

MT

2017 _____ 1100

Seat No.

MT - MATHEMATICS (71) ALGEBRA - SEMI PRELIM - I - PAPER - 2 (E)

Time : 2 Hours

Model Answer Paper

Max. Marks : 40

A.1. Solve the following : (Any 5)	
(i) Two numbers are 'y' and $y - 3$. As per the given condition, $y(y - 3) = 42$ m $y^2 - 3y - 42 = 0$	1
(ii) The roots of the quadratic equation are -3 and -11. Let $r = -3$ and $s = -11$ $\therefore r + s = -3 + (-11) = -3 - 11 = -14$ and $r.s = -3 \times -11 = 33$ We know that, $x^2 - (r + s)x + r.s = 0$ $\therefore x^2 - (-14)x + 33 = 0$ $\therefore x^2 + 14x + 33 = 0$ \therefore The required quadratic equation is $x^2 + 14x + 33 = 0$	$\frac{1}{2}$ $\frac{1}{2}$
(iii) If one of the root of the quadratic equation is $4 - 3\sqrt{2}$, then the other root is $4 + 3\sqrt{2}$ m $r = 4 - 3\sqrt{2}$ and $s = 4 + 3\sqrt{2}$ m $r + s = 4 - 3\sqrt{2} + 4 + 3\sqrt{2} = 8$ and $r.s = (4 - 3\sqrt{2}) \times (4 + 3\sqrt{2})$ $= (4)^2 - (3\sqrt{2})^2$ $= 16 - 9 \times 2$ $= 16 - 18$ $= -2$	$\frac{1}{2}$ $\frac{1}{2}$
(iv) $\bar{u} = \frac{df_1 u_1}{df_1}$ $= \frac{761}{500}$ $= 1.522$	$\frac{1}{2}$

	<p>Mean (\bar{x}) = $A + h\bar{u}$ = $52 + 15(1.522)$ = $52 + 22.83$ = 74.83</p>	$\frac{1}{2}$												
(v)	<p>Class width (h) = 10</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;">Speed in (km / hr)</th> <th style="padding: 5px;">No. of students</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">20 - 30</td> <td style="padding: 5px;">6</td> </tr> <tr> <td style="padding: 5px;">30 - 40</td> <td style="padding: 5px;">80 $\hat{=}$ f_1</td> </tr> <tr> <td style="padding: 5px;">40 - 50</td> <td style="padding: 5px;">156 $\hat{=}$ f_m</td> </tr> <tr> <td style="padding: 5px;">50 - 60</td> <td style="padding: 5px;">98 $\hat{=}$ f_2</td> </tr> <tr> <td style="padding: 5px;">60 - 70</td> <td style="padding: 5px;">60</td> </tr> </tbody> </table> <p>Here the maximum frequency $f_m = 156$ The corresponding class 40 - 50 is the modal class. $L = 40, f_m = 156, f_1 = 80, f_2 = 98, h = 10$</p>	Speed in (km / hr)	No. of students	20 - 30	6	30 - 40	80 $\hat{=}$ f_1	40 - 50	156 $\hat{=}$ f_m	50 - 60	98 $\hat{=}$ f_2	60 - 70	60	1
Speed in (km / hr)	No. of students													
20 - 30	6													
30 - 40	80 $\hat{=}$ f_1													
40 - 50	156 $\hat{=}$ f_m													
50 - 60	98 $\hat{=}$ f_2													
60 - 70	60													
(vi)	<p>Let total number of persons be x. No. of persons with blood group B = 600 persons Percentage of persons with blood group B = 30 %</p> <p>m $\frac{30}{100} \times x = 600$</p> <p>m $x = \frac{600 \times 100}{30}$</p> <p>m $x = 2000$</p> <p>m Total numbers of persons are 2000.</p>	1												
A.2. Solve the following : (Any 4)														
(i)	<p>$y^2 + 6y + 9 = 0$ Comparing with $ay^2 + by + c = 0$ we have $a = 1, b = 6, c = 9$</p> <p>U = $b^2 - 4ac$ = $(6)^2 - 4(1)(9)$ = $36 - 36$ = 0</p>	$\frac{1}{2}$												
m	<p>U = 0 Hence roots of the quadratic equation are real and equal.</p>	$\frac{1}{2}$												
		1												

