MT

2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - II : PAPER - 5

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1.	**(A) Solve the following : (Any 4)**	
(1)	d(A, B) = Greater co–ordinates – smaller co–ordinates = 5 – (–2) = 5 + 2 $\therefore$ d(A, B) = 7	1
(2)	line l || line m $\angle$BMN + $\angle$MND = 180° (Interior angles Theorem) $\therefore$ x + 50 = 180° $\therefore$ x = 130°	1
(3)	$\angle$TPQ is an exterior angle of ΔPQR $\angle$TPQ = $\angle$PQR + $\angle$PRQ (Remote Interior angles theorem) $\therefore$ 100 = x + 35 $\therefore$ x = 65	1
(4)	sin A = cos (A – 30) $\therefore$ cos (90 – A) = cos (A – 30) $\therefore$ 90 – A = A – 30 $\therefore$ 90 + 30 = A + A $\therefore$ 2A = 120 $\therefore$ A = 60	1
(5)	Radius (r) = 21 cm $\therefore$ Diameter = 42 cm Perimeter of semicircle = πr + d = $\frac{22}{7} \times 21 + 42$ = 66 + 42 $\therefore$ Perimeter of semicircle = 108 cm	1
(6)	In ΔABC, AB > AC (Given) $\therefore$ $\angle$C > $\angle$B (In a triangle, Angle opposite to greater side is greater)	1

A.1. **(B) Solve the following : (Any 2)**

(1) In ΔADB and ΔBEA (Given) ½

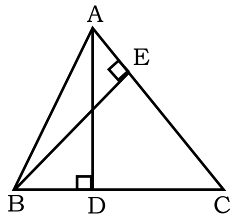
$\angle ADB = \angle BEA = 90^\circ$

Hypotenuse AB ≅ hypotenuse AB (Common side) ½

side BD ≅ side AE (Given)

$\therefore$ $\Delta ADB \cong \Delta BEA$ (Hypotenuse side theorem) ½

$\therefore$ seg AD ≅ seg BE (c.s.c.t) ½



(2) $\angle A = 30^\circ$

$$\text{LHS} = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

$$= 2 \times \frac{1}{\sqrt{3}} \div \left[1-\left(\frac{1}{\sqrt{3}}\right)^2\right]$$ 1

$$= \frac{2}{\sqrt{3}} \div \left[1-\frac{1}{3}\right]$$

$$= \frac{2}{\sqrt{3}} \div \frac{2}{3}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

$$= \frac{3}{\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{3}$$

$$= \sqrt{3}$$

$\therefore$ LHS = RHS 1

(3)	Radius (r) = 28 cm	
	½	
	Total surface area of a cone = $\pi r (r + l)$	½
	$\therefore \quad 7128 = \frac{22}{7} \times 28 (28 + l)$	
	$\therefore \quad \frac{7128}{22 \times 4} = 28 + l$	½
	$\therefore \quad 81 = 28 + l$	
	$\therefore \quad l = 81 - 28$	
	$\therefore \quad l = 53$	
	∴ **Slant height of a cone is 53 cm**	½
A.2.	**(A) Solve the following MCQs :**	
(1)	(B) $\Delta PQR \sim \Delta CAB$	1
(2)	(A) 1	1
(3)	(A) 14π	1
(4)	(D) Infinite	1
A.2.	**(B) Solve the following : (Any 2)**	
(1)	PQ = PM + MQ (P - M - Q)	½
	$\therefore$ 25 = 15 + MQ	
	$\therefore$ MQ = 25 – 15	
	$\therefore$ MQ = 10	
	PR = PN + NR (P - N - R)	½
	$\therefore$ 20 = PN + 8	
	$\therefore$ PN = 20 – 8	
	$\therefore$ PN = 12	
	Now, $\frac{PM}{MQ} = \frac{15}{10} = \frac{3}{2}$ ...(i)	½
	and $\frac{PN}{NR} = \frac{12}{8} = \frac{3}{2}$ ...(ii)	
	$\therefore$ In ΔPRQ, $\frac{PM}{MQ} = \frac{PN}{NR}$ [From (i) and (ii)]	
	∴ **seg NM ∥ side QR** (Converse of basic proportionality theorem)	½

