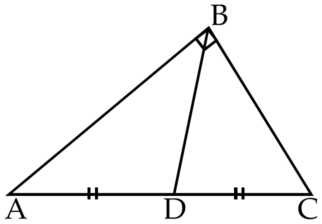
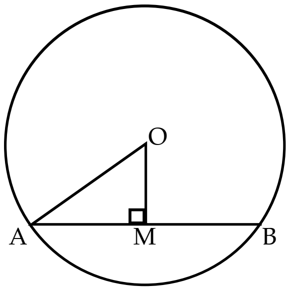
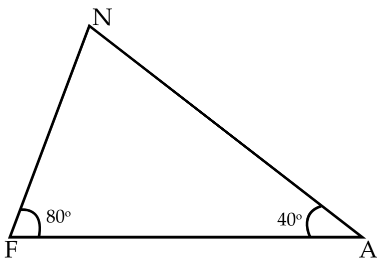
Q.P. SET CODE
A

MT - w

2018 ___ ___ 1100 - **MT - w** - MATHEMATICS (71) Geometry - SET - A (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1.(A)	**Solve ANY FOUR of the following :**	
(i)	If diagonals of a parallelogram are congruent, then it is a rectangle.	1
(ii)	If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is <u>180°</u> .	1
(iii)	In ΔABC, $\angle ABC = 90^\circ$ Seg BD is the median on hypotenuse AC.  $BD = \frac{1}{2} AC$ (Median drawn to the hypotenuse is half of it) $\therefore 7 = \frac{1}{2} AC$ $\therefore AC = 14$ cm	1
(iv)	A quadrilateral is a parallelogram if a pair of opposite sides is parallel and congruent.	1
(v)	Equation of the Y – axis is X = 0	1
(vi)	$\frac{\tan 40^\circ}{\cot 50^\circ}$ $= \frac{\tan 40^\circ}{\tan(90-50)^\circ}$ $= \frac{\tan 40^\circ}{\tan 40^\circ}$ $= 1$	1

A.1.(B)	**Solve ANY TWO of the following :**	
(i)	For a cone	
	Area of the base = 1386 sq. cm Height (h) = 28 cm	
	Volume of a cone = $\pi r^2 h$	1
	$= \frac{1}{3} \times 1386 \times 28$	½
	Volume of a cone = 12936 cm^3	½
(ii)	A circle with centre O	
	chord AB = 24 cm	
	seg OM $\perp$ chord AB	
	OM = 5 cm	
		
	AM $= \frac{1}{2}$ AB	½
	[Perpendicular drawn from the centre to the chord, bisects the chord]	
	$= \frac{1}{2} \times 24$	
	AM = 12 cm	½
	In ΔOMA, $\angle$OMA = 90°	
	$OA^2 = 5^2 + 12^2$ [Pythagoras theorem]	½
	$\therefore$ $OA^2 = 25 + 144$	
	$\therefore$ $OA^2 = 169$	
	$\therefore$ OA = 13 cm	
	$\therefore$ Radius of the circle is 13 cm	½
(iii)	In ΔFAN,	
	$\angle F = 80^\circ$	
	$\angle A = 40^\circ$	
		
	$\therefore$ $\angle N = 60^\circ$ [Remaining angle]	½
	$\therefore$ $\angle F > \angle N > \angle A$	½
	$\therefore$ AN > AF > FN [In a triangle, side opposite to greater angle is greater]	½
	$\therefore$ Greatest side is AN and the smallest side is FN	½

A.2.(A) Select the correct alternative answer and write it :

(i) (c) 15 — 1

(ii) (b) 0 — 1

(iii) (d) 99 cm — 1

(iv) (d) 5.5 units — 1

A.2.(B) Solve ANY TWO of the following :

(i)

$R(1, -1) = (x_1, y_1)$
$S(-2, k) = (x_2, y_2)$

Slope of line RS $= \dfrac{y_2 - y_1}{x_2 - x_1}$ — ½

$-2 = \dfrac{k-(-1)}{-2-1}$ — ½

$\therefore \quad -2 = \dfrac{k+1}{-3}$ — ½

$\therefore \quad (-2) \times (-3) = k + 1$
$\therefore \quad 6 = k + 1$
$\therefore \quad k = 6 - 1$
$\therefore \quad k = 5$ — ½