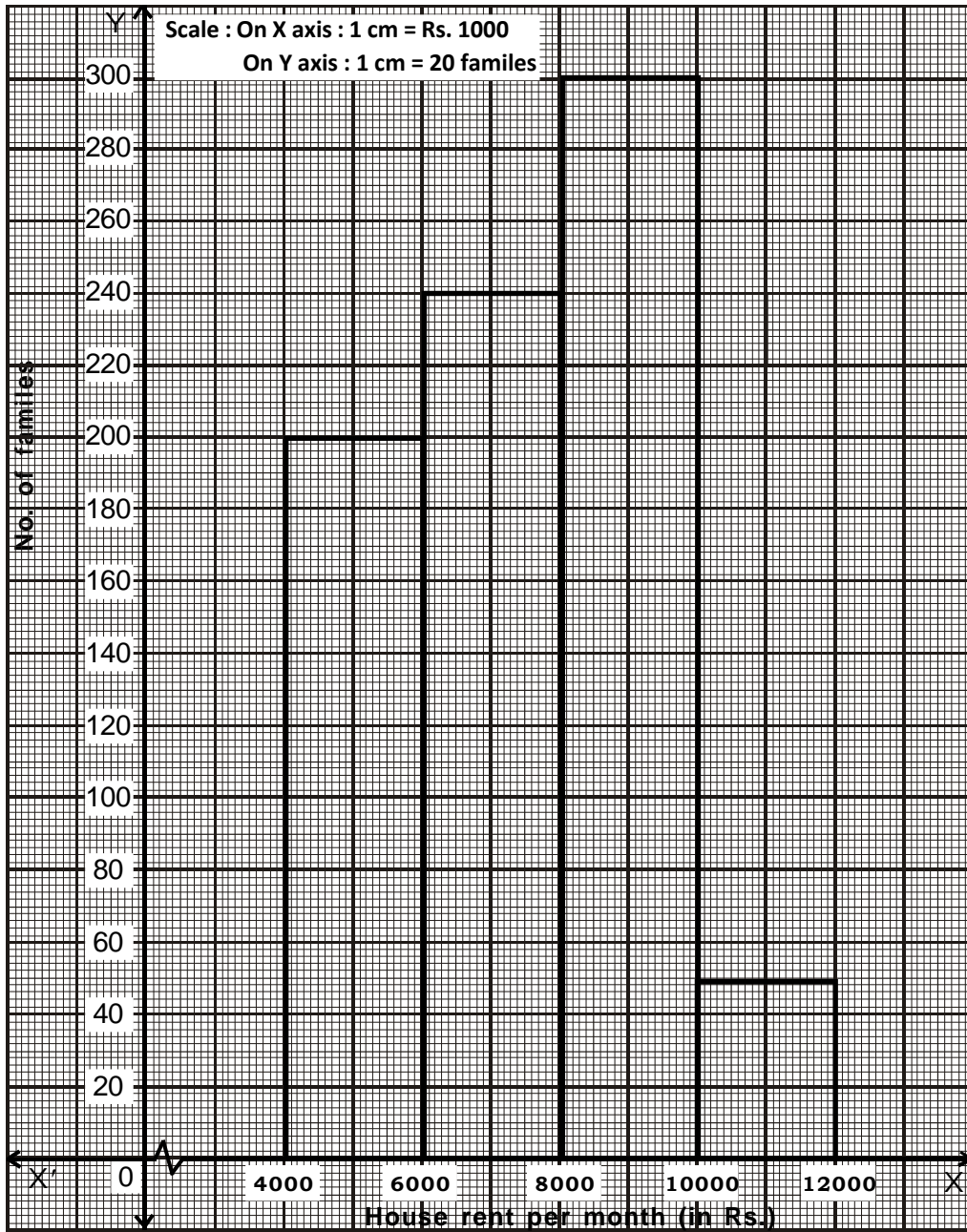
(ii)	Class width (h) = 2				
	Time in months (x_i)	Class Mark (f_i)	No. of disputes	$f_i x_i$	
	0 - 2	1	15	15	
	2 - 4	3	90	270	
	4 - 6	5	120	600	
	6 - 8	7	75	525	
	8 - 10	9	70	630	
	10 - 12	11	50	550	
	Total		420	2590	1
	Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{2590}{420} = \frac{37}{16}$				
m	Mean = 6.17 (approximately)				
m	Mean of time to settle a dispute is 6.17 months.				1
(iii)	If one of the root of the quadratic equation is $\sqrt{5} - \sqrt{3}$, then the other root is $\sqrt{5} + \sqrt{3}$.				
	Let $r = \sqrt{5} - \sqrt{3}$ and $s = \sqrt{5} + \sqrt{3}$				
m	$r + s = \sqrt{5} - \sqrt{3} + \sqrt{5} + \sqrt{3} = 2\sqrt{5}$				$\frac{1}{2}$
	and $r.s = (\sqrt{5} - \sqrt{3}) \times (\sqrt{5} + \sqrt{3})$				
	$= (\sqrt{5})^2 - (\sqrt{3})^2$				$\frac{1}{2}$
	$= 5 - 3$				
	$= 2$				
	We know that,				
	$x^2 - (r + s)x + r.s = 0$				
m	$x^2 - 2\sqrt{5}x + 2 = 0$				
m	The required quadratic equation is $x^2 - 2\sqrt{5}x - 2 = 0$				1
(iv)	Bowling speed (kms / hr.)	No. of players			
	85 - 100	9 $\hat{=}$ f_1			
	100 - 115	11 $\hat{=}$ f_m			
	115 - 130	8 $\hat{=}$ f_2			
	130 - 145	5			
	Here the maximum frequency $f_m = 11$.				
	The corresponding class 100 - 115 is the modal class.				
	$L = 100, f_m = 11, f_1 = 9, f_2 = 8, h = 15$.				1
	Mode = $L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2} \right) \times h$				
	$= 100 + \left(\frac{11 - 9}{2(11) - 9 - 8} \right) \times 15$				