(2) For the sector, $r = 3.5$ cm, length of arc (l) = 2.2 cm ½

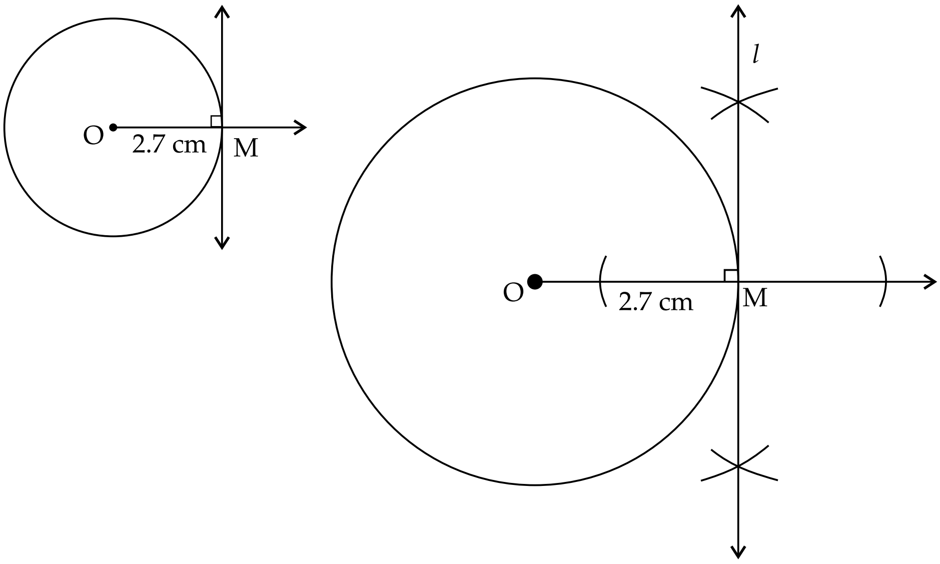
Area of the sector $= l \times \frac{r}{2}$

$= 2.2 \times \frac{3.5}{2}$ ½

$= 3.85 \text{ cm}^2$

$\therefore$ **Area of the sector is 3.85 cm²** 1

(3) Analytical figure:



1 mark for drawing circle
1 mark for drawing Tangent
line l is the required tangent to the circle passing through point M on the circle.

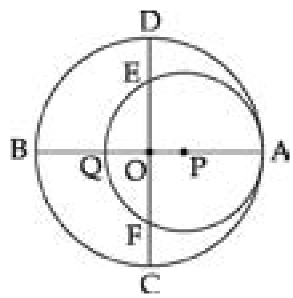
A.3. (A) Solve the following activity : (Any 2)

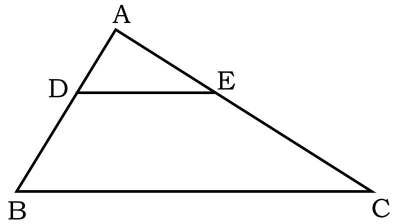
(1) Activity :

$2\text{ AX} = 3\text{ BX} \quad \therefore \frac{\text{AX}}{\text{BX}} = \frac{\boxed{3}}{\boxed{2}}$ ½

$\therefore \frac{\text{AX} + \text{BX}}{\text{BX}} = \frac{\boxed{3} + \boxed{2}}{\boxed{2}}$ ...(By componendo)

$\therefore \frac{\text{AB}}{\text{BX}} = \frac{\boxed{5}}{\boxed{2}}$ ...(i) ½

	$\therefore$ $\Delta BCA \sim \Delta BYX$ ...(By AA test for similarity)	
	$\therefore$ $\frac{BA}{BX} = \frac{AC}{XY}$ ...(c.s.s.t.)	½
	$\therefore$ $\frac{\boxed{5}}{\boxed{2}} = \frac{AC}{9}$ ...[From (i)]	
	$\therefore$ AC = 22.5 units	½
(2)	$x = a \cot\theta - b \operatorname{cosec}\theta$ ...(i)	½
	$y = a \cot\theta + b \operatorname{cosec}\theta$ ...(ii)	
	Adding (i) and (ii),	
	$x + y = 2a \cot\theta$	
	$\therefore \cot\theta = \frac{x+y}{2a}$ ...(iii)	½
	Subtracting (ii) from (i)	
	$y - x = 2b \operatorname{cosec}\theta$	
	$\therefore \operatorname{cosec}\theta = \frac{y-x}{2b}$ ...(iv)	½
	$\operatorname{cosec}^2\theta - \cot^2\theta = 1$	
	$\therefore \frac{(y-x)^2}{4b^2} - \frac{(y+x)^2}{4a^2} = 1$	
	or $\left(\frac{y-x}{b}\right)^2 - \left(\frac{y+x}{b}\right)^2 = 4$	½
(3)	Let the radius of the bigger circle be R and the radius of the smaller circle be r. OA, OB, OC and OD are radii of the bigger circle.	½
	$\therefore$ OA = OB = OC = OD = R PQ = PA = r OQ = OB – BQ = R – 9 OE = OD – DE = R – 5	
		