(ii)

Area of a triangle $= \dfrac{1}{2}$ base height — ½

A (PQR) $= \dfrac{1}{2} \times RQ \times PS$

$= \dfrac{1}{2} \times 6 \times 6$

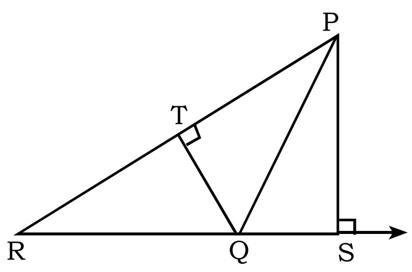
A (PQR) = 18 sq. units — ½

Also, A (PQR) $= \dfrac{1}{2} \times PR \times QT$

$\therefore \quad 18 = \dfrac{1}{2} \times 12 \times QT$ — ½

$QT = \dfrac{18 \times 2}{12}$

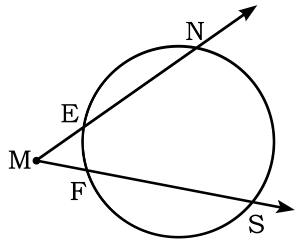
$\therefore$ **QT = 3 units** — ½



(iii) $m\angle NMS = \frac{1}{2}[m(\text{arc NS}) - m(\text{arc EF})]$ ½

$= \frac{1}{2}(125 - 37)$ ½

$= \frac{1}{2} \times 88$ ½



$\therefore$ $m\angle NMS = 44$ ½

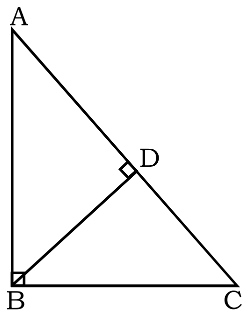
A.3.(A) Carry out ANY TWO of the following activites :

(i) Given : In Δ ABC, $\angle B = 90^\circ$ 2

seg BD $\perp$ hypotenuse AC

To prove : $\Delta ADB \sim \Delta BDC$

Proof :



In ΔADB and ΔABC

$\angle A \cong \angle A$ common angle

$\angle ADB \cong \angle ABC$ (each 90°)

$\therefore$ $\Delta ADB \sim \Delta ABC$ ...(i) AA test of similarity

In Δ BDC and Δ ABC

$\angle C \cong \angle C$ common angle

$\angle BDC \cong \angle ABC$ (each 90°)

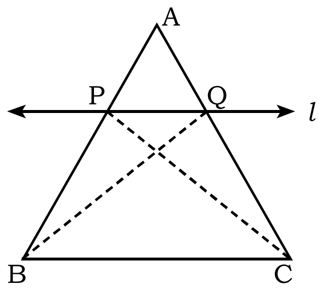
$\therefore$ $\Delta BDC \sim \Delta ABC$ ...(ii) AA test of similarity

$\therefore$ $\Delta ADB \sim \Delta BDC$ from (i) and (ii)

(ii) Given : In Δ ABC, 2

Line l || side BC

To prove : $\frac{AP}{PB} = \frac{AQ}{QC}$



Construction : Draw seg PC and seg BQ

Proof :

$\frac{A(\Delta APQ)}{A(\Delta PQB)} = \frac{AP}{PB}$ ....(i) Triangles with equal heights

$$\frac{A(\Delta APQ)}{A(\Delta PQC)} = \frac{\boxed{AQ}}{\boxed{QC}} \quad ....(ii) \quad \boxed{\text{Triangles with equal heights}}$$

seg PQ | | side BC (Given)

$\therefore$ Δ PQB and Δ PQC have equal heights.

