MT

2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - II : PAPER - 4

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1.	**(A) Solve the following : (Any 4)**	
(1)	d(A, B) = Greater co–ordinates – smaller co–ordinates = 5 – (–2) = 5 + 2 ∴ d(A, B) = 7	1
(2)	line l || line m $\angle$BMN + $\angle$MND = 180° (Interior angles Theorem) ∴ $x + 50 = 180°$ ∴ $x = 130°$	1
(3)	$\angle$TPQ is an exterior angle of ΔPQR $\angle$TPQ = $\angle$PQR + $\angle$PRQ (Remote Interior angles theorem) ∴ $100 = x + 35$ ∴ $x = 65$	1
(4)	sin A = cos (A – 30) ∴ cos (90 – A) = cos (A – 30) ∴ 90 – A = A – 30 ∴ 90 + 30 = A + A ∴ 2A = 120 ∴ A = 60	1
(5)	Radius (r) = 21 cm ∴ Diameter = 42 cm Perimeter of semicircle = πr + d $= \frac{22}{7} \times 21 + 42$ = 66 + 42 ∴ Perimeter of semicircle = 108 cm	1
(6)	In ΔABC, AB > AC (Given) ∴ $\angle$C > $\angle$B (In a triangle, Angle opposite to greater side is greater)	1

A.1. (B) Solve the following : (Any 2)

(1) In ΔADB and ΔBEA (Given) ½

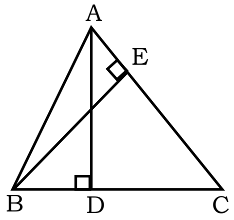
$\angle ADB = \angle BEA = 90^\circ$

Hypotenuse AB ≅ hypotenuse AB (Common side) ½

side BD ≅ side AE (Given)

$\therefore$ $\Delta ADB \cong \Delta BEA$ (Hypotenuse side theorem) ½

$\therefore$ seg AD ≅ seg BE (c.s.c.t) ½



(2) $\angle A = 30^\circ$

$$\text{LHS} = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

$$= 2 \times \frac{1}{\sqrt{3}} \div \left[1-\left(\frac{1}{\sqrt{3}}\right)^2\right]$$ 1

$$= \frac{2}{\sqrt{3}} \div \left[1-\frac{1}{3}\right]$$

$$= \frac{2}{\sqrt{3}} \div \frac{2}{3}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

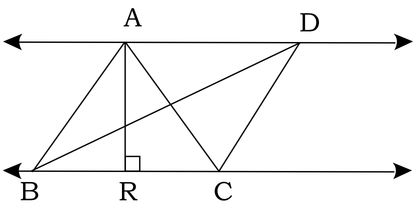
$$= \frac{3}{\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{3}$$

$$= \sqrt{3}$$

$\therefore$ LHS = RHS 1

(3)	Radius (r) = 28 cm	½
	Total surface area of a cone = $\pi r (r + l)$	
	$\therefore \quad 7128 = \frac{22}{7} \times 28 (28 + l)$	
	$\therefore \quad \frac{7128}{22 \times 4} = 28 + l$	½
	$\therefore \quad 81 = 28 + l$	
	$\therefore \quad l = 81 - 28$	½
	$\therefore \quad l = 53$	
	$\therefore$ slant height of a cone is 53 cm	½
A.2.	**(A) Solve the following MCQs :**	
(1)	(A) $\frac{AB}{AC} = \frac{AX}{AY}$	1
(2)	(A) Angle of Elevation	1
(3)	(A) 14π	1
(4)	(B) 80°	1
A.2.	**(B) Solve the following : (Any 2)**	
(1)		
	line AD ‖ line BC ...(Given)	½
	$\therefore$ ΔABC and ΔBCD lie between the same two parallel lines AD and BC.	
	$\therefore$ Their heights are equal.	½
	Also, they have a common base BC	
	$\therefore$ $A(\Delta ABC) = (\Delta BCD)$ (Triangles having equal base and equal height)	½
	$\therefore$ $\boxed{\frac{A(\Delta ABC)}{A(\Delta BCD)} = 1}$	½

(2) For a sector, r = 18 cm, θ = 80 ½

Length of an arc $= \frac{\theta}{360} \times 2\pi r$

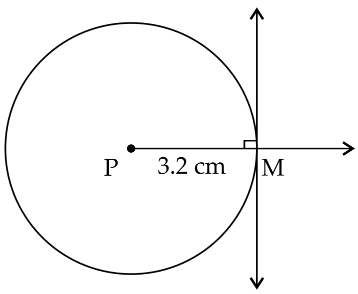
$= \frac{80}{360} \times 2 \times 3.14 \times 18$ ½