$$\begin{aligned}
 &= 100 + \left(\frac{2}{22-17}\right) \times 15 \\
 &= 100 + \left(\frac{30}{5}\right) \\
 &= 100 + 6 \\
 &= 106
 \end{aligned}$$

m Mode of bowling speed is 106 km/hr.

1

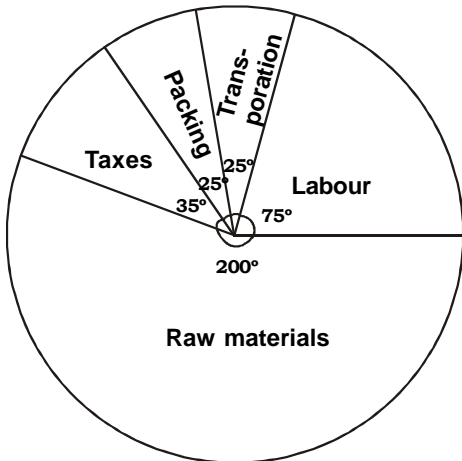
(v)



2

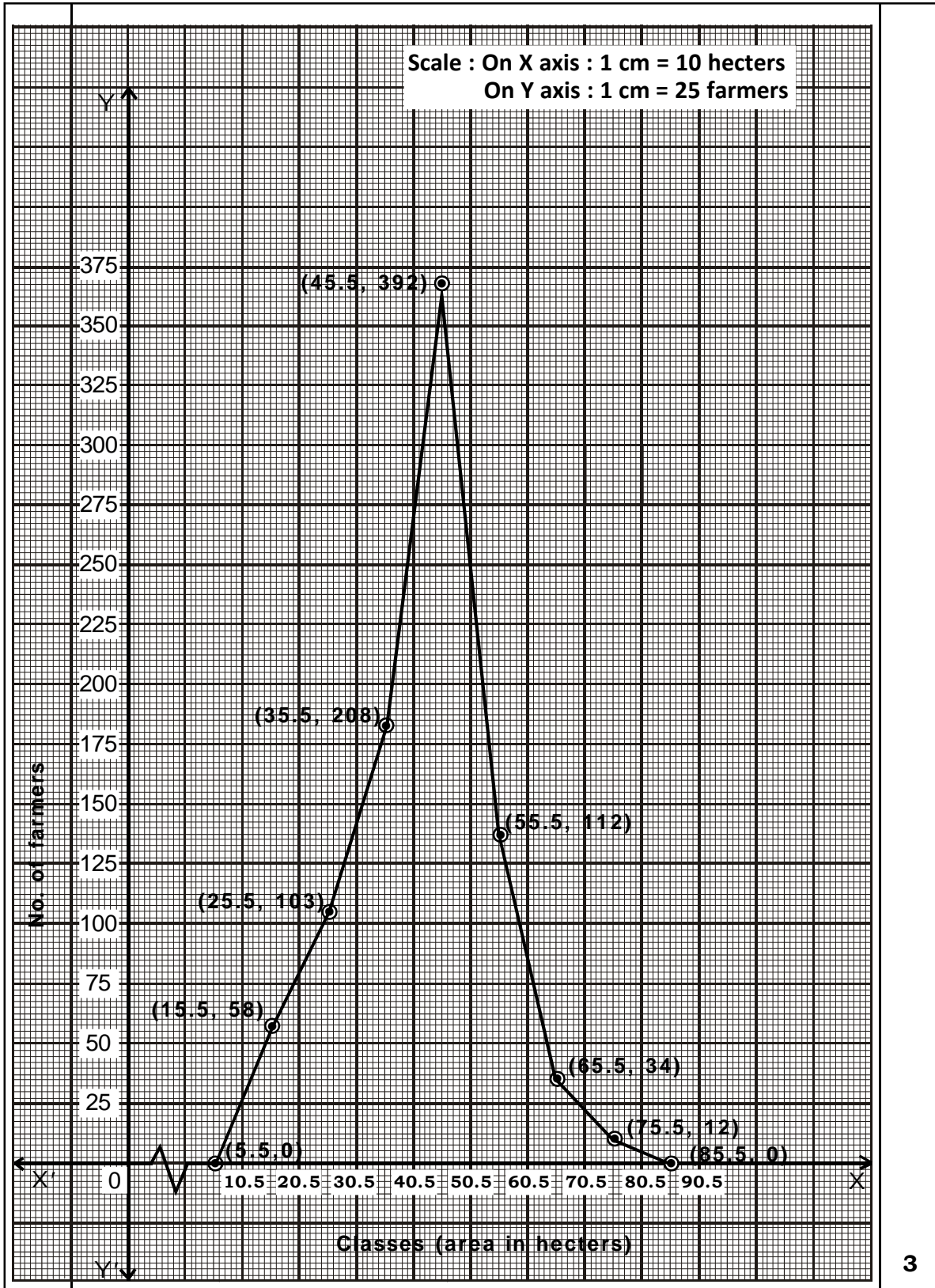
(vi)	Salesman	Measure of central angle	Sales done in Rs.	
	A	90°	18000	
	B	120°	$\frac{120}{360} \times 72000 = 24000$	
	C	80°	$\frac{80}{360} \times 72000 = 16000$	
	D	70°	$\frac{70}{360} \times 72000 = 14000$	
	Total	360°	72000	2
A.3. Solve the following : (Any 3)				
(i)	$k^2x^2 - 2(k-1)x + 4 = 0$ Comparing with $ax^2 + bx + c = 0$ we have $a = k^2$, $b = -2(k-1)$, $c = 4$			$\frac{1}{2}$
	$U = b^2 - 4ac$ $= [-2(k-1)]^2 - 4(k^2)(4)$ $= 4(k^2 - 2k + 1) - 16k^2$ $= 4k^2 - 8k + 4 - 16k^2$ $= 4k^2 - 8k + 4 - 16k^2$			$\frac{1}{2}$
	$U = -12k^2 - 8k + 4$			1
	∴ The roots of given equation are real and equal.			
	∴ U must be zero.			
	∴ $-12k^2 - 8k + 4 = 0$			
	∴ $-4(3k^2 + 2k - 1) = 0$			
	∴ $3k^2 + 3k - k - 1 = \frac{0}{-4}$			
	∴ $3k(k+1) - 1(k+1) = 0$			
	∴ $(k+1)(3k-1) = 0$			$\frac{1}{2}$
	∴ $k+1 = 0$ or $3k-1 = 0$			
	∴ $k = -1$ or $3k = 1$			
	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> $k = -1$ or $k = \frac{1}{3}$ </div>			$\frac{1}{2}$