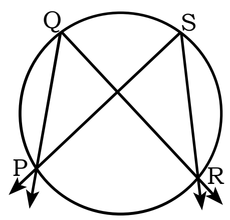
Also they have common base PQ

$\therefore$ $A\left(\boxed{\Delta PQB}\right) = A\left(\boxed{\Delta PQC}\right)$ ....(iii)

$\therefore$ $\frac{AP}{PB} = \frac{AQ}{QC}$ $\boxed{\text{from (i), (ii) \& (iii)}}$

(iii) Given :

$\angle$ PQR and $\angle$ PSR are inscribed in same arc PQR and intercepts same arc PR



To prove : $\angle PQR \cong \angle PSR$

Proof :

$\angle PQR = \frac{1}{2}\ \boxed{m(\text{arc } PR)}$ ....(i)

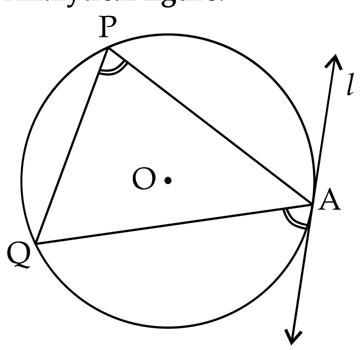
$\angle PSR = \frac{1}{2}\ \boxed{m(\text{arc } PR)}$ ....(ii)

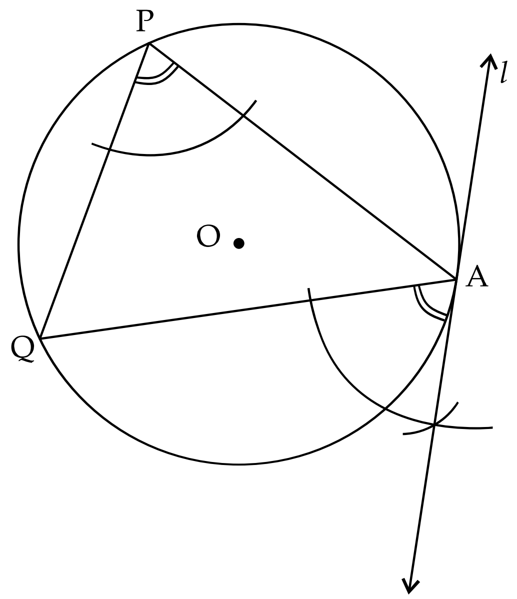
} $\boxed{\text{Inscribed Angle theorem}}$

$\therefore$ $\angle PQR \cong \angle PSR$ $\boxed{\text{from (i) \& (ii)}}$

A.3.(B) Solve ANY TWO of the following :

(i) **Analytical figure:**





1 mark for drawing circle and Δ APQ
1 mark for drawing tangent at A.
Line l is the required tangent to the circle at point A.

(ii) Let A(a, 0) be a point equidistant from P(2, –5) and Q(–2, 9).

∴ d(P, A) = d(Q, A)

Using distance formula,

$\sqrt{(a-2)^2 + [0-(-5)]^2} = \sqrt{[a-(-2)]^2 + (0-9)^2}$ 1

Squaring both the sides we get,

$(a-2)^2 + 5^2 = (a+2)^2 + (-9)^2$

∴ $a^2 - 4a + 4 + 25 = a^2 + 4a + 4 + 81$

∴ $a^2 - 4a - a^2 - 4a = 81 - 25$ ½

∴ $-8a = 56$

∴ $a = \frac{56}{-8}$

∴ $a = -7$

∴ **(–7, 0) is a point on X-axis equidistant from P(2, –5) and Q(–2, 9).** ½

(iii) L.H.S. $= \sec^2\theta + \csc^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$ $\left(\because \sec\theta = \dfrac{1}{\cos\theta}, \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}\right)$ ½

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \,.\, \sin^2\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$ ½

$= \sec^2\theta \,.\, \operatorname{cosec}^2\theta$ 1

= R.H.S.

$\therefore$ $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \,.\, \operatorname{cosec}^2\theta$

A.4. Solve ANY THREE of the following :

(i) AB and CD represents the height of two buildings at distance of 15 m, i.e. BC = 15 m, AB = 12 m ĎDAE is the angle of elevation.

