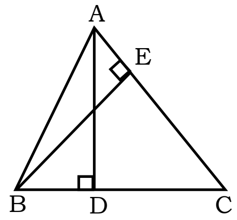
MT

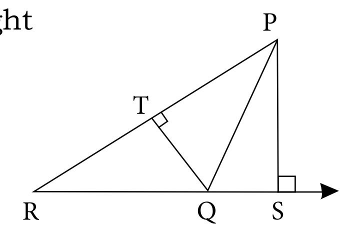
2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - II : PAPER - 3

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1.	**(A) Solve the following : (Any 4)**	
(1)	d(A, B) = Greater co–ordinates – smaller co–ordinates = 5 – (–2) = 5 + 2 $\therefore$ d(A, B) = 7	1
(2)	line l || line m $\angle$BMN + $\angle$MND = 180° (Interior angles Theorem) $\therefore$ $x + 50 = 180^\circ$ $\therefore$ $x = 130^\circ$	1
(3)	$\angle$TPQ is an exterior angle of ΔPQR $\angle$TPQ = $\angle$PQR + $\angle$PRQ (Remote Interior angles theorem) $\therefore$ $100 = x + 35$ $\therefore$ $x = 65$	1
(4)	sin A = cos (A – 30) $\therefore$ cos (90 – A) = cos (A – 30) $\therefore$ 90 – A = A – 30 $\therefore$ 90 + 30 = A + A $\therefore$ 2A = 120 $\therefore$ A = 60	1
(5)	Radius (r) = 21 cm $\therefore$ Diameter = 42 cm Perimeter of semicircle = πr + d $= \frac{22}{7} \times 21 + 42$ = 66 + 42 $\therefore$ Perimeter of semicircle = 108 cm	1
(6)	In ΔABC, AB > AC (Given) $\therefore$ $\angle$C > $\angle$B (In a triangle, Angle opposite to greater side is greater)	1

A.1.	**(B) Solve the following : (Any 2)**	
(1)	In ΔADB and ΔBEA (Given)	½
	$\angle ADB = \angle BEA = 90^\circ$	
	Hypotenuse AB $\cong$ hypotenuse AB (Common side)	½
	side BD $\cong$ side AE (Given)	
	$\therefore$ $\Delta ADB \cong \Delta BEA$ (Hypotenuse side theorem)	½
	$\therefore$ seg AD $\cong$ seg BE (c.s.c.t) 	½
(2)	$\angle A = 30^\circ$	
	LHS $= \dfrac{2\tan A}{1-\tan^2 A}$	½
	$= \dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$	
	$= 2 \times \dfrac{1}{\sqrt{3}} \div \left[1-\left(\dfrac{1}{\sqrt{3}}\right)^2\right]$	½
	$= \dfrac{2}{\sqrt{3}} \div \left[1-\dfrac{1}{3}\right]$	
	$= \dfrac{2}{\sqrt{3}} \div \dfrac{2}{3}$	½
	$= \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2}$	
	$= \dfrac{3}{\sqrt{3}}$	
	$= \dfrac{3}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$	
	$= \dfrac{3\sqrt{3}}{3}$	
	$= \sqrt{3}$	
	$\therefore$ LHS = RHS	½

(3)	Radius (r) = 28 cm	½
	Total surface area of a cone = $\pi r (r + l)$	
	$\therefore \quad 7128 = \frac{22}{7} \times 28 (28 + l)$	½
	$\therefore \quad \frac{7128}{22 \times 4} = 28 + l$	
	$\therefore \quad 81 = 28 + l$	
	$\therefore \quad l = 81 - 28$	½
	$\therefore \quad l = 53$	
	$\therefore$ slant height of a cone is 53 cm	½
A.2.	**(A) Solve the following MCQs :**	
(1)	(D) $4\sqrt{2}$	1
(2)	(C) $\sec^2\theta$	1
(3)	(A) 14π	1
(4)	(C) $\angle BCX$	1
A.2.	**(B) Solve the following : (Any 2)**	
(1)	Area of a triangle = $\frac{1}{2}$ × base × height 	½
	A (ΔPQR)= $\frac{1}{2} \times RQ \times PS$	
	$= \frac{1}{2} \times 6 \times 6$	½
	A (ΔPQR)= 18 sq. units	½
	Also, A (ΔPQR) = $\frac{1}{2} \times PR \times QT$	
	$\therefore \quad 18 = \frac{1}{2} \times 12 \times QT$	
	$QT = \frac{18 \times 2}{12}$	