(ii)	Class width (h) = 20, Assumed mean (A) = 150																																				
	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 20%;">Daily commission (in Rs.)</th> <th style="width: 15%;">Class Mark (x_i)</th> <th style="width: 15%;">d_i = x_i - A</th> <th style="width: 20%;">No. of salesman (f_i)</th> <th style="width: 30%;">f_id_i</th> </tr> </thead> <tbody> <tr> <td>100 - 120</td> <td>110</td> <td>- 40</td> <td>20</td> <td>- 800</td> </tr> <tr> <td>120 - 140</td> <td>130</td> <td>- 20</td> <td>45</td> <td>- 900</td> </tr> <tr> <td>140 - 160</td> <td>150 = A</td> <td>0</td> <td>22</td> <td>0</td> </tr> <tr> <td>160 - 180</td> <td>170</td> <td>20</td> <td>9</td> <td>180</td> </tr> <tr> <td>180 - 200</td> <td>190</td> <td>40</td> <td>4</td> <td>160</td> </tr> <tr> <td>Total</td> <td></td> <td></td> <td>100</td> <td>- 1360</td> </tr> </tbody> </table>	Daily commission (in Rs.)	Class Mark (x_i)	d _i = x _i - A	No. of salesman (f _i)	f _i d _i	100 - 120	110	- 40	20	- 800	120 - 140	130	- 20	45	- 900	140 - 160	150 = A	0	22	0	160 - 180	170	20	9	180	180 - 200	190	40	4	160	Total			100	- 1360	1
Daily commission (in Rs.)	Class Mark (x_i)	d _i = x _i - A	No. of salesman (f _i)	f _i d _i																																	
100 - 120	110	- 40	20	- 800																																	
120 - 140	130	- 20	45	- 900																																	
140 - 160	150 = A	0	22	0																																	
160 - 180	170	20	9	180																																	
180 - 200	190	40	4	160																																	
Total			100	- 1360																																	
	$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$																																				
m	$\bar{d} = \frac{-1360}{100}$																																				
m	$\bar{d} = - 13.6$																																				
	$\text{Mean } (\bar{x}) = A + \bar{d}$ $= 150 + (- 13.6)$ $= 150 - 13.6$ $= 136.4$	1																																			
m	Mean of daily commission of salesman is Rs. 136.4	1																																			
(iii)	$x^4 - 29x^2 + 100 = 0$																																				
m	$(x^2)^2 - 29x^2 + 100 = 0$																																				
	Substituting $x^2 = m$ we get,																																				
	$m^2 - 29m + 100 = 0$																																				
m	$m^2 - 4m - 25m + 100 = 0$	1																																			
m	$m(m - 4) - 25(m - 4) = 0$																																				
m	$m - 4 = 0 \text{ or } m - 25 = 0$																																				
m	$m = 4 \text{ or } m = 25$																																				
	Resubstituting $m = x^2$ we get,																																				
	$x^2 = 4 \text{ or } x^2 = 25$	1																																			
	Taking square root on both the sides, we get,																																				
	$x = \pm 2 \text{ or } x = \pm 5$	1																																			