	As the chords QA and EF of the circle with centre P intersect in the interior of the circle, so by the property of internal division of	

	two chords of a circle,	½
	OQ × OA = OE × OB	
	$\boxed{R-9} \times R = \boxed{R-5} \times \boxed{R-5}$	
	$R^2 - 9R = R^2 - 10R + 25$	
	$R = \boxed{25 \text{ units}}$	½
	AQ = 2r = AB – BQ	
	2r = 50 – 9 = 41	
	$r = \frac{41}{2} = \boxed{r = 20.5}$	½
A.3.	**(B) Solve the following : (Any 2)**	
(1)	In ΔABC, DE | | BC	½
	$\therefore \frac{AD}{DB} \; \frac{AE}{EC}$ (Basic proportionality theorem)	
	$\therefore \frac{1.8}{5.4} = \frac{AE}{7.2}$	½
	$\therefore$ AE = $\frac{1.8 \times 7.2}{5.4}$ = 2.4	½
	AE = 2.4 cm	½
		
(2)	$\cot\theta = \frac{40}{9}$	½
	$\text{cosec}^2\theta = 1 + \cot^2\theta$	
	$= 1 + \left(\frac{40}{9}\right)^2$	½
	$= 1 + \frac{1600}{81}$	
	$= \frac{81 + 1600}{81}$	½
	$\therefore \text{cosec}^2\theta = \frac{1681}{81}$	
	$\therefore \boxed{\text{cosec}\,\theta = \frac{41}{9}}$ (Taking square roots)	

$\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$

$\therefore \quad \sin\theta = 1 \div \dfrac{41}{9}$

$\therefore \quad \sin\theta = 1 \times \dfrac{9}{41}$

$\therefore \quad \boxed{\sin\theta = \dfrac{9}{41}}$ ½

(3) Amount of oil in an oil can = Volume of an oil can ½
= $l \times b \times h$
= $20 \times 20 \times 30$ ½
= $12{,}000\ cm^3$ or cu.cm ½

The amount of oil an oil can can hold is $12{,}000\ cm^3$ or cu.cm. ½

A.4. Solve the following questions : (Any 3)

(1) Proof : In ΔABC and ΔDAC, ½

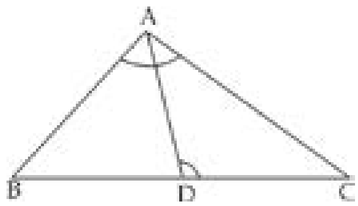
$\angle BAC \cong \angle ADC$ ...(Given) ½

$\angle BCA \cong \angle ACD$ (Common Angle)

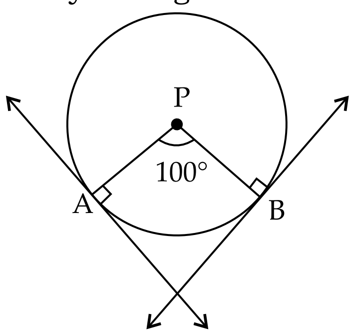
$\therefore \Delta ABC \sim \Delta DAC$ (By AA test of similarity)