$\therefore$ ĎDAE = 30°

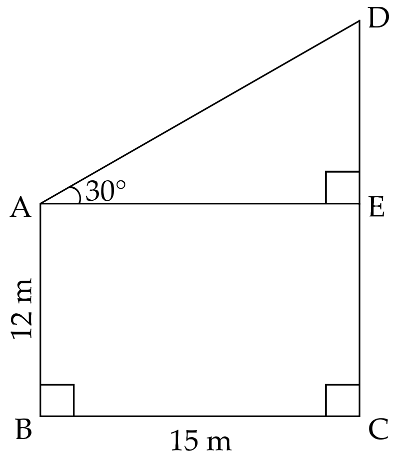
□ABCE is a rectangle. ...(By definition) ½

$\therefore$ AB = EC = 12 m

BC = AE = 15 m ...(Opposite sides of rectangle) ½

In ΔAED, ĎAED = 90

$\therefore$ tan ĎDAE $= \dfrac{DE}{AE}$ ...(By definition)



$\therefore$ tan 30 $= \dfrac{DE}{15}$

$\therefore$ $\dfrac{1}{\sqrt{3}} = \dfrac{DE}{15}$

$\therefore$ DE $= \dfrac{15}{\sqrt{3}}$ ½

$= \dfrac{15\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= \dfrac{15\sqrt{3}}{3}$

$= 5\sqrt{3}$

$= 5 \times 1.73$ ½

$\therefore$ DE = 8.65 m

CD = CE + DE ...(C – E – D)

= 12 + 8.65

CD = 20.65 m

y Height of the second building is 20.65 m.

(ii)

For the conical part

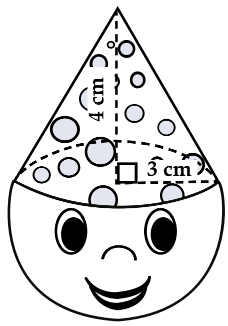
Radius (r) = 3 cm

Height (h) = 4 cm

Let l be the slant height of conical part

For the hemisphere

Radius (r) = 3 cm



$l^2 = r^2 + h^2$ ½

$\therefore \quad = 3^2 + 4^2$

$= 9 + 16$

$\therefore \quad l^2 = 25$

$\therefore \quad l = 5$ cm (Taking square roots) ½

Surface area of toy = Curved surface area of the cone + Curved surface area of the hemisphere ½