(iv)	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 30%;">Component</th> <th style="width: 20%;">Expenditure</th> <th style="width: 50%;">Measure of central angle</th> </tr> </thead> <tbody> <tr> <td>Raw material</td> <td style="text-align: center;">800</td> <td>$\frac{800}{1440} \times 360^\circ = 200^\circ$</td> </tr> <tr> <td>Labour</td> <td style="text-align: center;">300</td> <td>$\frac{300}{1440} \times 360^\circ = 75^\circ$</td> </tr> <tr> <td>Transportation</td> <td style="text-align: center;">100</td> <td>$\frac{100}{1440} \times 360^\circ = 25^\circ$</td> </tr> <tr> <td>Packing</td> <td style="text-align: center;">100</td> <td>$\frac{100}{1440} \times 360^\circ = 25^\circ$</td> </tr> <tr> <td>Taxes</td> <td style="text-align: center;">140</td> <td>$\frac{140}{1440} \times 360^\circ = 35^\circ$</td> </tr> <tr> <td>Total</td> <td style="text-align: center;">1440</td> <td style="text-align: center;">360°</td> </tr> </tbody> </table>	Component	Expenditure	Measure of central angle	Raw material	800	$\frac{800}{1440} \times 360^\circ = 200^\circ$	Labour	300	$\frac{300}{1440} \times 360^\circ = 75^\circ$	Transportation	100	$\frac{100}{1440} \times 360^\circ = 25^\circ$	Packing	100	$\frac{100}{1440} \times 360^\circ = 25^\circ$	Taxes	140	$\frac{140}{1440} \times 360^\circ = 35^\circ$	Total	1440	360°	1
Component	Expenditure	Measure of central angle																					
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Total	1440	360°																					
	 <p style="text-align: center;">Raw materials</p>	2																					
(v)	<p>r and s are the roots of a quadratic equation</p> <p>$\therefore r + s = 5$ and $r^3 + s^3 = 35$ [Given]</p> <p>We know that,</p> $x^2 - (r + s)x + r \cdot s = 0 \quad \dots\dots(i)$ <p>Also, $r^3 + s^3 = (r + s)^3 - 3r \cdot s(r + s)$</p> <p>m $35 = (5)^3 - 3r \cdot s(5)$ [$\because r + s = 5$ and $r^3 + s^3 = 35$]</p> <p>m $35 = 125 - 15r \cdot s$</p> <p>m $15r \cdot s = 125 - 35$</p> <p>m $15r \cdot s = 90$</p> <p>m $r \cdot s = \frac{90}{15}$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>																					