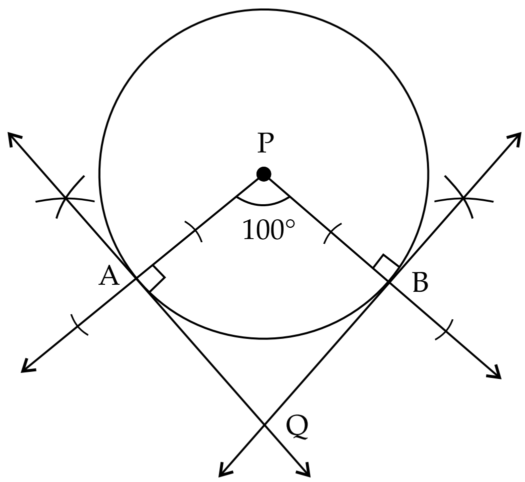
$\therefore \dfrac{CA}{CD} = \dfrac{CB}{CA}$ ...(c.s.s.t.) ½

$CA^2 = CB \times CD$ ½



(2) Analytical figure:





Line AQ and line BQ are tangents to the circle at points A and B respectively.

1 mark for drawing circle and $\angle$ APB

2 mark for drawing Tangent at A and B

(3) Proof: R.H.S. =

$$\frac{1}{\sec\theta - \tan\theta} = 1 \div (\sec\theta + \tan\theta)$$

$$= 1 \div \left[\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right]$$

$$= 1 \div \left[\frac{1-\sin\theta}{\cos\theta}\right]$$

$$= 1 \times \frac{\cos\theta}{1-\sin\theta}$$

$\therefore \quad \frac{1}{\sec\theta - \tan\theta} = \frac{\cos\theta}{1-\sin\theta} \quad \ldots\text{(i)}$ ½

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \quad \cos^2\theta = 1 - \sin^2\theta$

$\therefore \quad \cos\theta \times \cos\theta = (1 - \sin\theta)(1 + \sin\theta)$ ½

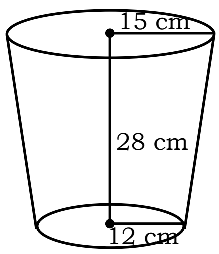
$\therefore \quad \frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

$\therefore \quad \frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta-\cos\theta}{\cos\theta-(1-\sin\theta)}$ (By theorem on equal ratios) ½

$\therefore \quad \frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta-\cos\theta}{\cos\theta-1+\sin\theta}$ ...(ii) [From (i)]

$\therefore \quad \mathbf{\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \frac{1}{\sec\theta-\tan\theta}}$ 1

(4) r_1 = 15 cm , r_2 = 12 cm, h = 28 cm



Capacity of the bucket = Volume of frustum ½

$= \frac{1}{3} \pi h \ (r_1^2 + r_2^2 + r_1 \times r_2)$ ½

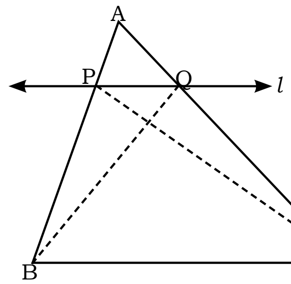
$= \frac{1}{3} \times \frac{22}{7} \times 28\ (15^2 \times 12^2 + 15 \times 12)$ ½