$= \pi rl + 2\pi r^2$ ½

$= \pi r\,(l + 2r)$

$= 3.14 \times 3\,(5 + 2 \times 3)$

$= 3.14 \times 3\,(5 + 6)$

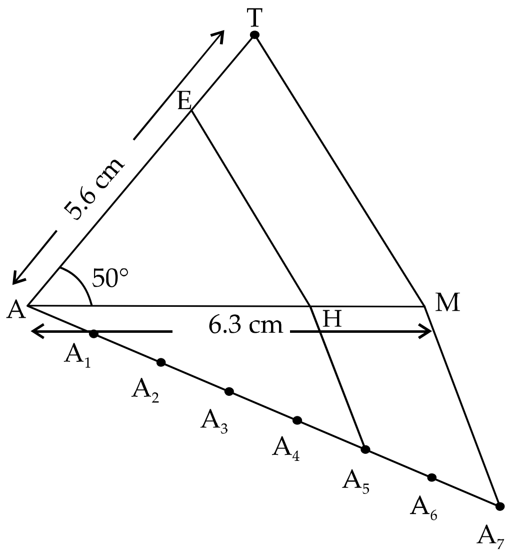
$= 3.14 \times 3 \times 11$ ½

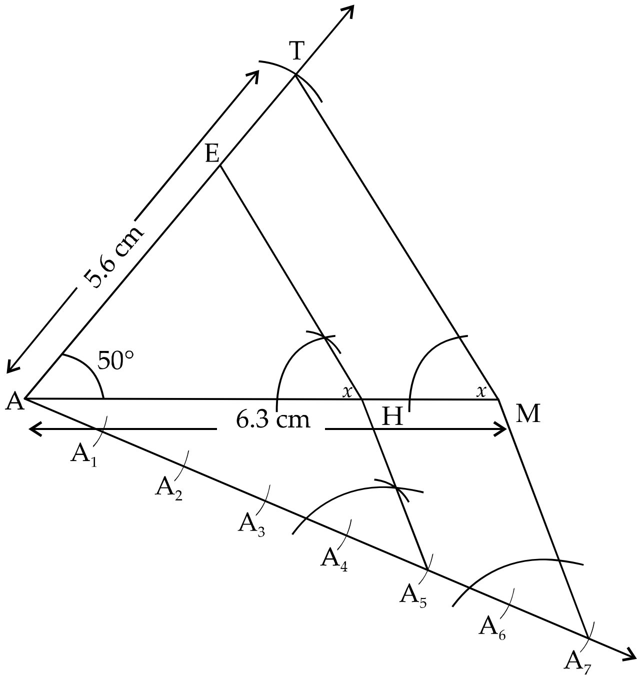
$= 3.14 \times 33$

$= 103.62\ \text{cm}^2$

$\therefore$ **Surface area of toy is 103.62 cm²** ½

(iii) **Analytical figure:**





1 mark for drawing ΔAMT
1 mark for constructing $\angle AA_7M \cong \angle AA_5H$
1 mark for constructing $\angle AMT \cong \angle AHE$

(iv) In ΔCPA and ΔCQB

$\angle C \cong \angle C$ ...(Common angle)

$\angle APC \cong \angle BQC$ ...(Each is 90^o)

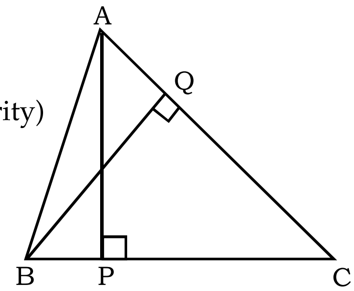
$\therefore$ ΔCPA ~ ΔCQB ...(By AA test for similarity) 1

$\therefore \dfrac{AP}{BQ} = \dfrac{AC}{BC}$ ...(c.s.s.t.) ½

$\therefore \dfrac{7}{8} = \dfrac{AC}{12}$ ½

$\therefore \dfrac{7 \times 12}{8} = AC$ ½

AC = 10.5 units ½

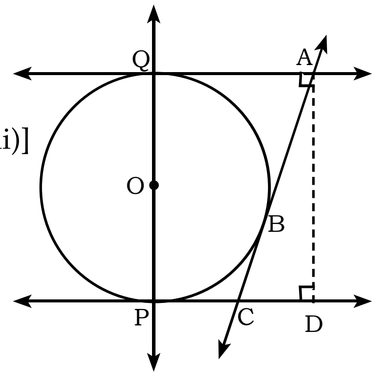


A.5. Solve ANY ONE of the following :

(i)

$AQ = AB = 7.2$ cm ...(i) [Tangent segment theorem]

$CP = CB = 5$ cm ...(ii)



$AC = AB + BC$ [A–B–C]

$\therefore$ $AC = 7.2 + 5$ [From (i) and (ii)]

$\therefore$ $AC = 12.5$ cm ...(iii) ½

In □ AQPD,

$m\angle AQP = 90^\circ$ }
$m\angle QPD = 90^\circ$ } [Tangent theorem]

$m\angle ADP = 90^\circ$ [Construction]

$m\angle QAD = 90^\circ$ [Remaining angle]

$\therefore$ □ AQPD is a rectangle [By definition] ½

$QA = PD = 7.2$ cm ...(iv) }
$PQ = AD$ ...(v) } [opposite sides of a rectangle are congruent]

$PD = PC + CD$ [P–C–D] ½

$\therefore$ $7.2 = 5 + CD$

$\therefore$ $CD = 7.2 - 5$

$\therefore$ $CD = 2.2$ cm ...(iv) ½

In ΔACD

$m\angle ADP = 90^\circ$ [Construction]

$\therefore$ $AC^2 = AD^2 + CD^2$ ½

$\therefore$ $(12.2)^2 = AD^2 + (2.2)^2$

$\therefore$ $AD^2 = (12.2)^2 - (2.2)^2$

$\therefore$ $AD^2 = (12.2 + 2.2)(12.2 - 2.2)$ ½

$\therefore$ $AD^2 = 14.4 \quad 10$

$\therefore$ $AD^2 = 144$

$\therefore$ $AD = 12$ cm [Taking square roots] ½

$\therefore$ $PQ = 12$ cm [From (v)]