	m	$r \cdot s = 6$	1																											
	∴	$x^2 - (r+s)x + r.s = 0$ [From (i)]																												
	m	$x^2 - 5x + 6 = 0$																												
	m	The required quadratic equation is $x^2 - 5x + 6 = 0$.	1																											
A.4. Solve the following : (Any 2)																														
	(i)	Let the two consecutive even natural numbers be x and $x + 2$. As per the given condition.																												
		$x^2 + (x + 2)^2 = 100$																												
	∴	$x^2 + x^2 + 4x + 4 = 100$	1																											
	m	$2x^2 + 4x + 4 - 100 = 0$																												
	m	$2x^2 + 4x - 96 = 0$																												
		Dividing throughout by 2 we get,																												
		$x^2 + 2x - 48 = 0$																												
	m	$x^2 - 6x + 8x - 48 = 0$	1																											
	m	$x(x - 6) + 8(x - 6) = 0$																												
	m	$(x - 6)(x + 8) = 0$																												
	m	$x - 6 = 0$ or $x + 8 = 0$																												
	m	$x = 6$ or $x = -8$	1																											
		x is an natural number																												
	m	$x \neq -8$																												
		Hence $x = 6$																												
		And $x + 2 = 6 + 2 = 8$																												
	m	The two consecutive even natural numbers are 6 and 8 respectively.	1																											
	(ii)	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 33%;">Classes (Monthly exp.)</th> <th style="width: 33%;">Frequency (f_i) (No. of households)</th> <th style="width: 33%;">Cumulative frequency less than type</th> </tr> </thead> <tbody> <tr> <td>150 - 225</td> <td>65</td> <td>65</td> </tr> <tr> <td>225 - 300</td> <td>171</td> <td>236 \ddot{E} c.f.</td> </tr> <tr> <td>300 - 375</td> <td>196 \ddot{E} f</td> <td>432</td> </tr> <tr> <td>375 - 450</td> <td>75</td> <td>507</td> </tr> <tr> <td>450 - 525</td> <td>53</td> <td>560</td> </tr> <tr> <td>525 - 600</td> <td>26</td> <td>586</td> </tr> <tr> <td>600 and above</td> <td>14</td> <td>600</td> </tr> <tr> <td>Total</td> <td>600 \ddot{E} N</td> <td></td> </tr> </tbody> </table>	Classes (Monthly exp.)	Frequency (f_i) (No. of households)	Cumulative frequency less than type	150 - 225	65	65	225 - 300	171	236 \ddot{E} c.f.	300 - 375	196 \ddot{E} f	432	375 - 450	75	507	450 - 525	53	560	525 - 600	26	586	600 and above	14	600	Total	600 \ddot{E} N		1
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600 and above	14	600																												
Total	600 \ddot{E} N																													

	<p>Here total frequency = $df_i = N = 600$</p> <p>m $\frac{N}{2} = \frac{600}{2} = 300$</p> <p>Cumulative frequency (less than type) which is just greater than 300 is 432. Therefore corresponding class 300 - 375 is median class.</p> <p>$L = 300, N = 600, c.f. = 236, f = 196, h = 75$</p> <p>Median = $L + \left(\frac{N}{2} - c.f.\right) \frac{h}{f}$</p> $= 300 + \left(\frac{600}{2} - 236\right) \frac{75}{196}$ $= 300 + (300 - 236) \frac{75}{196}$ $= 300 + (64) \frac{75}{196}$ $= 300 + 16 \left(\frac{75}{49}\right)$ $= 300 + \frac{1200}{49}$ $= 300 + 24.49$ $= 324.49$ <p>m Median of monthly expenditure is Rs. 324.49.</p>	<p>1</p> <p>1</p> <p>1</p>																																
(iii)	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center;">Area in hectares</th> <th style="text-align: center;">Continuous classes</th> <th style="text-align: center;">Class mark</th> <th style="text-align: center;">No. of farmers</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">11 - 20</td> <td style="text-align: center;">10.5 - 20.5</td> <td style="text-align: center;">15.5</td> <td style="text-align: center;">58</td> </tr> <tr> <td style="text-align: center;">21 - 30</td> <td style="text-align: center;">20.5 - 30.5</td> <td style="text-align: center;">25.5</td> <td style="text-align: center;">103</td> </tr> <tr> <td style="text-align: center;">31 - 40</td> <td style="text-align: center;">30.5 - 40.5</td> <td style="text-align: center;">35.5</td> <td style="text-align: center;">208</td> </tr> <tr> <td style="text-align: center;">41 - 50</td> <td style="text-align: center;">40.5 - 50.5</td> <td style="text-align: center;">45.5</td> <td style="text-align: center;">392</td> </tr> <tr> <td style="text-align: center;">51 - 60</td> <td style="text-align: center;">50.5 - 60.5</td> <td style="text-align: center;">55.5</td> <td style="text-align: center;">112</td> </tr> <tr> <td style="text-align: center;">61 - 70</td> <td style="text-align: center;">60.5 - 70.5</td> <td style="text-align: center;">65.5</td> <td style="text-align: center;">34</td> </tr> <tr> <td style="text-align: center;">71 - 80</td> <td style="text-align: center;">70.5 - 80.5</td> <td style="text-align: center;">75.5</td> <td style="text-align: center;">12</td> </tr> </tbody> </table>	Area in hectares	Continuous classes	Class mark	No. of farmers	11 - 20	10.5 - 20.5	15.5	58	21 - 30	20.5 - 30.5	25.5	103	31 - 40	30.5 - 40.5	35.5	208	41 - 50	40.5 - 50.5	45.5	392	51 - 60	50.5 - 60.5	55.5	112	61 - 70	60.5 - 70.5	65.5	34	71 - 80	70.5 - 80.5	75.5	12	<p>1</p>
Area in hectares	Continuous classes	Class mark	No. of farmers																															
11 - 20	10.5 - 20.5	15.5	58																															
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51 - 60	50.5 - 60.5	55.5	112																															
61 - 70	60.5 - 70.5	65.5	34																															
71 - 80	70.5 - 80.5	75.5	12																															