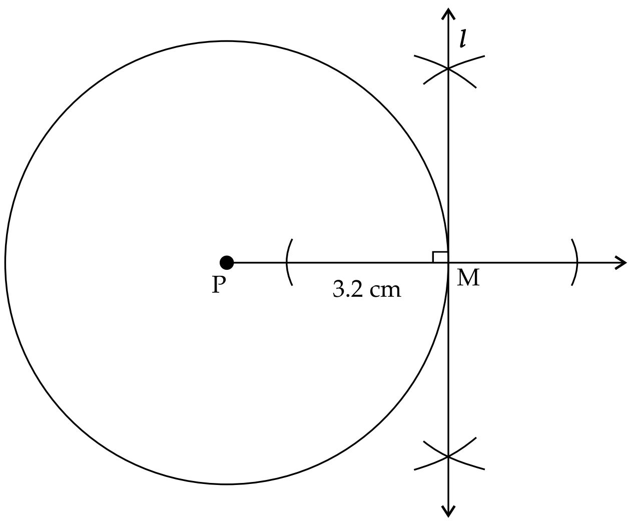
$= 3.14 \times 8$

$= 25.12$ cm ½

∴ **Length of the arc is 25.12 cm** ½

(3) Analytical figure



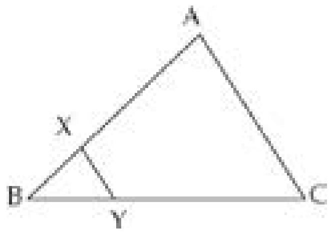


1 mark for drawing circle
1 mark for drawing Tangent
line l is the required tangent to the circle passing through point M on the circle.

A.3. **(A) Solve the following activity : (Any 2)**

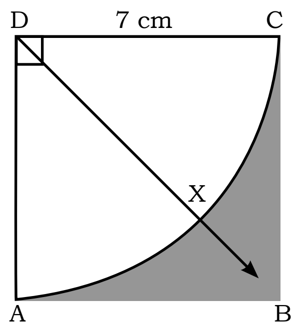
(1) Activity :

2 AX = 3 BX ∴ $\frac{AX}{BX} = \frac{\boxed{3}}{\boxed{2}}$ ½



∴ $\frac{AX + BX}{BX} = \frac{\boxed{3} + \boxed{2}}{\boxed{2}}$ ...(By componendo)

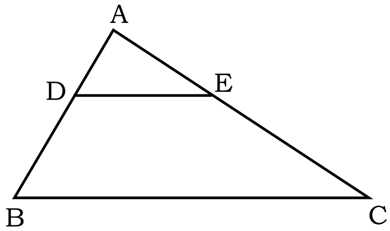
	$\therefore \quad \frac{AB}{BX} = \frac{\boxed{5}}{\boxed{2}}$ ...(i)	½
	$\therefore \quad \Delta BCA \sim \Delta BYX$ ...(By $\boxed{AA}$ test for similarity)	
	$\therefore \quad \frac{BA}{BX} = \frac{AC}{XY}$ ...(c.s.s.t.)	½
	$\therefore \quad \frac{\boxed{5}}{\boxed{2}} = \frac{AC}{9}$ ...[From (i)]	
	$\therefore \quad AC = \boxed{22.5 \text{ units}}$	½
(2)	$\tan\theta + \frac{1}{\tan\theta} = 2$	½
	squaring both sides	
	$\boxed{\left[\tan\theta \frac{1}{\tan\theta}\right]^2} = 4$	½
	$\tan^2\theta + \boxed{2\left[\tan\theta \times \frac{1}{\tan\theta}\right]} + \frac{1}{\tan^2\theta} = 4$	½
	$\therefore \quad \tan^2\theta + \frac{1}{\tan^2\theta} = \boxed{2}$	½
(3)	Area of a square $= \boxed{(\text{Side})^2}$	½
	$= \boxed{(7)^2}$	
	$= 49 \text{ cm}^2$	
	Area of sector (D – AXC) $= \boxed{\frac{\theta}{360°} \times \pi \times r^2}$	½
	$= \frac{\boxed{90}}{360} \times \frac{22}{7} \times \boxed{7^2}$	
	$= 38.5 \text{ cm}^2$	½
	A (shaded region) = A (Square) – A (Sector)	
	$= \boxed{49} \text{ cm}^2 - \boxed{38.5} \text{ cm}^2$	
	$= \boxed{10.5} \text{ cm}^2$	½



