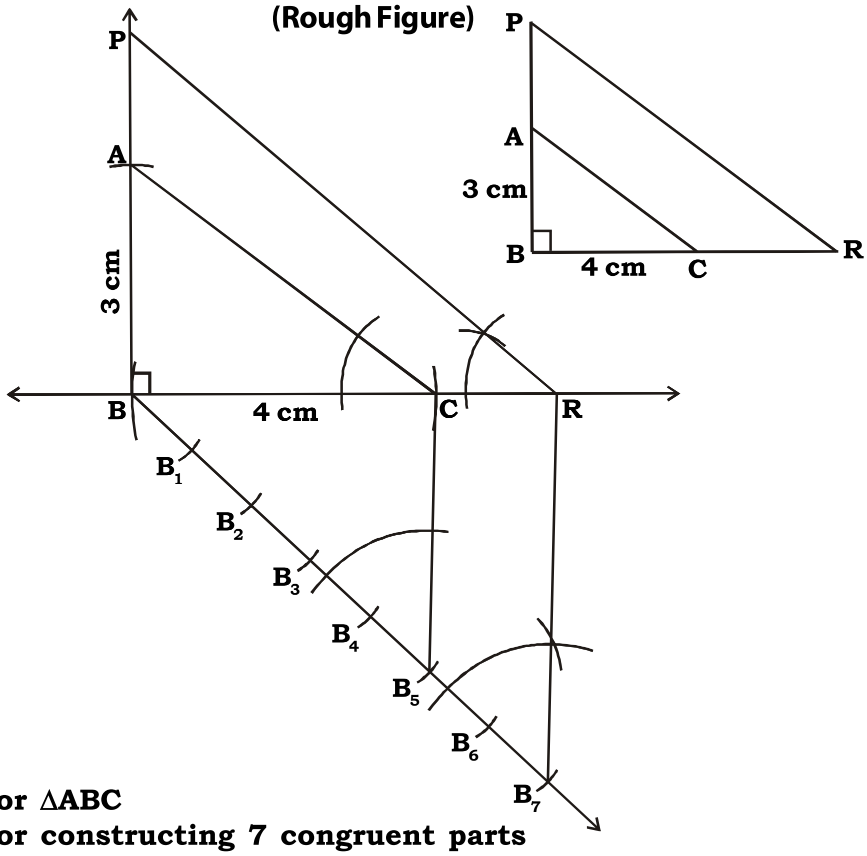
Also, they have same base DE. ½

∴ A(ΔBDE) = A(ΔCDE) ……(iii) [Areas of two triangles having equal bases and equal heights are equal] ½

∴ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{A(\Delta ADE)}{A(\Delta CDE)}$ ……(iv) [From (i), (ii) and (iii)] ½

∴ $\frac{AD}{DB} = \frac{AE}{EC}$ [From (i), (ii) and (iv)] ½

(ii)

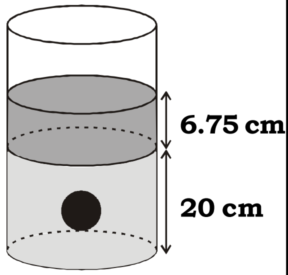


1 mark for ΔABC
1 mark for constructing 7 congruent parts
1 mark for constructing $\angle CB_5B \cong \angle RB_7B$
1 mark for constructing ∠ACB ≅ ∠PRB
1 mark for required ΔPRB

(iii) Radius of the cylinder (r) = 12 cm
A spherical iron ball is dropped into the cylinder
and the water level rises by 6.75 cm ½
∴ Volume of water displaced = volume of the iron ball ½
Height of the raised water level (h) = 6.75 m ½
Volume of water displaced $= \pi r^2 h$ ½

$= \pi \times 12 \times 12 \times 6.75 \text{ cm}^3$

$\therefore$ Volume of iron ball $= \pi \times 12 \times 12 \times 6.75 \text{ cm}^3$ ½



But, Volume of iron ball $= \frac{4}{3}\pi r^3$

$\therefore \quad \pi \times 12 \times 12 \times 6.75 = \frac{4}{3} \times \pi \times r^3$ ½

$\therefore \quad \frac{12 \times 12 \times 6.75 \times 3}{4} = r^3$ ½

$\therefore \quad r^3 = 3 \times 12 \times 6.75 \times 3$

$\therefore \quad r^3 = 3 \times 3 \times 3 \times 4 \times 6.75$ ½

$\therefore \quad r^3 = 3 \times 3 \times 3 \times 27$

$\therefore \quad r = \sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$ ½

[Taking cube roots]

$\therefore \quad r = 3 \times 3$

$\therefore \quad r = 9$

$\therefore$ **Radius of the iron ball is 9 cm.** ½

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