A.5. Solve the following : (Any 2)

- (i) Let the number of days required by father alone to do a certain piece of work be 'x' days and by Tinu alone is (x + 9) days
 m No. of days required by tinu along is (x + 9) days
 Also number of days required by both to complete the same work is 6 days.

$$\text{Work done by father in 1 day} = \frac{1}{x}$$

$$\text{Work done by Tinu in 1 day} = \frac{1}{x + 9}$$

$$\text{Work done by both in 1 day} = \frac{1}{6}$$

As per the given condition,

$$\frac{1}{x} + \frac{1}{x + 9} = \frac{1}{6}$$

$$m \quad \frac{x + 9 + x}{x(x + 9)} = \frac{1}{6}$$

$$m \quad \frac{2x + 9}{x^2 + 9x} = \frac{1}{6}$$

$$m \quad 6(2x + 9) = 1(x^2 + 9x)$$

$$m \quad 12x + 54 = x^2 + 9x$$

$$m \quad 0 = x^2 + 9x - 12x - 54$$

$$m \quad 0 = x^2 - 3x - 54$$

$$m \quad x^2 - 3x - 54 = 0$$

$$m \quad x^2 - 9x + 6x - 54 = 0$$

$$m \quad x(x - 9) + 6(x - 9) = 0$$

$$m \quad (x - 9)(x + 6) = 0$$

$$m \quad x - 9 = 0 \quad \text{or} \quad x + 6 = 0$$

$$m \quad x = 9 \quad \text{or} \quad x = -6$$

∴ The number of days cannot be negative

$$m \quad x \neq -6$$

$$\text{Hence } x = 9$$

$$m \quad x + 9 = 9 + 9 = 18$$

m Tinu alone requires 18 days to complete the work.

1

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(ii)	Class width (h) = 4, Assumed mean (A) = 25					
	No. of trees	Class Mark (x_i)	$d_i = x_i - A$	$u_i = \frac{d_i}{h}$	No. of people (f_i)	$f_i u_i$
	7 - 11	9	- 16	- 4	5	- 20
	11 - 15	13	- 12	- 3	9	- 27
	15 - 19	17	- 8	- 2	13	- 26
	19 - 23	21	- 4	- 1	21	- 21
	23 - 27	25 $\hat{=}$ A	0	0	16	0
	27 - 31	29	4	1	15	15
	31 - 35	33	8	2	12	24
	35 - 39	37	12	3	9	27
	Total				100	- 28
	$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$					2
	m	$\bar{u} = \frac{-28}{100}$				1
	m	$\bar{u} = - 0.28$				1
		$\text{Mean } (\bar{x}) = A + h\bar{u}$ $= 25 + 4 (- 0.28)$ $= 25 - 1.12$ $= 23.88 \text{ years}$				1
	m	Mean of age is 23.88 years.				1
(iii)						
	No. of words typed per minute	Continuous Classes	Frequency No. of typists			
	30 - 39	29.5 - 39.5	2			
	40 - 49	39.5 - 49.5	8			
	50 - 59	49.5 - 59.5	15			
	60 - 69	59.5 - 69.5	12			
	70 - 79	69.5 - 79.5	3			
						1