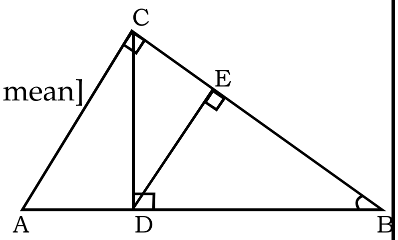
Radius of the circle $= \frac{PQ}{2} = \frac{12}{2} = 6$

$\therefore$ **Radius of the circle is 6 cm** ½

(ii)

In ΔABC, m $\angle ACB = 90^{\circ}$ [Given]

seg $CD \perp$ hypotenuse AB [Given]



$CD^2 = AD \times DB$ [Property of geometric mean] ½

$\therefore$ $CD^2 \times AC = AD \times DB \times AC$ ...(i) ½

In ΔACB and ΔDEB

$\angle ACB \cong \angle DEB$ [Each 90°]

$\angle ABC \cong \angle DBE$ [Common angle]

$\therefore$ $\Delta ACB \sim \Delta DEB$ ...(ii) [AA test of similarity] 1

$\therefore$ $\dfrac{AC}{DE} = \dfrac{AB}{DB}$ [c.s.s.t.] ½

$DB \times AC = AB \times DE$ ...(iii) ½

Substituting equation (iii) in (i)

$CD^2 \times AC = AD \times DB \times AC$ ½

$\therefore$ $CD^2 \times AC = AD \times AB \times DE$ ½

A.6. Solve ANY ONE of the following :

(i)

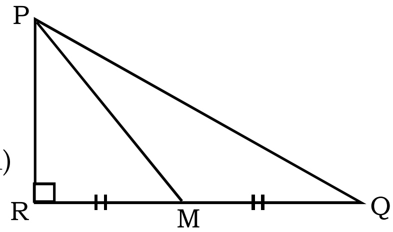
Proof :

In ΔPRQ, $\angle PRQ = 90^{\circ}$ ...(Given)

$\therefore$ $PQ^2 = PR^2 + QR^2$ ...(i) (By Pythagoras theorem) ½

$\therefore$ $QR = 2RM$ ...(ii) (M is the midpoint of seg QR)

$\therefore$ $PQ^2 = PR^2 + (2\ RM)^2$ ...[From (i) and (ii)]



$\therefore$ $PQ^2 = PR^2 + 4\ RM^2$ ...(iii) ½

In ΔPRM, $\angle PRM = 90^{\circ}$

$\therefore$ $PM^2 = PR^2 + RM^2$ ...(Pythagoras theorem)

$\therefore$ $RM^2 = PM^2 - PR^2$ ...(iv) ½

$\therefore$ $PQ^2 = PR^2 + 4\ (PM^2 - PR^2)$ ...[From (iii) and (iv)]

$\therefore$ $PQ^2 = PR^2 + 4\ PM^2 - 4\ PR^2$

$\therefore$ $\boxed{PQ^2 = 4\ PM^2 - 3\ PR^2}$ ½

(ii)

For a frustum

radius of bigger circle (r_1) = 14 cm

radius of smaller circle (r_2) = 6 cm

height (h) = 6 cm

Slant height of the frustum (l) = $\sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{6^2 + (14-6)^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$ cm ...(Taking square roots)

(i) Curved surface area of the frustum $= \pi (r_1 + r_2) l$

$= 3.14 \times (14 + 6) \times 10$

$= 3.14 \times 20 \times 10$

$= 628 \text{ cm}^2$

Curved surface area of the frustum is 628 cm² 1

(ii) Total surface area of the frustum

$= \pi (r_1 + r_2) l + \pi r_1^2 + \pi r_2^2$

$= 628 + \pi (14^2 + 6^2)$

$= 628 + \pi (196 + 36)$

$= 628 + 3.14 \times 232$

$= 628 + 728.48$

$= 1{,}356.48 \text{ cm}^2$

Total surface area of the frustum is 1356.48 cm² 1

(iii) Volume of the frustum $= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 \times r_2)$

$= \frac{1}{3} \times 3.14 \times 6 (14^2 + 6^2 + 14 \times 6)$

$= 3.14 \times 2 (196 + 36 + 84)$

$= 3.14 \times 2 \times 316$

$= 1{,}984.48 \text{ cm}^3$

$\therefore$ **Volume of the frustum is 1,984.48 cm³** 1

❖❖❖❖❖