A.3. (B) Solve the following : (Any 2)

(1) In ΔABC, DE || BC ½

$\therefore \dfrac{AD}{DB} = \dfrac{AE}{EC}$ (Basic proportionality theorem)



$\therefore \dfrac{1.8}{5.4} = \dfrac{AE}{7.2}$ ½

$\therefore AE = \dfrac{1.8 \times 7.2}{5.4} = 2.4$ ½

AE = 2.4 cm ½

(2)

$\sin\theta = \dfrac{7}{25}$ ½

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \left(\dfrac{7}{25}\right)^2 + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \dfrac{49}{625}$ ½

$\therefore \cos^2\theta = \dfrac{625 - 49}{625}$

$\therefore \cos^2\theta = \dfrac{576}{625}$

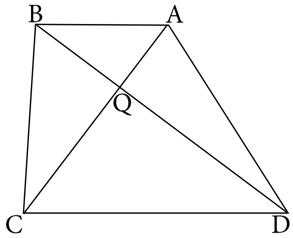
$\therefore \boxed{\cos\theta = \dfrac{24}{25}}$ (Taking square roots) ½

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

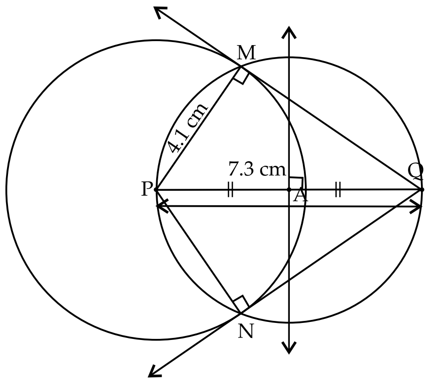
$\therefore \tan\theta = \dfrac{7}{25} \div \dfrac{24}{25}$ ½

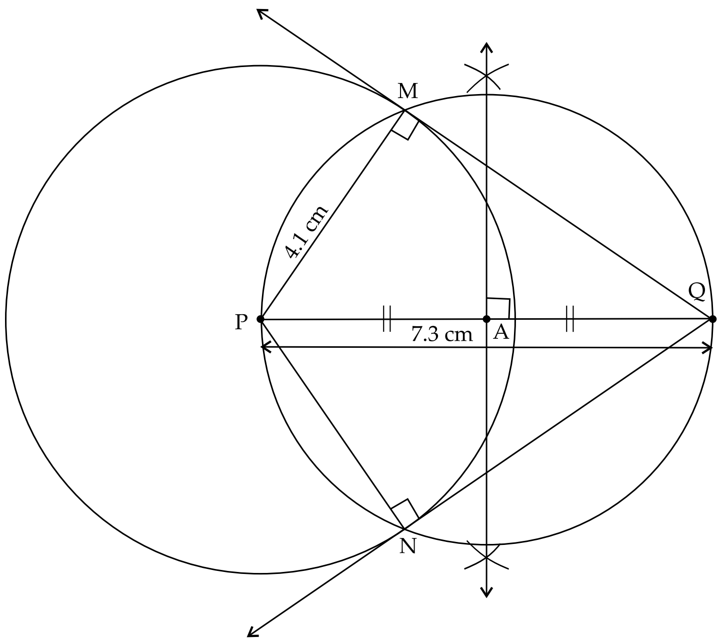
$\therefore \tan\theta = \dfrac{7}{25} \times \dfrac{25}{24}$

$\therefore \boxed{\tan\theta = \dfrac{7}{24}}$

(3)	Amount of oil in an oil can = Volume of an oil can	½
	= $l \times b \times h$	
	= $20 \times 20 \times 30$	½
	= 12,000 cm^3 or cu.cm	½
	The amount of oil an oil can can hold is 12,000 cm^3 or cu.cm.	½
A.4.	**Solve the following questions : (Any 3)**	
(1)	Proof : 2QA = QC $\therefore \frac{QA}{QC} = \frac{1}{2}$ ......(i)	½
	2QB = QD $\therefore \frac{QB}{QD} = \frac{1}{2}$ ......(ii) 	½
	$\therefore \frac{QA}{QC} = \frac{QB}{QD}$ (from (i) and (ii))	
	In ΔAQB and ΔCQD,	½
	$\frac{QA}{QC} = \frac{QB}{QD}$ (proved)	
	$\angle AQB \cong \angle DQC$ (opposite angles)	½
	$\therefore \Delta AQB \sim \Delta DQC$ (SAS test of similarity)	
	$\therefore \frac{AQ}{CQ} = \frac{QB}{QD} = \frac{AB}{CD}$ (correponding sides are proportional)	
	But $\frac{AQ}{CQ} = \frac{1}{2}$	½
	$\therefore \frac{AB}{CD} = \frac{1}{2}$	
	$\therefore$ 2AB = CD	½

(2) Analytical figure:





Line MQ and line NQ are the required tangents to the circle from point Q.