∴ **QT = 3 units** ½

(2) For the sector, $r = 10$ cm, $\theta = 54°$ ½

Area of the sector $= \dfrac{\theta}{360} \times \pi r^2$

$= \dfrac{54}{360} \times 3.14 \times 10 \times 10$ ½

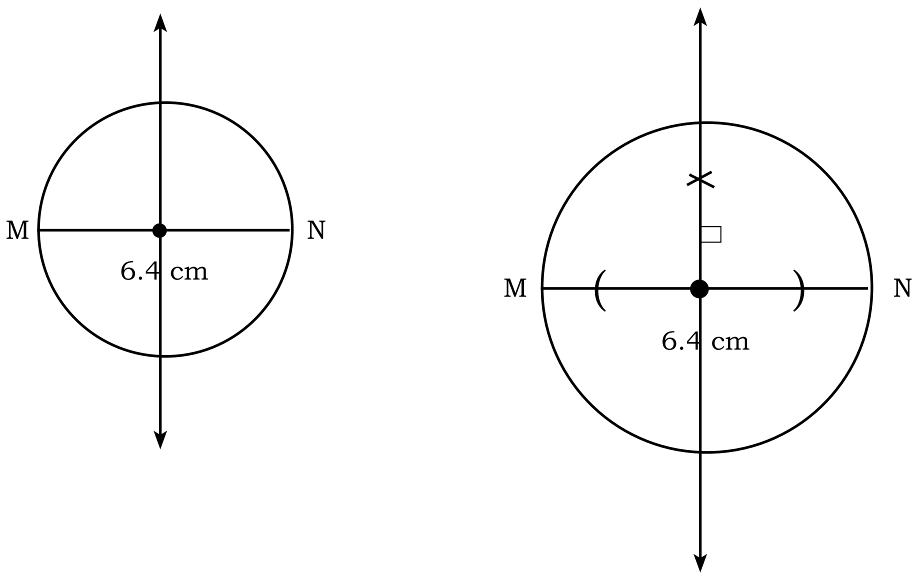
$= \dfrac{3}{20} \times 314$ ½

$= \dfrac{942}{20}$

$= 47.1$ cm²

∴ **Area of the sector is 47.1 cm²** ½

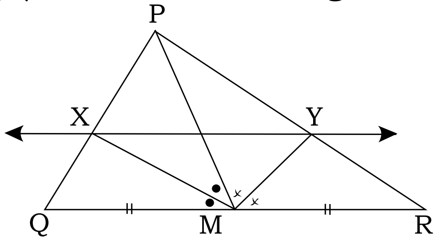
(3) Analytical figure:



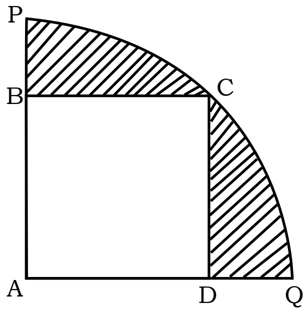
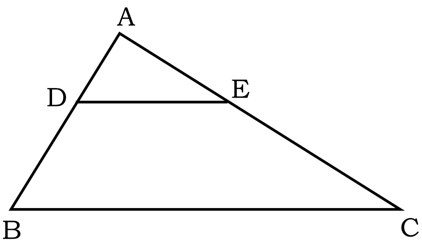
1 mark for drawing circle
1 mark for drawing Perpendicular

A.3. (A) Solve the following activity : (Any 2)

(1)



	Complete the proof by filling the boxes. In ΔPMQ, ray MX bisects $\angle$PMQ $\therefore \quad \boxed{\frac{PM}{MQ}} = \boxed{\frac{PX}{XQ}}$ ...(i) (Property of an angle bisector) In ΔPMR, ray MY bisects $\angle$PMR $\therefore \quad \boxed{\frac{PM}{MR}} = \boxed{\frac{PY}{YR}}$ ...(ii) (Property of an angle bisector) But, $\frac{PM}{MQ} = \frac{PM}{MR}$ (M is the midpoint QR, hence MQ = MR) $\therefore \quad \frac{PX}{XQ} = \frac{PY}{YR}$ $\therefore$ XY ∥ QR $\boxed{\text{Converse of Basic proportionality theorem}}$	½ ½ ½ ½
(2)	LHS = $\sqrt{\frac{1+\sin x}{1-\sin x}}$ $= \sqrt{\frac{1+\sin x}{1-\sin x} \times \frac{\boxed{1+\sin x}}{\boxed{1+\sin x}}}$ $= \sqrt{\frac{\boxed{(1+\sin x)^2}}{\boxed{\cos^2 x}}}$ $= \frac{1+\sin x}{\cos x}$ $= \frac{1}{\boxed{\cos x}} + \frac{\boxed{\sin x}}{\boxed{\cos x}}$ $= \sec x + \tan x$ $\therefore$ LHS = RHS	 ½ ½ ½ ½