$= \frac{22 \times 4}{3} \times 549$ ½

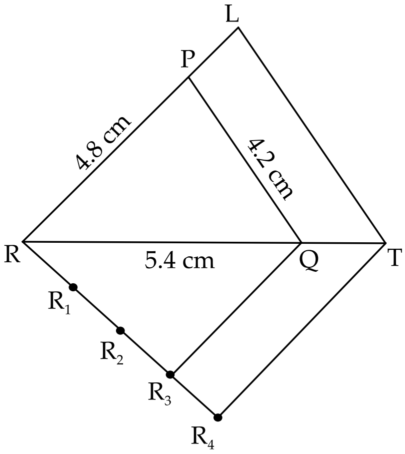
= 88 × 183 ½

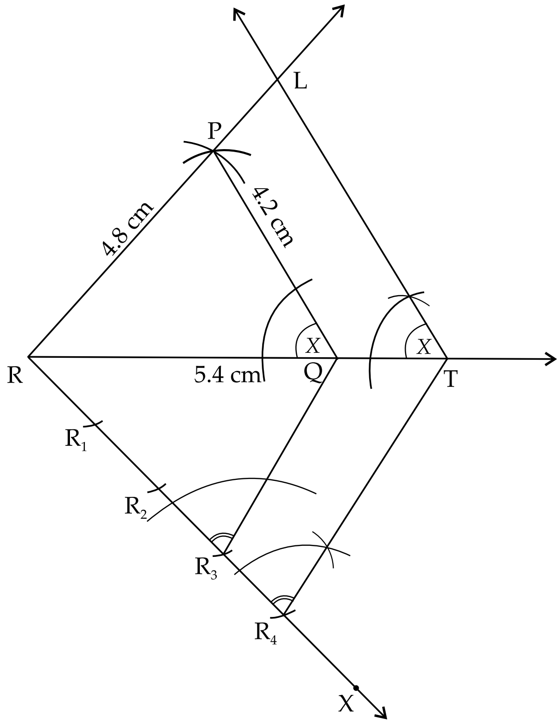
= 16104 cm^3 = 16.104 litre

$\therefore$ capacity of the bucket is 16.104 litre. ½

A.5.	**Solve the following questions : (Any 1)**	
(1)	In Δ ABC line l || line BC and line l intersects AB and AC in point P and Q respectively To Prove : $\frac{AP}{PB} = \frac{AQ}{QC}$  	½
	Proof : ΔADE and ΔBDE have a common vertex E and their bases AD and BD lie on the same line AB.	½
	∴ Their heights are equal .	
	∴ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{AD}{DB}$ ...(i) (Triangles having equal height)	½
	ΔADE and ΔCDE have a common vertex D and their bases AE and EC lie on the same line AC.	
	∴ Their heights are equal.	
	∴ $\frac{A(\Delta ADE)}{A(\Delta CDE)} = \frac{AE}{CE}$ ...(ii) (Triangles having equal height)	½
	line DE || side BC ...(Given)	
	ΔBDE and ΔCDE are between the same two parallel lines DE and BC.	½
	∴ Their heights are equal.	
	Also, they have same base DE.	
	∴ A(ΔBDE) = A(ΔCDE) ...(iii)	½
	(Areas of two triangles having equal base and equal height are equal)	
	∴ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{A(\Delta ADE)}{A(\Delta CDE)}$ ...(iv) [From (iii)]	½
	∴ $\mathbf{\frac{AD}{DB} = \frac{AE}{EC}}$ ...[From (i), (ii) and (iv)]	½

(2) Analytical figure:





ΔLTR is the required triangle similar to the ΔPQR.
1 mark for drawing triangle
1 mark for drawing $\angle RR_3Q = \angle RR_4T$
2 marks for drawing $\angle RQP = \angle RTL$.

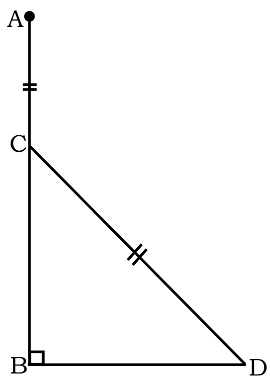
A.6. Solve the following questions : (Any 1)

(1) As shown in figure, suppose AB is the tree. It was broken at 'C' and its top touched at 'D'. (½)

$\angle CDB = 30°$, BD = 10 m, BC = x m

CA= CD = y m

In right angled ΔCDB, (½)

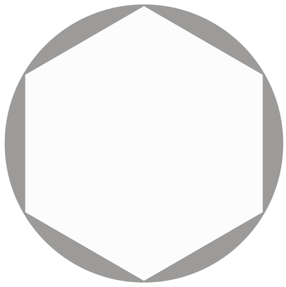


$$\tan 30° = \frac{BC}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{10}$$

$$x = \frac{10}{\sqrt{3}}$$ (½)

$$y = \frac{20}{\sqrt{3}}$$ (½)

	$x + y = \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}}$	
	$= \frac{30}{\sqrt{3}}$	½
	$x + y = 10\sqrt{3}$	
	$\therefore$ height of the tree was $10\sqrt{3}$ m	½
(2)	side of the hexagon = 14 cm	½
	A(hexagon) = $6 \times \frac{\sqrt{3}}{4} \times (\text{side})^2$ 	
	$= 6 \times \frac{\sqrt{3}}{4} \times 14^2$	½
	= 509.208 cm^2	
	A(circle) = πr^2	½
	$= \frac{22}{7} \times 14 \times 14$	
	= 616 cm^2	½
	The area of the region between the circle and the hexagon	½
	= A(circle) – A(hexagon)	
	= 616 – 509. 208	½