1 mark for drawing circle and seg PQ

1 mark for drawing perpendicular bisector of seg PQ

1 mark for drawing tangents.

(3) Proof: LHS $= \dfrac{\tan^3\theta - 1}{\tan\theta - 1}$ ½

$= \dfrac{\tan^3\theta - 1^3}{\tan\theta - 1}$ ½

$= \dfrac{(\tan\theta - 1)\,(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)}$ ½

$= \tan^2\theta + 1 + \tan\theta$ ($\because$ Q $\tan^2\theta + 1 = \sec^2\theta$) ½

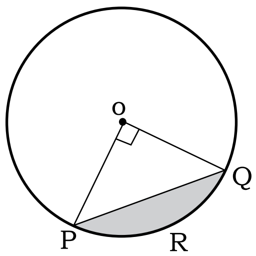
$= \sec^2\theta + \tan\theta$

= R.H.S. ½

$\therefore$ $\mathbf{\dfrac{\tan^3\theta - 1}{\tan\theta - 1} = \sec^2\theta + \tan\theta}$ ½

(4) For segment PRQ, $\theta = \angle POQ = 90°$ ½

A (segment PRQ) $= 114\ \text{cm}^2$



A (segment PQR) $= r^2 \left[\dfrac{\pi\theta}{360} - \dfrac{\sin\theta}{2}\right]$ ½

$\therefore$ $114 = r^2 \left[\dfrac{3.14 \times 90}{360} - \dfrac{\sin 90}{2}\right]$ ½

$\therefore$ $114 = r^2 \left[\dfrac{3.14}{4} - \dfrac{1}{2}\right]$

$\therefore$ $114 = r^2 \left[\dfrac{1.57 - 1}{2}\right]$ ½

$\therefore$ $114 = r^2 \left[\dfrac{0.57}{2}\right]$

$\therefore$ $\dfrac{114 \times 2}{0.57} = r^2$ ½

$\therefore$ $r^2 = \dfrac{114 \times 2 \times 100}{57}$

$\therefore$ $r^2 = 2 \times 2 \times 10 \times 10$

$\therefore$ $r = 20$ (Taking square roots)

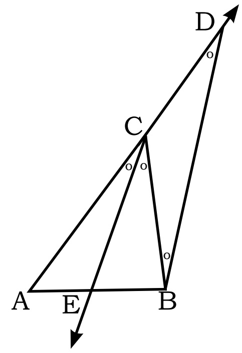
$\therefore$ **Radius of the circle is 20 cm** ½

A.5. Solve the following questions : (Any 1)

(1) (1) Given :

In ΔABC, ray AD is the bisector of $\angle BAC$ such that B - D - C.

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$



Construction : Draw a line passing through C, parallel to line AD and intersecting line BA at point E, B - A - E. 1½

Proof : In ΔBEC,

line AD || side CE ...(Construction)

$\frac{BD}{DC} = \frac{AB}{AE}$ ... (i) (By Basic Proportionately Theorem) ½

line AD || line CE ... (Construction)

$\therefore$ On transversal BE,

$\angle BAD \cong \angle AEC$ ...(ii) (Corresponding angle theorem) ½

Also, on transversal AC,

$\angle DAC \cong \angle ACE$ ...(iii) (alternate angle theorem) ½

But, $\angle BAD \cong \angle DAC$ ...(iv)

($\because$ ray AD bisects $\angle BAC$)

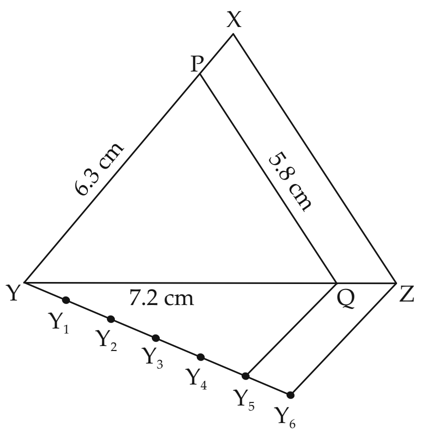
In ΔAEC, $\angle AEC \cong \angle ACE$ ...[From (ii), (iii) and (iv)]