(3)		
	Area of square = (side)^2 = $\boxed{20}^2$ = $\boxed{400 \text{ sq cm}}$ ...(i)	
	Radius of sector A – PCQ = Length of diagonal of square ABCD	
	= $20\sqrt{2}$ cm	½
	Area of the shaded region	
	= Area of sector A – PCQ – Area of square ABCD	½
	= A (A – PCQ) – A (□ABCD)	
	= $\left(\frac{\theta}{360} \times \pi \times r^2\right) - \boxed{\text{side}}^2$	
	= $\frac{90}{360} \times 3.14 \times (20\sqrt{2})^2 - (20)^2$	½
	= $\boxed{628} - \boxed{400}$	
	= $\boxed{228}$	½
A.3.	**(B) Solve the following : (Any 2)**	
(1)	In ΔABC, DE || BC	½
	$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (Basic proportionality theorem)	
		
	$\therefore \frac{1.8}{5.4} = \frac{AE}{7.2}$	½
	$\therefore$ AE = $\frac{1.8 \times 7.2}{5.4}$ = 2.4	½
	AE = 2.4 cm	½

(2) $\cot\theta = \frac{40}{9}$ ½

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$= 1 + \left(\frac{40}{9}\right)^2$

$= 1 + \frac{1600}{81}$ ½

$= \frac{81+1600}{81}$

$\therefore \quad \operatorname{cosec}^2\theta = \frac{1681}{81}$

$\therefore \quad \boxed{\mathbf{cosec}\,\theta = \frac{41}{9}}$ ...(Taking square roots) ½

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\therefore \quad \sin\theta = 1 \div \frac{41}{9}$

$\therefore \quad \sin\theta = 1 \times \frac{9}{41}$

$\therefore \quad \boxed{\mathbf{sin}\,\theta = \frac{9}{41}}$ ½

(3) Amount of oil in an oil can = Volume of an oil can ½

= $l \times b \times h$

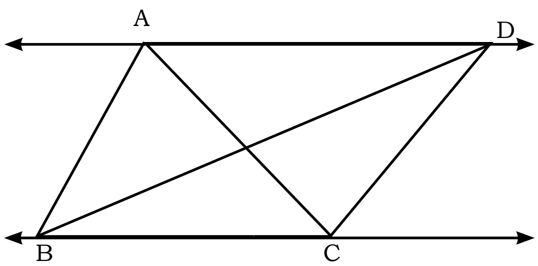
= 20 × 20 × 30 ½

= 12,000 cm^3 or cu.cm ½

The amount of oil an oil can can hold is 12,000 cm^3 or cu.cm. ½

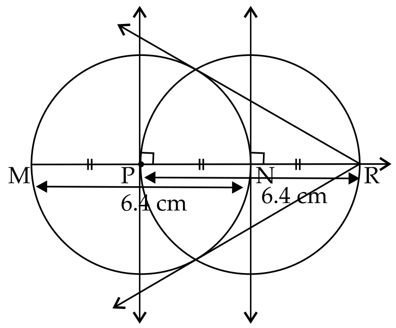
A.4. Solve the following questions : (Any 3)

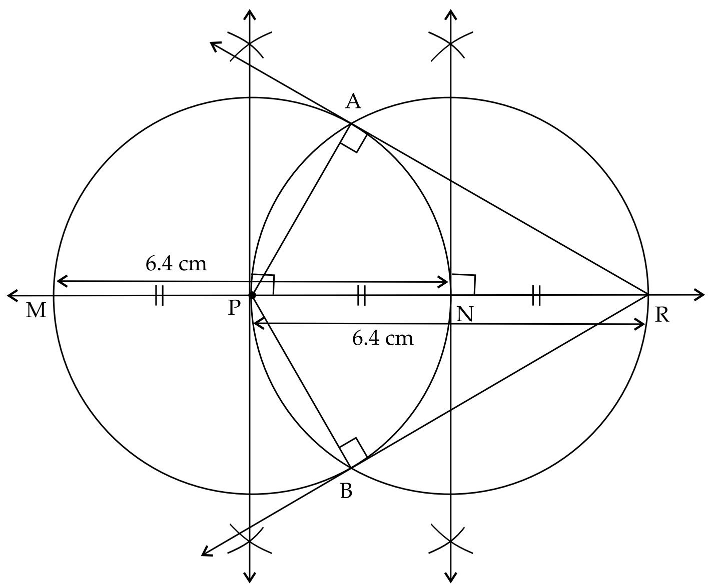
(1)



□ABDE is a parallelogram. ½
∴ AD || BC and AB || DC
Consider ΔABC and ΔBDC. ½
Both the triangles are drawn in two parallel lines. Hence the distance between the two parallel lines is the height of both triangles. ½
In ΔABC and ΔBDC, common base is BC and heights are equal.
Hence, $A(\Delta ABC) = A(\Delta BDC)$ ½
In ΔABC and ΔABD, AB is common base and heights are equal.
∴ $A(\Delta ABC) = (\Delta ABD)$ 1

(2) Analytical figure:





1 mark for drawing circle
1 mark for drawing Perpendicular bisector
1 mark for drawing tangents
line RA and line RB are the required tangents to the circle from point R.

(3) Proof: LHS = $\sec^6 x - \tan^6 x$ ½

$= (\sec^2 x)^3 - (\tan^2 x)^3$

$= (\sec^2 x - \tan^2 x)^3 + 3\sec^2 x \tan^2 x.(\sec^2 x - \tan^2 x)$ ½

...[$\because$ $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$] ½

$= (1)^3 + 3\sec^2 x \,.\, \tan^2 x\,(1)$ $\left(\begin{array}{l}1 + \tan^2 x = \sec^2 x \\ \therefore \sec^2 x - \tan^2 x = 1\end{array}\right)$ ½

$= 1 + 3\sec^2 x \,.\, \tan^2 x$ ½

= R.H.S.

$\therefore$ **$\sec^6 x - \tan^6 x = 1 + 3\sec^2 x \times \tan^2 x$** ½

(4) For the sphere, $r = 9$ cm ½

For the wire, Thickness (diameter) = 4 mm

$\therefore$ Radius $(r_1) = \frac{4}{2}$ mm = 2 mm = $\frac{2}{10}$ cm ...[1 cm = 10 mm]

Let the length of wire be h_1 ½

Wire is made by melting the sphere,

Volume of the wire = Volume of the sphere

$\therefore \pi r_1^2 h_1 = \frac{4}{3} \times \pi r^3$ ½

$\therefore \pi \times \frac{2}{10} \times \frac{2}{10} \times h_1 = \frac{4}{3}\pi \times 9 \times 9 \times 9$ ½

$\therefore h_1 = \frac{4 \times \pi \times 9 \times 9 \times 9 \times 10 \times 10}{3 \times \pi \times 2 \times 2}$

$\therefore h_1 = 24{,}300$ cm ½

$\therefore h_1 = 243$ m ...[$\because$1 m = 100 cm]

$\therefore$ **Length of the wire formed is 243 m.** ½

A.5. Solve the following questions : (Any 1)

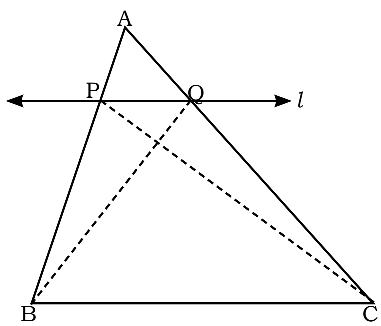
(1) Given:

In ΔABC,

(i) line l || side BC

(ii) Line l intersects sides AB and AC at points D and E respectively.

A-D-B, A-E-C



To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw seg BE and seg CD.

Proof : ΔADE and ΔBDE have a common vertex E and their bases 1

AD and BD lie on the same line AB.

$\therefore$ Their heights are equal .

$\therefore$ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{AD}{DB}$ ...(i) (Triangles having equal height)

ΔADE and ΔCDE have a common vertex D and their bases AE and EC lie on the same line AC.

$\therefore$ Their heights are equal.

$\therefore$ $\frac{A(\Delta ADE)}{A(\Delta CDE)} = \frac{AE}{CE}$ ...(ii) (Triangles having equal height) 1

line DE || side BC ...(Given)

ΔBDE and ΔCDE are between the same two parallel lines DE and BC.

$\therefore$ Their heights are equal.

Also, they have same base DE.