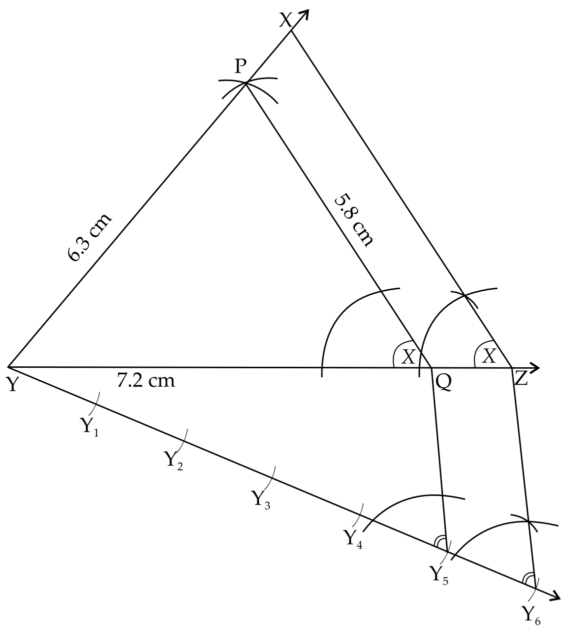
$\therefore$ seg AC $\cong$ seg AE ...(Converse of Isosceles triangle theorem) ½

$\therefore$ AC = AE ...(v)

$\therefore$ $\mathbf{\frac{BD}{DC} = \frac{AB}{AC}}$ ...[From (i) and (v)] ½

(2) Analytical figure:

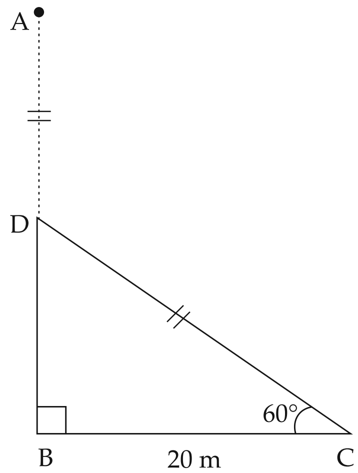




Δ XYZ is the required triangle similar to the ΔPYQ.
½ mark for analytical
1 mark for drawing triangle
½ mark for drawing 7 arcs
1 mark for drawing $\angle YY_5Q = \angle YY_6Q$
1 mark for drawing $\angle YQP = \angle YZX$.

A.6. Solve the following questions : (Any 1)

(1) AB represents the
height of the tree. ½
Tree breaks at D.
AD represents the
broken part of the tree
which takes the position DC. ½



$\therefore$ AD = DC
$\angle DCB = 60°$
BC = 20 m
In ΔDBC, $\angle DBC = 90°$

$\therefore \tan 60° = \frac{DB}{BC}$ (By definition)

	$\therefore \quad \sqrt{3} = \frac{DB}{20}$	½
	$\therefore \quad DB = 20\sqrt{3}$	
	$= 20\,(1.73)$	
	$\therefore \quad DB = 34.60$ m	½
	$\cos 60° = \frac{BC}{DC}$ (By definition)	
	$\therefore \quad \frac{1}{2} = \frac{20}{DC}$	
	$\therefore \quad DC = 40$ m	
	$\therefore \quad AD = DC = 40$ m	
	$\therefore \quad AB = AD + DB$ (A – D – B)	½
	$\therefore \quad AB = 40 + 34.60$	
	$\therefore \quad AB = 74.60$ m	
	$\therefore$ **The height of the tree is 74.60 m.**	½
(2)	For the cylindrical bucket, diameter = 28 cm $\therefore$ Radius (r) = 14 cm. height (h) = 20 cm Volume of sand in the bucket = Volume of the bucket $= \pi r^2 h$	½
	For conical shape sand height (h_1) = 14 cm Let the radius be r_1 Sand from the bucket is emptied to form a cone.	½
	$\therefore$ Volume of sand in the conical shape = Volume of the sand in the bucket	½
	$\therefore \quad \frac{1}{3}\pi r_1^2 h_1 = \pi r^2 h$	
	$\therefore \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{22}{7} \times 14 \times 14 \times 20$	½
	$\therefore \quad \frac{1}{3} \times \pi r_1^2 \times 14 = \frac{22}{7} \times 14 \times 14 \times 20$	½
	$\therefore \quad \pi r_1^2 = \frac{22}{7} \times 14 \times 14 \times 20 \times \frac{3}{14}$	
	$\pi r_1^2 = 2{,}640$ sq.cm	
	$\therefore$ **Area of base of the cone is 2640 cm²**	½

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