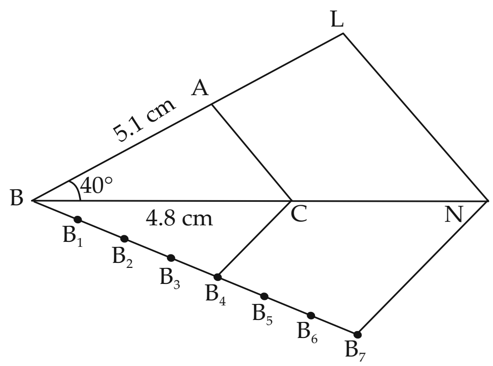
$\therefore$ A(ΔBDE) = A(ΔCDE) ...(iii) 1

(Areas of two triangles having equal base and equal height are equal)

$\therefore$ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{A(\Delta ADE)}{A(\Delta CDE)}$ ...(iv) [From (iii)]

$\therefore$ $\mathbf{\frac{AD}{DB} = \frac{AE}{EC}}$ ...[From (i), (ii) and (iv)] 1

(2) Analytical figure: 1



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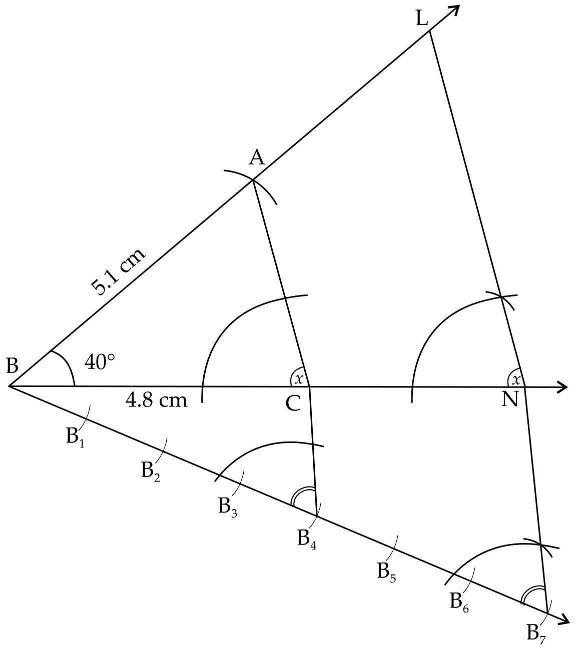
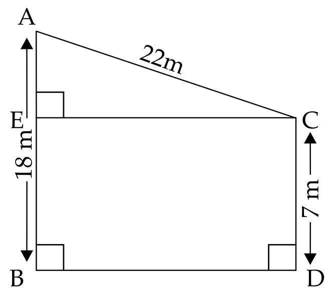


½ mark for analytical figure
1 mark for drawing triangle
½ mark for drawing 7 arcs.
1 mark for drawing $\angle BB_4C = \angle BB_7N$
1 mark for drawing $\angle BCA = \angle BNL$.
ΔLBN is the required triangle similar to the ΔABC.

A.6. Solve the following questions : (Any 1)

(1) AB and CD represents the height of the two poles. ½



AC is the length of the wire of length 22 m joining top A and top C of two poles. ½
AC = 22 m
$\angle$ACE is the angle made by the wire with the horizontal. ½

□EBDC is a rectangle	(By definition)
BE = CD = 7 m	(Opposite sides of rectangle)
AB = AE + BE	(A – E – B)
∴ 18 = AE + 7	

	$\therefore$ $18 - 7 = AE$	
	$\therefore$ $AE = 11$ m	
	In $\triangle$AEC, $\angle AEC = 90°$	½
	$\therefore$ $\sin \angle ACE = \frac{AE}{AC}$ (By definition)	
	$\therefore$ $\sin \angle ACE = \frac{11}{22}$	
	$\therefore$ $\sin \angle ACE = \frac{1}{2}$	½
	But, $\sin 30° = \frac{1}{2}$	
	$\therefore$ $\sin \angle ACE = \sin 30°$	
	$\therefore$ $\angle ACE = 30°$	
	$\therefore$ **The angle made by the wire with the horizontal is 30°.**	½
(2)	For frustum shaped tub,	½
	$r_1 = 20$ cm, $r_2 = 15$ cm, $h = 21$ cm	
	Quantity of water that can be contained in the tub	
	= Inner volume of tub	½
	$= \frac{1}{3} \pi \times h (r_1^2 + r_2^2 + r_1 \times r_2)$	
	$= \frac{1}{3} \times \frac{22}{7} \times 21 (20^2 + 15^2 + 20 \times 15)$	½
	$= 22 (400 + 225 + 300)$	
	$= 22 \times 925$	
	$= 20{,}350$ cm^3	½
	$= \frac{20350}{1000}$ litres [$\because$ 1 litres = 1000 cm^3]	
	= 20.35 litres	
	$\therefore$ **Quantity of water that can be contained in the tub is 20.35 litres**	1

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